Transceiver Optimization for Block-Based Multiple Access Through ISI Channels

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***Abstract*—In this paper, we describe a formulation of the minimum mean square error (MMSE) joint transmitter-receiver design problem for block-based multiple access communication over intersymbol interference (ISI) channels. Since the direct formulation of this problem turns out to be nonconvex, we develop various alternative convex formulations using techniques of linear matrix inequalities (LMIs) and second-order cone programming (SOCP). In particular, we show that the optimal MMSE transceiver design problem can be reformulated as a semidefinite program (SDP), which can be solved using highly efficient interior point methods. When the channel matrices are diagonal (as in cyclic prefixed multicarrier systems), we show that the optimal MMSE transceivers can be obtained by subcarrier allocation and optimal power loading to each subcarrier for all the users. Moreover, the optimal subcarrier allocation and power-loading can be computed fairly simply (in polynomial time) by the relative ratios of the magnitudes of the subchannel gains corresponding to all subcarriers. We also prove that any two users can share no more than *one* subcarrier in the optimal MMSE transceivers. By exploiting this property, we design an efficient strongly polynomial time algorithm for the determination of optimal powerloading and subcarrier allocation in the two-user case.**

***Index Terms*—Frequency division multiple access, intersymbol interference, orthogonal frequency division multiplexing, time division multiple access.**

I. Introduction

THE communication of data through intersymbol interference (ISI) channels can often be simplified by transmitting the data in a block-based fashion [6]. In particular, if the blocks are designed so that they do not interfere with each other at the receiver, then effective detection can be performed on a block-by-block basis. Within this family of block-by-block communication schemes, the most commonly used schemes are the multicarrier modulation based discrete multitone (DMT) [1], [4], [16] and orthogonal frequency division multiplexing (OFDM) [20], [21] schemes. These schemes employ the (inverse) fast Fourier transform [(I)FFT] at the transmitter and the receiver (along with a cyclic prefix) to effectively diagonalize the channel matrix, resulting in a lowcost, high-performance implementation. Many of the digital subscriber line (xDSL) systems for wired media use DMT, while proposed digital audio broadcasting (DAB) and digital video broadcasting (DVB) wireless systems use OFDM.

To achieve the capacity of a spectrally-shaped Gaussian channel in a single user multicarrier system, it is well known [3] that one must use appropriate bit and power allocation among the subcarriers in a way that corresponds to the classic water-filling distribution [7]. However, achieving capacity in the multiuser case requires more sophisticated resource allocation [2], [18]. In particular, simply using time division (TDMA) or frequency division (FDMA) nonoverlapping resource allocation schemes in an arbitrary fashion will result in multiuser rates far below capacity [25]. Unfortunately, the optimal resource allocation can be difficult to compute exactly [26]. Furthermore, to attain reliable performance at the rates predicted in [2] and [18], we may need to employ joint (or at least successive) detection at the receiver, which may result in an unacceptably high computational load. To simplify the receiver, one can impose a structure, such as frequency division, on the transmitted signals and retain high rates [24] (see also [22] for a dual problem of minimizing the transmitted power for given data and error rates). The alternative approach taken in this paper is to devise optimal transmitter resource allocation for multiple access systems with a *linear* receiver. To do so, we jointly optimize the transmitter and receiver (the "transceiver") to minimize the mean square error (MSE) of the receiver output, under the constraint of finite transmission power.

We will adopt the general framework of block-based symbol-spread multicarrier communication schemes [12]. This framework includes as special cases many of the popular communication schemes such as direct sequence code division multiple access (DS-CDMA), multicarrier DS-CDMA (MC-DS-CDMA), and orthogonal frequency division multiple access (OFDMA). Within this framework, the optimal single-user transmitter design problem, in terms of information rate, was studied recently in [12], under the assumption that maximum likelihood detection was computationally feasible. The single-user joint transmitter and linear receiver design under the minimum mean square error (MMSE) criterion has also been addressed [13], [14], where it was shown to lead to an analytic solution for the optimal linear precoder and equalizer pair. The joint MMSE transmitter and receiver design was also

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considered in [23] in the context of the multi-input multi-output (MIMO) channel. The system considered in [23] is essentially a multiplexing system in which the users' data sequences are jointly precoded and transmitted over a common channel. The power of this multiplexed transmission is controlled by a single power constraint. In contrast, the system we consider is a block-based multiple access system in which the users' data sequences are precoded separately and transmitted over distinct channels, and the transmission power of each user can be independently controlled. In fact, the system in [23] is algebraically equivalent to a single-user version of our scheme. Therefore, our work can be considered the multiuser extension of the work in [23].

