How to do Convolution — A Recipe

$$y[n] = \sum_k x[k]h[n-k]$$

Like cooking, you must do all the steps to get it right!

1. Graph $x[k]$, $h[k]$ and $\tilde{u}[k] = h[-k]$

2. Start with n large and negative

3. Graph $\tilde{u}[k-n] = h[n-k]$, and use your graph of $x[k]$ and the functional descriptions of x and h to write down a formula for $w_n[k]$

4. increase n until your formula for $w_n[k]$ is no longer valid. Record the value of n at this change as one of the N_j's in previous formula for $y[n]$.

5. Repeat steps 3 and 4 until all functional forms of $w_n[k]$ and all N_j's have been identified. Usually this requires n to be large and positive

6. In each interval find $f_j[n] = \sum_k w_n[k]$.

EXAMPLE 2 3

An LT system has mpulse response

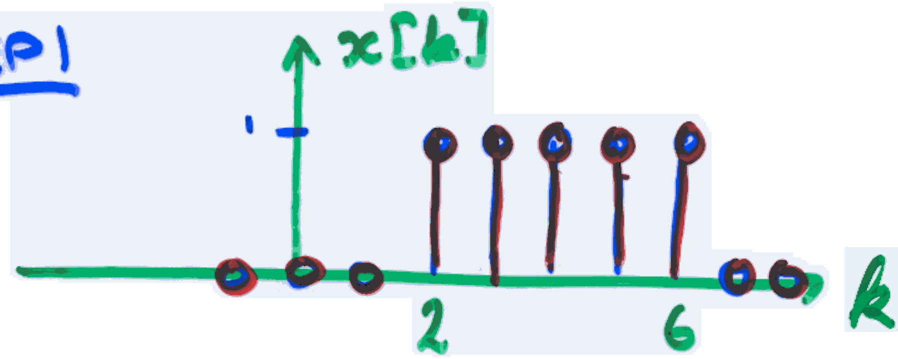
$$h[n] = u[n] \quad u[n-0]$$

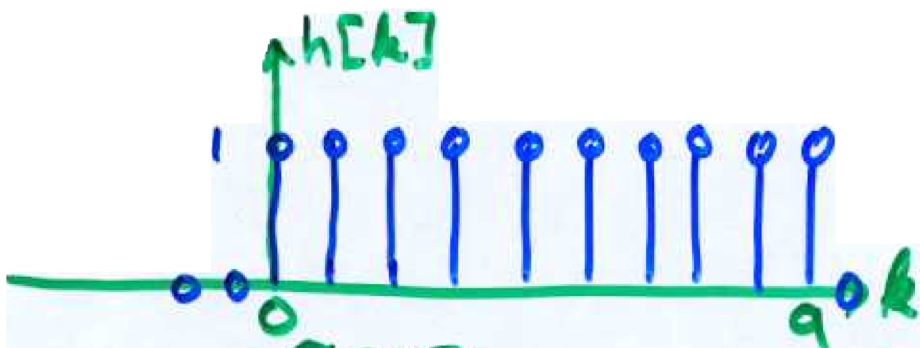
and nput

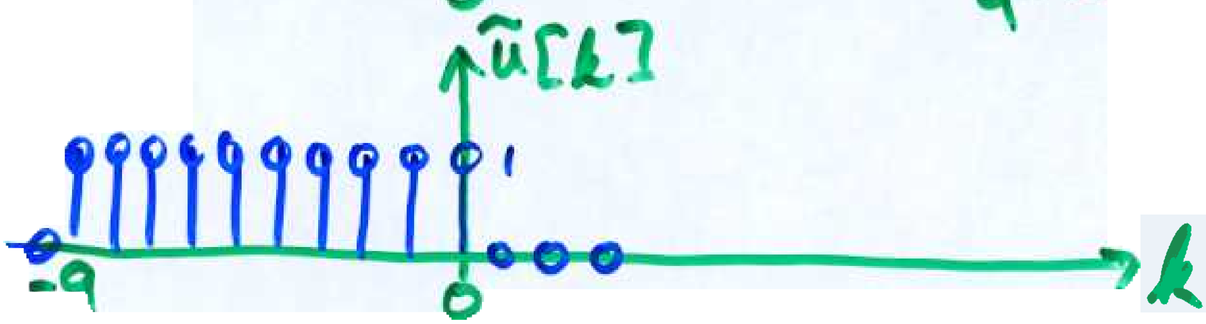
$$x[n] = u[n-2] \quad u[n-7]$$

Find the output $y[n]$

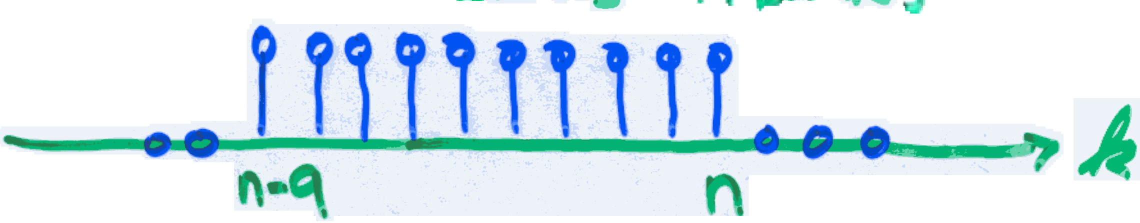
STEP 1





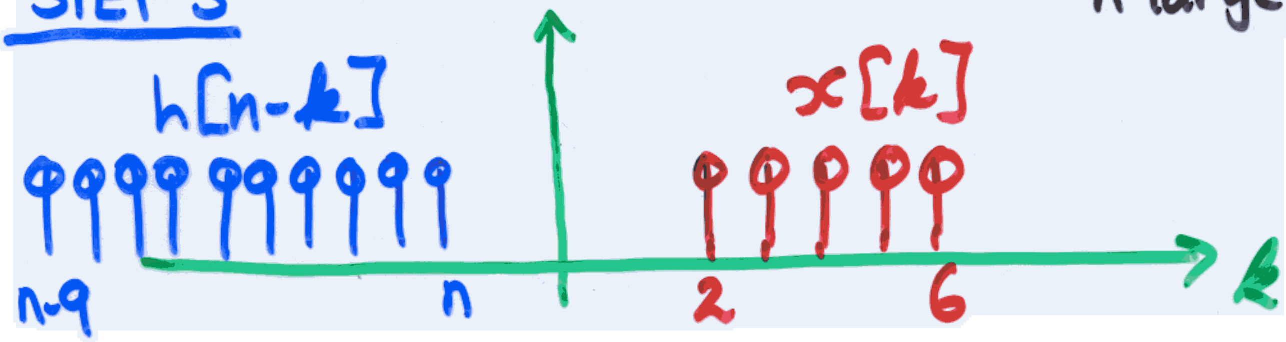


$$\tilde{u}[k-n] = h[n \; k]$$



STEP 3

n large



$$w_n[k] \qquad x[k]\,h[n-k]$$

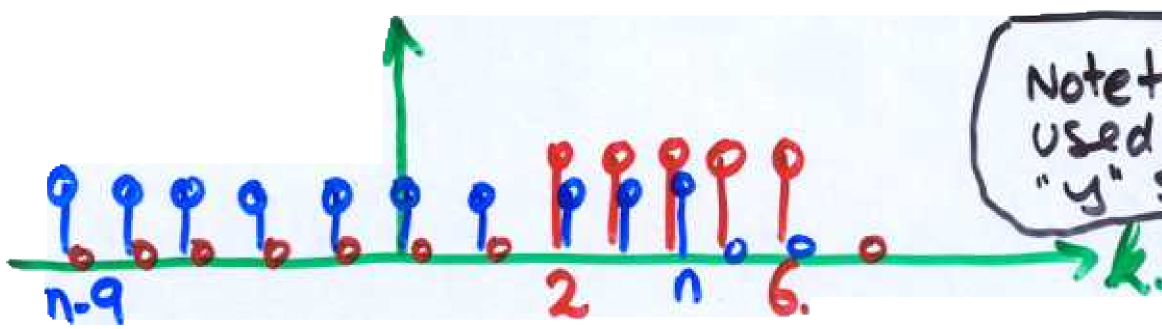
Hence for n large and negative,

$$w_n[k] = 0$$

STEP 4

The above representation (functional form of $w_n[k]$ is true, [redacted] for $n < 2$.

