Stabilization of an unstable system.

- In certain cases, an unstable system can be made stable by feedback.
- This is used in advanced fighter aircraft design
- Sketch the root locus of a unity feedback system with

$$G(s) = \frac{0.5}{s-4} \quad ; \quad \tilde{H}(s) = \frac{s+2}{s(s+12)}$$

$$\Rightarrow \quad G(s)H(s) = \frac{0.5K(s+2)}{s(s-4)(s+12)}$$

STEP 1: poles of $G(s)\tilde{H}(s)$ are at $s = -12, 0, 4$
$\Rightarrow G(s)$ is unstable !

STEP 2: zeros of $G(s)\tilde{H}(s)$ at $s = -2$ and two zeros at ∞

STEP 3: Asymptotes: $N - M = 2 \Rightarrow$ two paths to ∞

Angles: $90^\circ, 270^\circ$

Centroid/Intersection: $\dfrac{(-12+0+4)-(-2)}{3-1} = -3$

STEP 4: Break away point

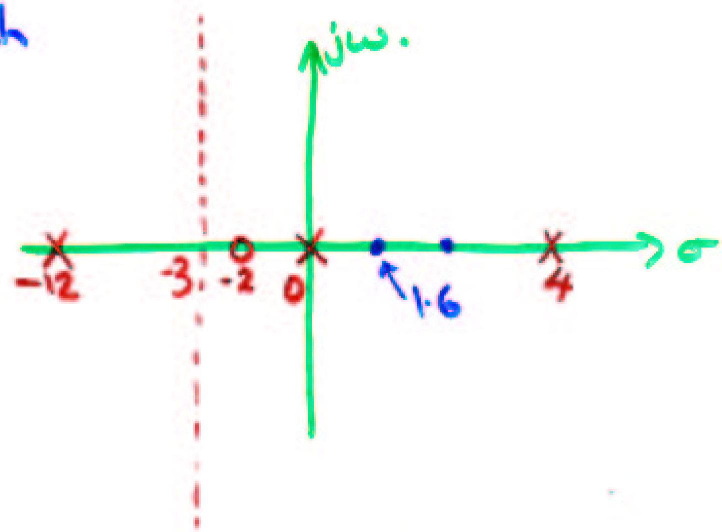
$$\frac{d}{ds}\left(\frac{s(s+12)(s-4)}{0.5K(s+2)}\right)=0$$

$$\Rightarrow \quad s^3+7s^2+16s-48=0$$

$$\Rightarrow \quad s \approx 1.61$$

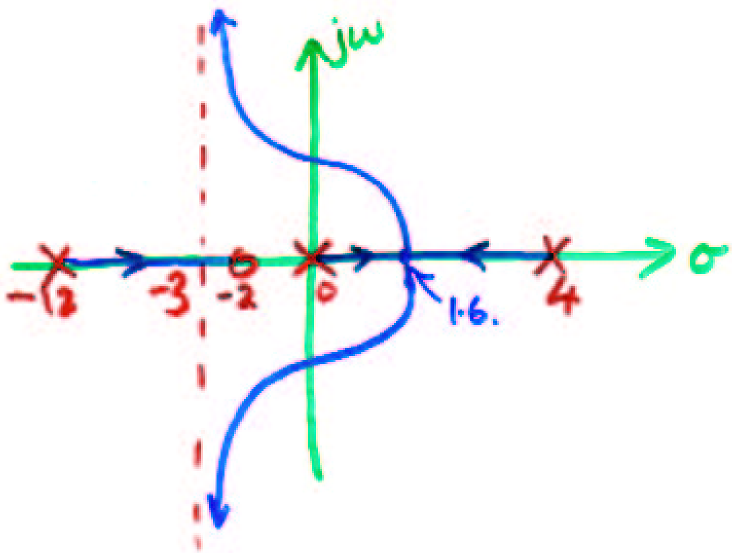
corresponding gain ≈ 5.56

Sketch



Hence pole at -12 moves to zero at -2.

Poles at 0 and 4 move towards eachother, collide and break away at $s = 1.6$ and then approach ∞ along the asymptotes



⇒ For high enough gain, system becomes stable.
What is the minimum gain required?

- Solve for K and ω_c such that

$$A(s) = (s^2 + \omega_c^2)\,\tilde{A}(s)$$

$$G(s)\tilde{H}(s) = \frac{P(s)}{Q(s)} = \frac{0.5\,(s+2)}{s(s-4)(s+12)}$$

$$A(s) = KP(s) + Q(s)$$

$$= s^3 + 8s^2 + (0.5K - 48)s + K.$$

Polynomial division

$$
\begin{array}{r|l}
 & \qquad\qquad\qquad\quad s \quad + 8 \\
s^2+\omega_c^2 & s^3 + 8s^2 + (0.5K-48)s + K \\
 & -(s^3 + 0 + \omega_c^2 s + 0) \\
\hline
 & 0 + 8s^2 + (0.5K-48-\omega_c^2)s + K \\
 & -(0 + 8s^2 + 0 + 8\omega_c^2) \\
\hline
 & 0 + 0 + (0.5K-48-\omega_c^2)s + K - 8\omega_c^2
\end{array}
$$

$$\Rightarrow A(s) = (s^2+\omega_c^2)(s+8) + \underbrace{(0.5K-48-\omega_c^2)s + K-8\omega_c^2}_{\text{Residual}}$$

For residual = 0, we require

$$0.5K-48-\omega_c^2 = 0 \quad \text{(A)}$$

$$K-8\omega_c^2 = 0 \quad \text{(B)}$$

$$\Rightarrow \quad K = 8\omega_c^2.$$

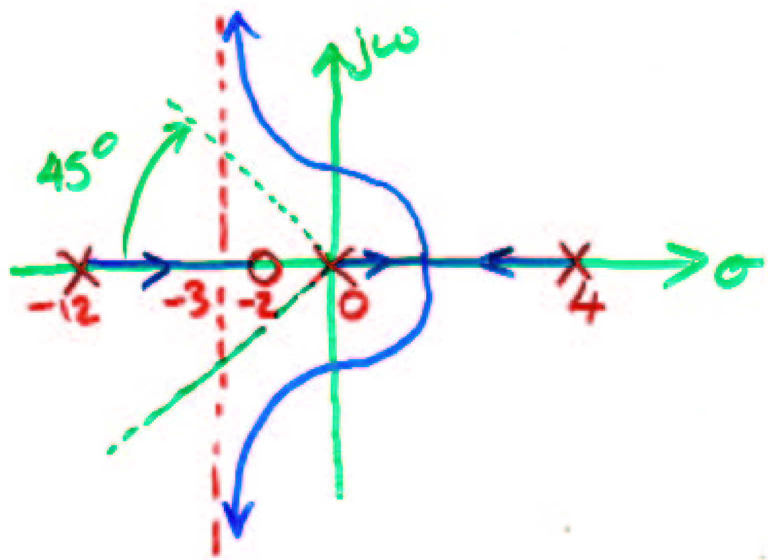
$$\Rightarrow \quad 3\omega_c^2 = 48$$

$$\Rightarrow \quad \omega_c = \pm 4$$

$$\Rightarrow \quad K = 128.$$

$\Rightarrow$ System is stable for all $K > 128$

- Now find the smallest value of K so that the damping factor $\geqslant \frac{1}{\sqrt{2}}$



Root locus is never completely contained in the damping cone

$\Rightarrow$ there is no K which will satisfy the constraint.

This example demonstrates the power of sketching the root locus

Automated methods would search for a K for a long time and not find one.

Unless you sketch the root locus you may be left wondering whether all you need to do is search harder!