In this paper, we present various formulations and algorithms for the MMSE transceiver design problem for a general block-based multiple access communication system. In particular, we consider the joint design of an optimal transmitter/receiver pair under the constraint of fixed finite transmission power. It turns out that the direct formulation of this problem is nonconvex, making it difficult to solve in practice. We develop herein an alternative convex formulation of the MMSE transceiver design problem using the linear matrix inequality (LMI) technique. When the channel matrices are diagonal (or jointly diagonalizable), as in the generalized OFDMA or DMT type systems [21], we show that the optimal MMSE transmitters can be realized by appropriately allocating subcarriers and power to each user. This result generalizes a result in [23] for what corresponds to the single-user case in our framework. One major consequence of this result is that it allows us to simplify the semidefinite programming (SDP) formulation of the transceiver design problem to a second-order cone program (SOCP), which can be solved by highly efficient interior point methods [11]. In addition, we show that any pair of users can share no more than *one* subcarrier in an optimal MMSE scheme. By exploiting this property, we design an efficient strongly polynomial time algorithm for the determination of optimal power loading and subcarrier allocation in the two-user case.

Throughout this paper, we assume that the channel matrices of all the users and the noise correlation [collectively known as the channel state information (CSI)] are known. This information is usually tracked and estimated by most practical receivers in order to facilitate decoding. For example, in xDSL and digital cable TV systems, the channel does not vary very often, and it is possible for the "central office" to acquire the CSI. The CSI can also be obtained by the base station in a quasistatic wireless multiple access scheme, where the channels undergo only slow changes. A recent work [9] on channel-adapted precoder design also assumed full knowledge of CSI and demonstrated improved system performance for an uplink CDMA system. Similarly, the methods presented in this paper are designed to exploit the CSI to efficiently determine how the users should adapt their transmission to the current environment in a jointly optimal manner. Our simulation results indicate that this joint adaptation results in a substantial improvement in the performance of the multiple access scheme. In the applications we have envisioned, the optimization of the transmitters will be performed at the central office or base station and will be communicated to the users via control channels.

The rest of this paper is organized as follows. Section II gives two convex formulations (SDP and SOCP) of the optimal MMSE transceiver design problem in the two-user case. The structure of optimal MMSE transceivers is analyzed when the channel matrices are all diagonal in Section III. This optimal structure gives rise to a strongly polynomial time algorithm for the determination of optimal power/subcarrier allocation. Section IV presents the generalization of the results of Section II to the case of more than two users. Section V presents some simulation results that compare the performance in a fading environment of the jointly optimal MMSE transceivers with that of an OFDMA scheme, which does not require CSI, and that of a scheme in which the CSI is used to design MMSE transceivers on a user-by-user basis. The simulation results indicate that the performance advantage of the jointly optimal scheme is substantial [a signal-to-noise ratio (SNR) gain of around 7 dB]. The final section (Section VI) contains some concluding remarks and suggestions of future work.

Our notational conventions are as follows: The n-dimensional Euclidean space is denoted by Re^n, and the non-negative orthant of Re^n is denoted by Re^n_+. Vectors and matrices will be represented by bold lowercase and uppercase letters, respectively, and the superscript ^{H} will denote the Hermitian transpose. The elements of these structures will be denoted with appropriate indices, e.g., $\mathbf{x}(k) = [\mathbf{x}]_k$ and $\mathbf{X}(i,j) = [\mathbf{X}]_{ij}$, whereas the rank of $\mathbf{X}$ will be denoted by $r(\mathbf{X})$. For a random vector $\mathbf{x}$, $E(\mathbf{x})$ will denote its mean, and $E(\mathbf{x}\mathbf{x}^H)$ will denote its correlation matrix. Moreover, for any symmetric matrix $\mathbf{X}$, the notation $\mathbf{X} \succeq \mathbf{0}$ (or $\mathbf{X} \succ \mathbf{0}$) signifies that $\mathbf{X}$ is positive semidefinite (or positive definite respectively), and the notation $\mathrm{tr}(\mathbf{X})$ denotes the trace of $\mathbf{X}$.

II. Joint MMSE Transmitter-Receiver Design: Two-User Case

Consider a quasisynchronous vector multiple access scheme with two users whose data vectors $\mathbf{s}_1$ and $\mathbf{s}_2$ are uncorrelated (see Fig. 1). The channel matrices $\mathbf{H}_1$ and $\mathbf{H}_2$, which are of size $p \times n$, are assumed to be known, and $\mathbf{n}$ is a zero mean additive Gaussian noise vector that is uncorrelated with $\mathbf{s}_1$ and $\mathbf{s}_2$ and has known correlation matrix $E(\mathbf{n}\mathbf{n}^H) = \mathbf{R}$. With square transmitter precoding matrices $\mathbf{F}_1$ and $\mathbf{F}_2$, the received signal takes the form

$$\mathbf{x} = \mathbf{H}_1\mathbf{F}_1\mathbf{s}_1 + \mathbf{H}_2\mathbf{F}_2\mathbf{s}_2 + \mathbf{n}. \tag{2.1}$$