REPEAT STEP 3 for $n \geq 2$

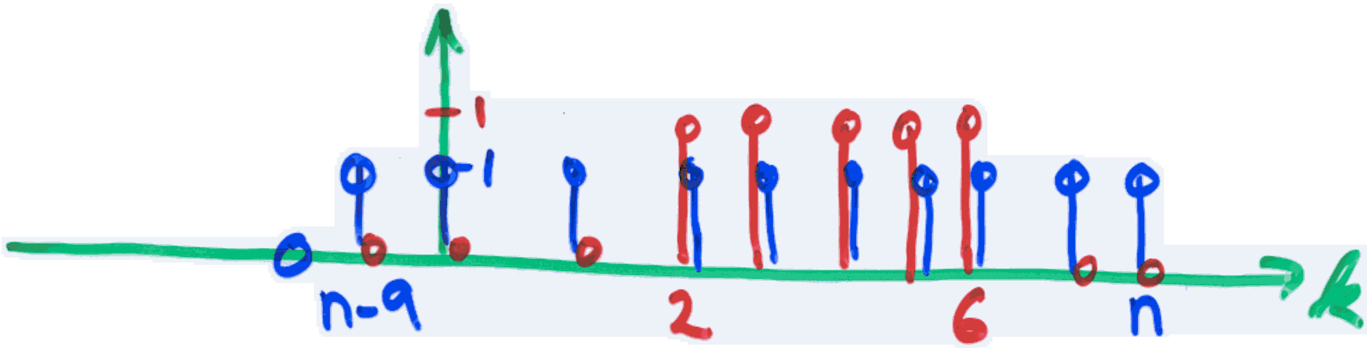


Note that I've used different "y" scales for clari

Hence $w_n[k]$ $\begin{cases} & 2 \leq k \leq n \\ 0 & \text{otherwise} \end{cases}$

[redacted]

Step 3 for $n \geq 7$



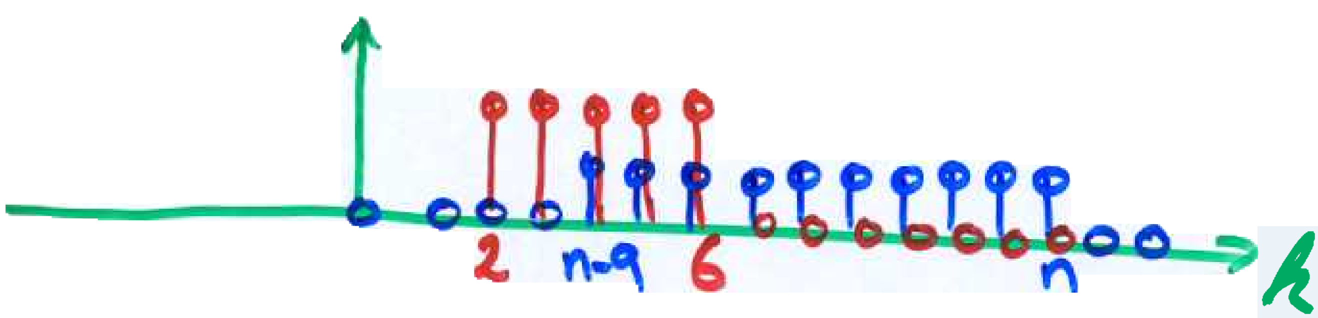
$$w_n[k] = \begin{cases} & 2 \leq k \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

This is valid until $n-9 > 2$

ie it's valid for $7 \leq n < 12$

N.B. Note that there is a little in the boundary, depending on whether you use $<$ or $\leq$

Step 3 for $n \geq 12$

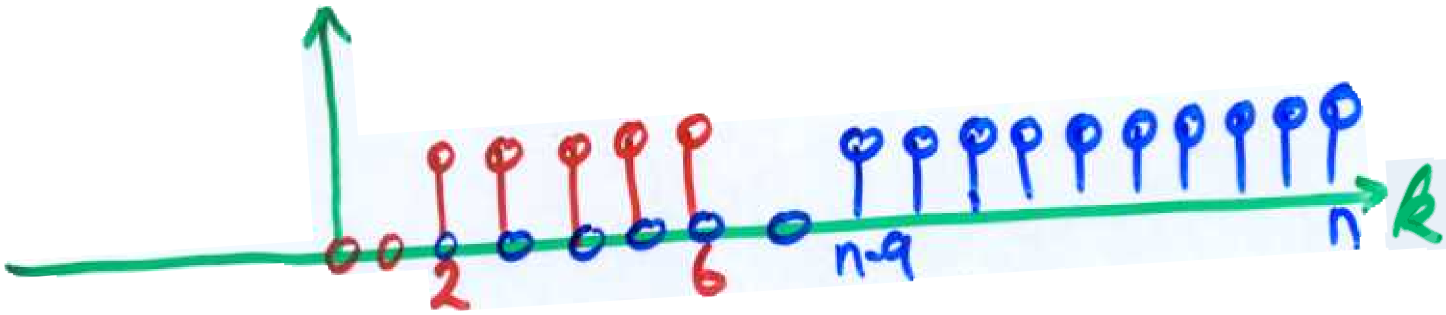


$$w_n[k] = \begin{cases} & n\ 9 \leq k \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

