

## Computation of convolution integral

Lets go straight to the recommended approach.

$$y(t) = \int x(\tau)h(t-\tau)d\tau$$

- Calculate output at time  $t_0$  due to inputs at all times  $\tau$
- Re do this for the next value of  $t$ .

Here are the basic steps for one value of  $t$ , say  $t_0$ .

a) Form  $w_{t_0}(\tau) = x(\tau)h(t_0-\tau)$   
for all  $\tau$ .

b)  $y(t_0) = \int w_{t_0}(\tau)d\tau$

Then repeat for more values of  $t$ .

- Again, the advantage of this method is that for many signals and systems, the functional form of  $w_t(\tau)$  remains the same for values of  $t$  in an interval.

Therefore we expect to write an answer of the form.

$$y(t) = \begin{cases} f_0(t) & t \leq t_0 \\ f_1(t) & t_0 < t \leq t_1 \\ \vdots & \vdots \\ f_{m-1}(t) & t_{m-1} < t \leq t_m \\ f_m(t) & t > t_m \end{cases}$$

where each  $f_i(t)$  is of the form

$$f(t) = \int w_t(\tau) d\tau$$

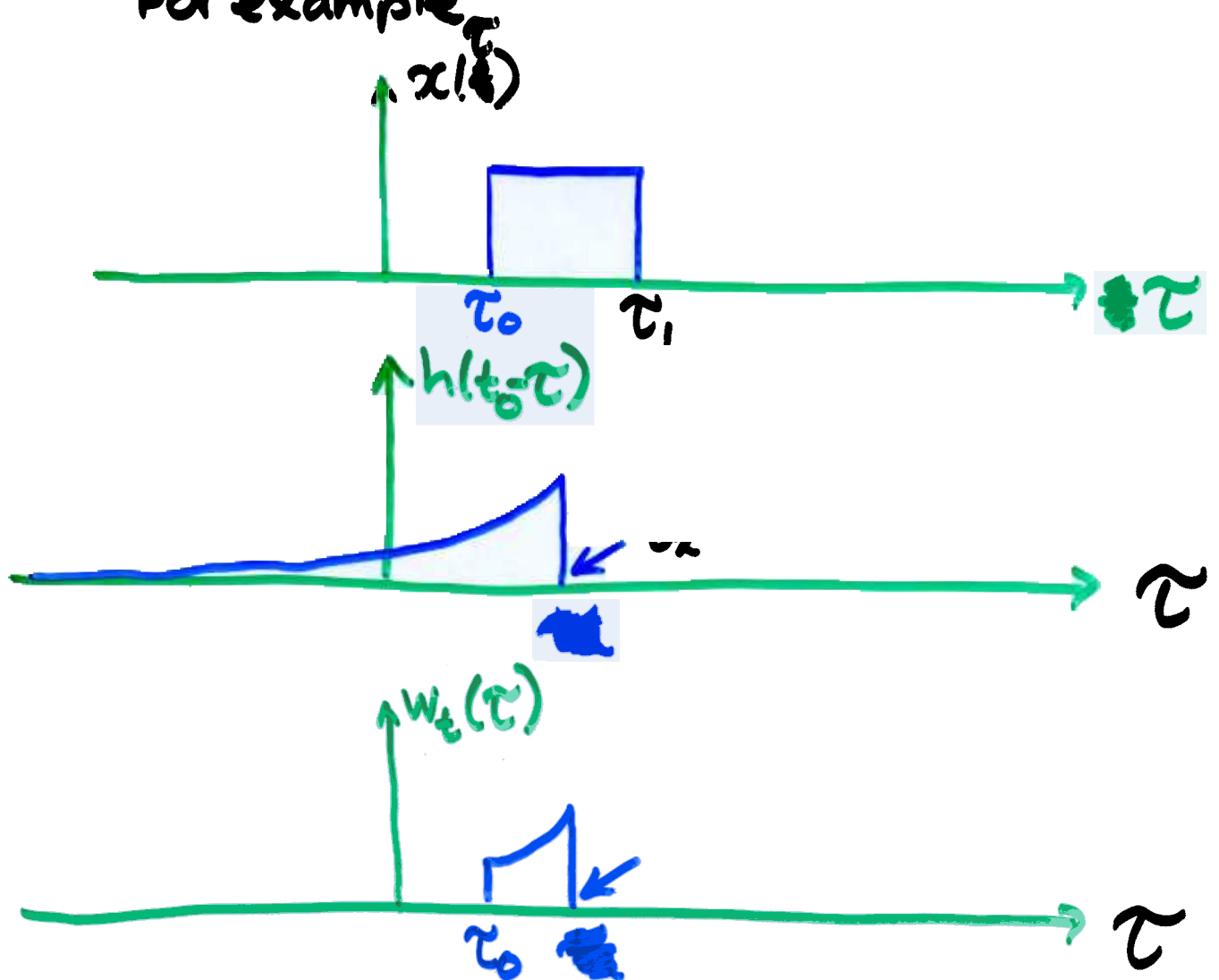
and over each range the functional form of  $w_t(\tau)$  is the same

Again  $w_t(\tau)$  can be computed in a convenient graphical manner

$$w_t(\tau) = x(\tau)h(t-\tau)$$

Just plot them + multiply

For example



Now  $y(t_0) = \int w_t(\tau) d\tau$

One tricky thing here is the formation of  $h(t_0 - \tau)$ .

Lets define  $\tilde{u}(\tau) = h(-\tau)$  (reflection)  
what is  $\tilde{u}(\tau - t_0)$  (shift to right by  $t_0$ )

$$\tilde{u}(\tau - t_0) = h(-(\tau - t_0)) = h(t_0 - \tau)$$