In our development, each data block $\mathbf{s}_i$ will be treated as white with identity correlation matrix. However, our results carry over to the colored case as well, because a whitening matrix $\mathbf{V}_i = \mathbf{R}_{s,i}^{-1/2}$ can be readily absorbed into the precoder $\mathbf{F}_i$, as long as the corresponding source correlation matrix $\mathbf{R}_{s,i}$ is known (and full rank). From the received signal $\mathbf{x}$, we wish to extract the transmitted signals $\mathbf{s}_i$, $i = 1, 2$. This can be accomplished in various ways. A popular (and arguably the simplest) approach is to use a linear receiver $\mathbf{G}_i$, whereby the equalized signal $\mathbf{G}_i\mathbf{x}$ is quantized according to the finite alphabet of $\mathbf{s}_i$, e.g., for BPSK

$$\hat{\mathbf{s}}_i = \mathrm{sign}\,(\mathbf{G}_i\mathbf{x})$$

where $\mathbf{G}_i$, $i = 1, 2$ is the block (matrix) equalizers. Of course, nonlinear receiver structures (for example, DFE type receivers)

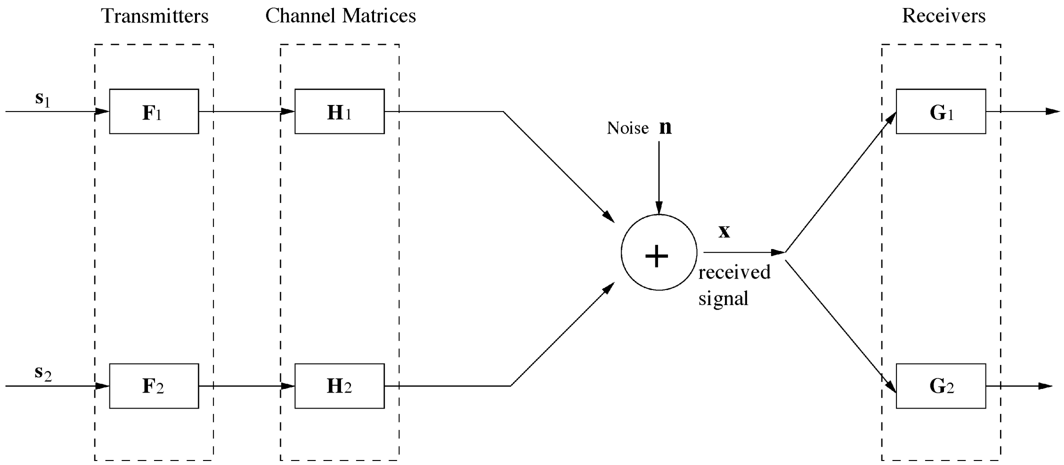

Fig. 1. Two-user multiple access scheme (uplink).

are also possible, but we will not discuss them in this paper. The objective of this section is to obtain efficiently solvable formulations for the (joint) optimization of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{G}_1$, and $\mathbf{G}_2$ to minimize the MSE at the equalizer output.

Since the precoder matrices are nominally square, $\mathbf{s}_1$ and $\mathbf{s}_2$ are nominally of length n, corresponding to a (maximum) symbol rate of n symbols per block per user. After the matrices $\mathbf{F}_1$ and $\mathbf{F}_2$ are designed, we usually have $r(\mathbf{F}_1) \le n$ and $r(\mathbf{F}_2) \le n$, where $r(\cdot)$ denotes the rank of a matrix. Since (at most) $r(\mathbf{F}_j)$ symbols from user j can be reliably recovered using the linear equalizer $\mathbf{G}_j$, the actual symbol rate of user j will be reduced to $r(\mathbf{F}_j)$ symbols per block, resulting in a transmission redundancy of $n-r(\mathbf{F}_j)$ and a coding rate of $r(\mathbf{F}_j)/n$. In other words, we do not set the coding rates before the design process. Instead, the coding rates are (implicitly) optimized along with the explicit optimization of the transceivers using the MMSE criterion. We will explain that point in more details in Section III-A.

The above vector multiple access channel model arises naturally in the so-called generalized multicarrier block transmission scheme [21]. In such block transmission, the input signal streams for the two users are divided into blocks or vectors via a serial-to-parallel converter, whereas at the receiver, the output signal block is processed on a block by block basis and then parallel-to-serial converted before decoding. To avoid inter block interference at the receiver, the precoded symbol vectors are usually either zero-padded or coded with a cyclic prefix. In the case of cyclic prefixing, the channel matrices $\mathbf{H}_i$ are circulant. By applying IFFT and FFT transformation to the data vectors $\mathbf{s}_1$, $\mathbf{s}_2$, and $\mathbf{x}$, as well as to the transmitter filter matrices, we can further diagonalize the channel matrices $\mathbf{H}_i$, in a way much like the well known single-user OFDM system. This process is illustrated in Fig. 2. In the case of zero-padding, the channel matrices are tall, Toeplitz, and full column rank. If an appropriate time-aliasing operation is used at the receiver, the channel matrix is again circulant, and the IFFT/FFT diagonalization procedure can be carried as in the cyclic prefix case. The model in (2.1) is also applicable in multiple input, multiple output (MIMO) block transmission schemes, and some of the techniques developed herein extend naturally to that case. However, for simplicity, we will focus our attention on the single input, single output case and merely observe the MIMO extensions in the conclusion. In the following section, we develop an SDP formulation of the MMSE transceiver design problem for general $\mathbf{H}_j$ and $\mathbf{R}$. In Section III, we will develop a more efficiently solvable SOCP formulation for the case of diagonal $\mathbf{H}_j$ and $\mathbf{R}$. Before we do so, we point out that similar models to that in Fig. 1 have been considered in [25], where the capacity region for the above multiaccess communication channel is evaluated using the tool of linear matrix inequalities and semidefinite programming.