valid until $n-9 > 6$

ie valid for

STEP 3 FOR $n \geqslant 16$



$w_n[k] = 0$

This is valid for all $n \geqslant 6$

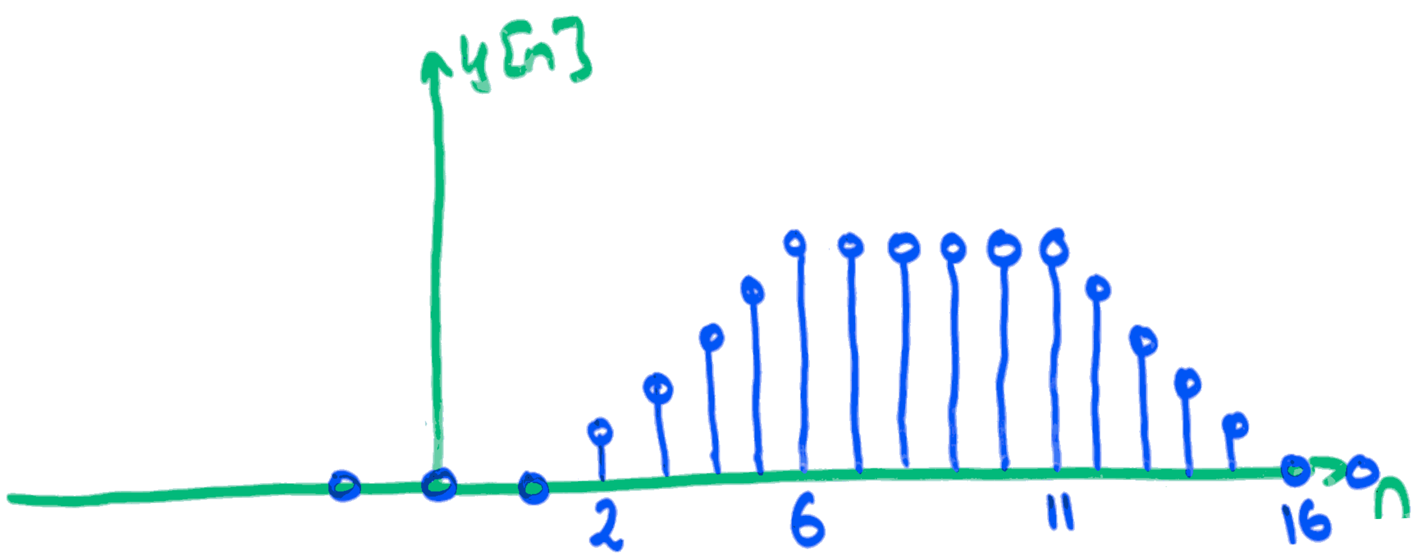
STEP

Summarise $w_n[k]$

$$w_n[k] = \begin{cases} \{\, 0 \quad \text{for all } k, & n < 2 \\ \left\{ \begin{array}{ll} & 2 \leq k \leq n \\ 0 & \text{otherwise} \end{array} \right., & 2 \leq n < 7 \\ \left\{ \begin{array}{ll} & 2 \leq k \leq 6 \\ 0 & \text{otherwise} \end{array} \right. & 7 \leq n < \\ \left\{ \begin{array}{ll} & n-9 \leq k \leq 6 \\ 0 & \text{otherwise} \end{array} \right., & 12 \leq n < \\ \{\, 0 \quad \text{for all } k, & n \geqslant \end{cases}$$

Now $y[n] = \sum_{k} w_n[k]$

$$\Rightarrow \quad y[n] = \begin{cases} 0 & , n < 2 \\ \sum_{k=2}^{n} 1 = n & , 2 \leq n < 7 \\ \sum_{k=2}^{6} = 5 & , 7 \leq n < 2 \\ \sum_{k=n-9}^{6} 1 = 6\ n & 2 \leq n < \\ 0 & , n \geq 6 \end{cases}$$



WARNING

- Its tempting to do a few examples and then come up with your own "tricks" which allow you to reduce the amount of work you have to do, and still give you the right answer
- It is very unlikely that these will work in all cases

ADVICE

- Once you have formed $y[n]$, sketch t
- Think about whether your answer makes sense Are there any strange features in your answer when the input and impulse response look sensible?