A. SDP Formulation of MMSE Transceiver Design

For the system in Fig. 1, let $\mathbf{e}_i$ denote the error vector (before making the hard decision) for user i, $i = 1, 2$. Then

$$\begin{aligned}\mathbf{e}_1 &= \mathbf{G}_1\mathbf{x} - \mathbf{s}_1\\ &= \mathbf{G}_1(\mathbf{H}_1\mathbf{F}_1\mathbf{s}_1 + \mathbf{H}_2\mathbf{F}_2\mathbf{s}_2 + \mathbf{n}) - \mathbf{s}_1\\ &= (\mathbf{G}_1\mathbf{H}_1\mathbf{F}_1 - \mathbf{I})\mathbf{s}_1 + \mathbf{G}_1\mathbf{H}_2\mathbf{F}_2\mathbf{s}_2 + \mathbf{G}_1\mathbf{n}.\end{aligned}$$

This further implies that

$$\begin{aligned}E\left(\mathbf{e}_1\mathbf{e}_1^H\right) = (\mathbf{G}_1\mathbf{H}_1\mathbf{F}_1 - \mathbf{I})(\mathbf{G}_1\mathbf{H}_1\mathbf{F}_1 - \mathbf{I})^H\\ +(\mathbf{G}_1\mathbf{H}_2\mathbf{F}_2)(\mathbf{G}_1\mathbf{H}_2\mathbf{F}_2)^H + \mathbf{G}_1\mathbf{R}\mathbf{G}_1^H\end{aligned}$$

where we have used the fact that the signals $\mathbf{s}_1$, $\mathbf{s}_2$ and the noise $\mathbf{n}$ are mutually uncorrelated:

$$E\left(\mathbf{s}_i\mathbf{s}_j^H\right) = \mathbf{0} \quad \text{and} \quad E(\mathbf{s}_i\mathbf{n}^H) = \mathbf{0}, \quad i,j = 1,2, \quad i \ne j$$

that the noise correlation matrix is known and that the source correlation matrices are normalized:

$$E(\mathbf{n}\mathbf{n}^H) = \mathbf{R} \quad \text{and} \quad E\left(\mathbf{s}_i\mathbf{s}_i^H\right) = \mathbf{I}, \quad i = 1,2.$$

Similarly, we have

$$\begin{aligned}E\left(\mathbf{e}_2\mathbf{e}_2^H\right) = (\mathbf{G}_2\mathbf{H}_2\mathbf{F}_2 - \mathbf{I})(\mathbf{G}_2\mathbf{H}_2\mathbf{F}_2 - \mathbf{I})^H\\ +(\mathbf{G}_2\mathbf{H}_1\mathbf{F}_1)(\mathbf{G}_2\mathbf{H}_1\mathbf{F}_1)^H + \mathbf{G}_2\mathbf{R}\mathbf{G}_2^H.\end{aligned}$$

Introducing the matrix

$$\mathbf{W} = \left(\mathbf{H}_1\mathbf{F}_1\mathbf{F}_1^H\mathbf{H}_1^H + \mathbf{H}_2\mathbf{F}_2\mathbf{F}_2^H\mathbf{H}_2^H + \mathbf{R}\right)^{-1} \qquad (2.2)$$

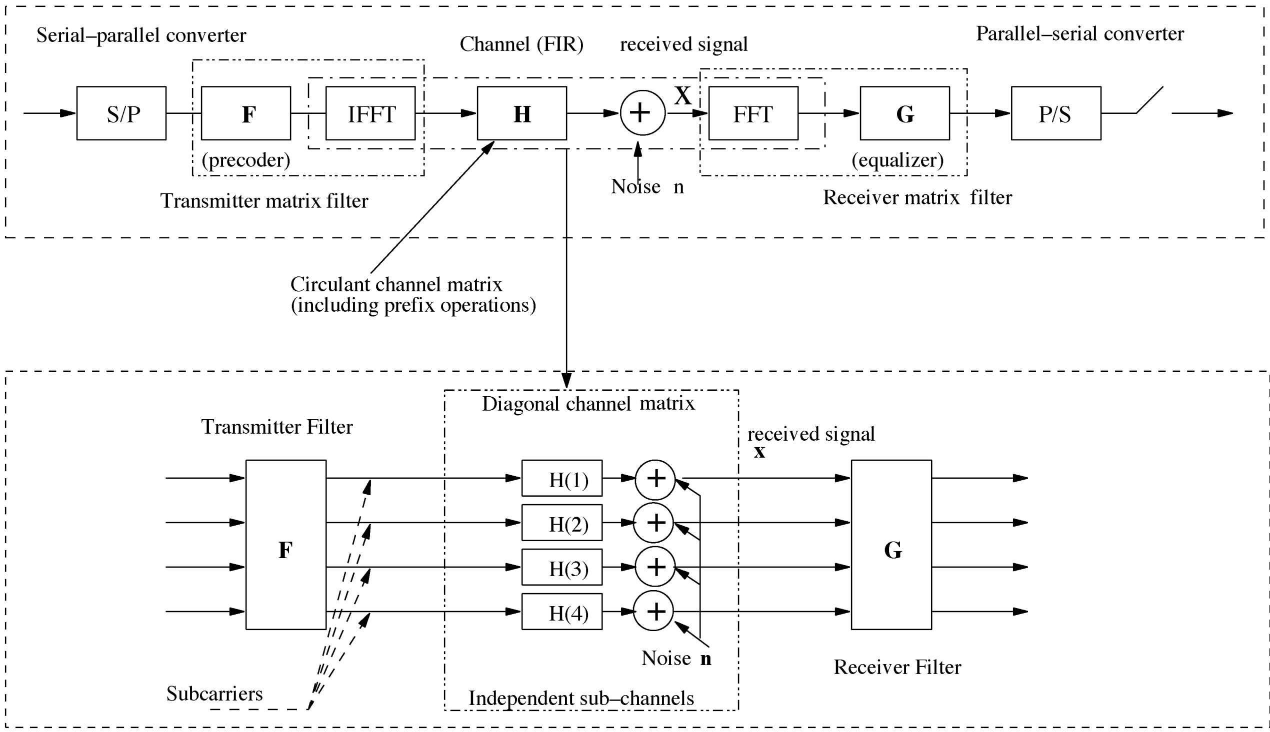

Fig. 2. OFDM communication system.

which can be seen as the inverse of the covariance matrix of $\mathbf{x}$ in (2.1), we can rewrite the error covariance matrices for users 1 and 2 as

$$E\left(\mathbf{e}_1\mathbf{e}_1^H\right) = \mathbf{G}_1\mathbf{W}^{-1}\mathbf{G}_1^H - \mathbf{G}_1\mathbf{H}_1\mathbf{F}_1 - (\mathbf{G}_1\mathbf{H}_1\mathbf{F}_1)^H + \mathbf{I} \tag{2.3}$$

$$E\left(\mathbf{e}_2\mathbf{e}_2^H\right) = \mathbf{G}_2\mathbf{W}^{-1}\mathbf{G}_2^H - \mathbf{G}_2\mathbf{H}_2\mathbf{F}_2 - (\mathbf{G}_2\mathbf{H}_2\mathbf{F}_2)^H + \mathbf{I}. \tag{2.4}$$

As is always the case in practice, there are power constraints on the transmitting matrix filters:

$$\operatorname{tr}\left(\mathbf{F}_1\mathbf{F}_1^H\right) \le p_1, \quad \operatorname{tr}\left(\mathbf{F}_2\mathbf{F}_2^H\right) \le p_2 \tag{2.5}$$

where $p_1 > 0$ and $p_2 > 0$ are user-specified bounds on the transmitting power for each user. Our goal is to design a set of transmitting matrix filters $\mathbf{F}_i$ satisfying the power constraints (2.5) and a set of matrix equalizers $\mathbf{G}_i$ such that the total MSE = $\operatorname{tr}(E(\mathbf{e}_1\mathbf{e}_1^H)) + \operatorname{tr}(E(\mathbf{e}_2\mathbf{e}_2^H))$ is minimized. In other words, we aim to solve

$$\begin{array}{ll} \text{minimize}_{\mathbf{F}_1,\mathbf{F}_2,\mathbf{G}_1,\mathbf{G}_2} & \operatorname{tr}\left(E\left(\mathbf{e}_1\mathbf{e}_1^H\right)\right) + \operatorname{tr}\left(E\left(\mathbf{e}_2\mathbf{e}_2^H\right)\right) \\ \text{subject to} & \operatorname{tr}\left(\mathbf{F}_1\mathbf{F}_1^H\right) \le p_1 \\ & \operatorname{tr}\left(\mathbf{F}_2\mathbf{F}_2^H\right) \le p_2 \end{array} \tag{2.6}$$

where $\operatorname{tr}(E(\mathbf{e}_1\mathbf{e}_1^H))$ and $\operatorname{tr}(E(\mathbf{e}_2\mathbf{e}_2^H))$ are given by (2.3) and (2.4), respectively. The receiver filters $\mathbf{G}_1$ and $\mathbf{G}_2$ in (2.6) are unconstrained. The objective function of (2.6) is a fourth-order polynomial in $\mathbf{G}_i$, $\mathbf{F}_i$, $i = 1, 2$. It can be easily checked (even for the case where the block length n is one; i.e., each $\mathbf{G}_i$, $\mathbf{F}_i$ is a scalar) that the Hessian matrix of this fourth-order polynomial is not positive semidefinite. Therefore, the objective function of (2.6) is nonconvex, and hence, it can be difficult to minimize due to the usual difficulties with local solutions and the selection of a stepsize and starting point. In what follows, we will reformulate (2.6) as a convex semidefinite program.

As the first step, we can eliminate $\mathbf{G}_1$ and $\mathbf{G}_2$ in (2.6) by first minimizing the total MSE with respect to $\mathbf{G}_1$ and $\mathbf{G}_2$, assuming $\mathbf{F}_1$ and $\mathbf{F}_2$ are fixed. The resulting receivers are the so-called *linear MMSE receivers*. More specifically, the linear MMSE equalizer $\mathbf{G}_1$ is defined as a matrix that, given the transmitting matrices $\mathbf{F}_1, \mathbf{F}_2$, minimizes the MSE for user 1 (or, equivalently, the total MSE since $\operatorname{tr}(E(\mathbf{e}_2\mathbf{e}_2^H))$ is independent of $\mathbf{G}_1$). By minimizing $\operatorname{tr}(E(\mathbf{e}_1\mathbf{e}_1^H))$ with respect to $\mathbf{G}_1$, we can obtain the linear MMSE equalizer for user 1 in a standard manner:

$$\mathbf{G}_1 = \mathbf{F}_1^H\mathbf{H}_1^H\mathbf{W}. \tag{2.7}$$

Substituting the MMSE equalizer (2.7) into (2.3) results in the following minimized (with respect to $\mathbf{G}_1$) MSE:

$$E\left(\mathbf{e}_1\mathbf{e}_1^H\right) = -\mathbf{F}_1^H\mathbf{H}_1^H\mathbf{W}\mathbf{H}_1\mathbf{F}_1 + \mathbf{I}. \tag{2.8}$$

Similarly, the MMSE equalizer $\mathbf{G}_2$ for user 2 is given by

$$\mathbf{G}_2 = \mathbf{F}_2^H\mathbf{H}_2^H\mathbf{W} \tag{2.9}$$

and the resulting minimized (with respect to $\mathbf{G}_2$) mean square error for user 2 is given by

$$E\left(\mathbf{e}_2\mathbf{e}_2^H\right) = -\mathbf{F}_2^H\mathbf{H}_2^H\mathbf{W}\mathbf{H}_2\mathbf{F}_2 + \mathbf{I}. \tag{2.10}$$

Substituting (2.8) and (2.10) into the total MSE gives rise to

$$\begin{aligned} \text{MSE} &= \operatorname{tr}\big(E\big(\mathbf{e}_1\mathbf{e}_1^H\big)\big) + \operatorname{tr}\big(E\big(\mathbf{e}_2\mathbf{e}_2^H\big)\big) \\ &= -\operatorname{tr}\big(\mathbf{F}_1^H\mathbf{H}_1^H\mathbf{W}\mathbf{H}_1\mathbf{F}_1\big) - \operatorname{tr}\big(\mathbf{F}_2^H\mathbf{H}_2^H\mathbf{W}\mathbf{H}_2\mathbf{F}_2\big) + 2n \\ &= -\operatorname{tr}\big(\mathbf{W}\mathbf{H}_1\mathbf{F}_1\mathbf{F}_1^H\mathbf{H}_1^H\big) - \operatorname{tr}\big(\mathbf{W}\mathbf{H}_2\mathbf{F}_2\mathbf{F}_2^H\mathbf{H}_2^H\big) + 2n \\ &= -\operatorname{tr}\big(\mathbf{W}\big(\mathbf{H}_1\mathbf{F}_1\mathbf{F}_1^H\mathbf{H}_1^H + \mathbf{H}_2\mathbf{F}_2\mathbf{F}_2^H\mathbf{H}_2^H\big)\big) + 2n \\ &= -\operatorname{tr}\big(\mathbf{W}(\mathbf{W}^{-1} - \mathbf{R})\big) + 2n \\ &= \operatorname{tr}(\mathbf{W}\mathbf{R}) + n \end{aligned} \tag{2.11}$$

where the second to last step follows from the definition of $\mathbf{W}$ in (2.2). Thus, by eliminating the variables $\mathbf{G}_1$ and $\mathbf{G}_2$, we obtain a formulation equivalent to (2.6)

> Design a pair of transmitting matrix filters $\mathbf{F}_1,\ \mathbf{F}_2$ satisfying the power constraints (2.5) such that the total mean squared error given by (2.11) is minimized.

Now, let us define two new matrix variables

$$\mathbf{U}_1 = \mathbf{F}_1\mathbf{F}_1^H \quad \text{and} \quad \mathbf{U}_2 = \mathbf{F}_2\mathbf{F}_2^H.$$

Then, the MMSE (2.11) can be expressed as

$$\text{MSE} = \text{tr}\left(\left(\mathbf{H}_1\mathbf{U}_1\mathbf{H}_1^H + \mathbf{H}_2\mathbf{U}_2\mathbf{H}_2^H + \mathbf{R}\right)^{-1}\mathbf{R}\right) + n$$

and the power constraints (2.5) can be expressed as

$$\text{tr}(\mathbf{U}_1) \le p_1 \quad \text{and} \quad \text{tr}(\mathbf{U}_2) \le p_2.$$

Consequently, the optimal joint MMSE transmitter-receiver design problem can be stated as

$$\begin{aligned} \text{minimize}_{\mathbf{U}_1,\mathbf{U}_2} \quad & g(\mathbf{U}_1,\mathbf{U}_2) := \text{tr}\Big(\big(\mathbf{H}_1\mathbf{U}_1\mathbf{H}_1^H + \mathbf{H}_2\mathbf{U}_2\mathbf{H}_2^H + \mathbf{R}\big)^{-1}\mathbf{R}\Big) \\ \text{subject to} \quad & \text{tr}(\mathbf{U}_1) \le p_1, \quad \text{tr}(\mathbf{U}_2) \le p_2 \\ & \mathbf{U}_1 \succeq \mathbf{0}, \quad \mathbf{U}_2 \succeq \mathbf{0}. \end{aligned} \tag{2.12}$$

Using the auxiliary matrix variable $\mathbf{W}$ (2.2) and the nature of our minimization problem, we can rewrite (2.12) in the following alternative (but equivalent) form:

$$\begin{aligned} \text{minimize}_{\mathbf{W},\mathbf{U}_1,\mathbf{U}_2} \quad & \text{tr}(\mathbf{W}\mathbf{R}) \\ \text{subject to} \quad & \text{tr}(\mathbf{U}_1) \le p_1, \quad \text{tr}(\mathbf{U}_2) \le p_2 \\ & \mathbf{W} \succeq \left(\mathbf{H}_1\mathbf{U}_1\mathbf{H}_1^H + \mathbf{H}_2\mathbf{U}_2\mathbf{H}_2^H + \mathbf{R}\right)^{-1} \\ & \mathbf{U}_1 \succeq \mathbf{0}, \quad \mathbf{U}_2 \succeq \mathbf{0}. \end{aligned} \tag{2.13}$$

The equivalence of (2.12) and (2.13) can be argued as follows: First, we recall the simple property from linear algebra that $\text{tr}(\mathbf{MN}) \ge 0$ for all $\mathbf{M} \succeq \mathbf{0}$ and $\mathbf{N} \succeq \mathbf{0}$. Since $\mathbf{R}$ is Hermitian symmetric and positive definite, we have $\text{tr}(\mathbf{WR}) \ge \text{tr}((\mathbf{H}_1\mathbf{U}_1\mathbf{H}_1^H + \mathbf{H}_2\mathbf{U}_2\mathbf{H}_2^H + \mathbf{R})^{-1}\mathbf{R})$ whenever $\mathbf{W} \succeq (\mathbf{H}_1\mathbf{U}_1\mathbf{H}_1^H + \mathbf{H}_2\mathbf{U}_2\mathbf{H}_2^H + \mathbf{R})^{-1}$. Since we are minimizing $\text{tr}(\mathbf{WR})$, a monotonicity argument ensures that the equality $\mathbf{W} = (\mathbf{H}_1\mathbf{U}_1\mathbf{H}_1^H + \mathbf{H}_2\mathbf{U}_2\mathbf{H}_2^H + \mathbf{R})^{-1}$ must hold at optimality. Notice that the constraint

$$\mathbf{W} \succeq \left(\mathbf{H}_1\mathbf{U}_1\mathbf{H}_1^H + \mathbf{H}_2\mathbf{U}_2\mathbf{H}_2^H + \mathbf{R}\right)^{-1}$$

can be rewritten, via Schur's complement [10, Th. 7.7.6, p. 472], as the following linear matrix inequality:

$$\begin{bmatrix} \mathbf{W} & \mathbf{I} \\ \mathbf{I} & \mathbf{H}_1\mathbf{U}_1\mathbf{H}_1^H + \mathbf{H}_2\mathbf{U}_2\mathbf{H}_2^H + \mathbf{R} \end{bmatrix} \succeq \mathbf{0}. \tag{2.14}$$

Therefore, we obtain the following semidefinite programming (SDP) formulation [19]:

$$\begin{aligned} \text{minimize}_{\mathbf{W},\mathbf{U}_1,\mathbf{U}_2} \quad & \text{tr}(\mathbf{W}\mathbf{R}) \\ \text{subject to} \quad & \text{tr}(\mathbf{U}_1) \le p_1, \quad \text{tr}(\mathbf{U}_2) \le p_2 \\ & \mathbf{W} \text{ satisfies (2.14)} \\ & \mathbf{U}_1 \succeq \mathbf{0}, \quad \mathbf{U}_2 \succeq \mathbf{0}. \end{aligned} \tag{2.15}$$

This SDP formulation makes it possible to efficiently solve the optimal transmitter design problem using interior point methods [19]. The advantage of the SDP formulation (2.15) over the formulation (2.6) is that the former is convex, whereas the latter is not. The convexity of (2.15) is due to the linear cost function and the fact that the constraints are in the form of linear matrix inequalities, which are also convex [19]. The convexity of (2.15) ensures that its global optimum can be found in polynomial time without the usual headaches of step size selection, algorithm initialization, or the risk of local minima. The arithmetic complexity of the interior point methods for solving the SDP (2.15) is $O(n^{6.5}\log(1/\epsilon))$, where $\epsilon > 0$ is the solution accuracy [19].

We remark that solving the optimization problem (2.15) requires CSI knowledge, i.e., one needs to have available $\mathbf{H}_1$, $\mathbf{H}_2$, and the noise correlation matrix $\mathbf{R}$. Since these quantities are usually estimated and available at the central office or base station, a natural implementation would be to perform the optimization there. Once the optimal $\mathbf{U}_1$ and $\mathbf{U}_2$ have been determined, they can be factorized (using, e.g., Cholesky factorization) as

$$\mathbf{U}_1 = \mathbf{F}_1\mathbf{F}_1^H \quad \text{and} \quad \mathbf{U}_2 = \mathbf{F}_2\mathbf{F}_2^H$$

to obtain optimal MMSE transmitter matrices $\mathbf{F}_1$ and $\mathbf{F}_2$. The corresponding optimal MMSE equalizers $\mathbf{G}_1$ and $\mathbf{G}_2$ can then be computed by substituting $\mathbf{F}_1$ and $\mathbf{F}_2$ into (2.7) and (2.9). The optimal transmitter matrices $\mathbf{F}_1$ and $\mathbf{F}_2$ can then be sent to the transmitters over control channels.

III. Diagonal Designs

When the channel matrices $\mathbf{H}_1$ and $\mathbf{H}_2$ are diagonal (as in OFDM systems) and the noise covariance matrix $\mathbf{R}$ is also diagonal, we can show (see Theorem 3.1 below) that the optimal transmitters are also diagonal and can be computed more efficiently [faster than solving the SDP (2.15)].

Theorem 3.1: If the channel matrices $\mathbf{H}_1$ and $\mathbf{H}_2$ are diagonal and the noise covariance matrix $\mathbf{R}$ is diagonal, then the optimal transmitters $\mathbf{U}_1$ and $\mathbf{U}_2$ are also diagonal. Consequently, the MMSE transceivers for a multiuser OFDM system can be implemented by optimally allocating power to each subcarrier for all the users.

Proof: The proof proceeds via a contradiction argument based on the fact [8, p. 402] that for a positive definite matrix $\mathbf{A}$

$$\text{tr}(\mathbf{A}^{-1}) \ge \sum_i \frac{1}{\mathbf{A}(i,i)} \tag{3.1}$$

holds, with equality holding if and only if $\mathbf{A}$ is diagonal. Let $\mathbf{U}_1^*$ and $\mathbf{U}_2^*$ be the optimal solution to (2.12), and suppose that they are not both diagonal. Let $\bar{\mathbf{U}}_1^*$ and $\bar{\mathbf{U}}_2^*$ be the diagonal parts of $\mathbf{U}_1^*$ and $\mathbf{U}_2^*$, respectively. Then, $\text{tr}(\bar{\mathbf{U}}_1^*) = \text{tr}(\mathbf{U}_1^*) \le p_1$ and $\text{tr}(\bar{\mathbf{U}}_2^*) = \text{tr}(\mathbf{U}_2^*) \le p_2$, and therefore, $\bar{\mathbf{U}}_1^*$ and $\bar{\mathbf{U}}_2^*$ are in the feasible set of (2.12). Let $\mathbf{A}^* = \mathbf{R}^{-1}(\mathbf{H}_1\mathbf{U}_1^*\mathbf{H}_1^H + \mathbf{H}_2\mathbf{U}_2^*\mathbf{H}_2^H + \mathbf{R})$, where $\mathbf{H}_1$, $\mathbf{H}_2$, and $\mathbf{R}$ are diagonal. Then, the diagonal part of $\mathbf{A}^*$, which is denoted $\bar{\mathbf{A}}^*$, is given by

$$\bar{\mathbf{A}}^* = \mathbf{R}^{-1}\left(\mathbf{H}_1\bar{\mathbf{U}}_1^*\mathbf{H}_1^H + \mathbf{H}_2\bar{\mathbf{U}}_2^*\mathbf{H}_2^H + \mathbf{R}\right).$$

Using the inequality (3.1), we obtain $\text{tr}((\bar{\mathbf{A}}^*)^{-1}) < \text{tr}((\mathbf{A}^*)^{-1})$, where the strict inequality holds since $\mathbf{U}_1^*$ and $\mathbf{U}_2^*$ are not both diagonal. Since $g(\bar{\mathbf{U}}_1^*, \bar{\mathbf{U}}_2^*) = \text{tr}((\bar{\mathbf{A}}^*)^{-1})$ and $g(\mathbf{U}_1^*, \mathbf{U}_2^*) = \text{tr}((\mathbf{A}^*)^{-1})$, it follows that $g(\bar{\mathbf{U}}_1^*, \bar{\mathbf{U}}_2^*) < g(\mathbf{U}_1^*, \mathbf{U}_2^*)$, which contradicts the assertion that the (nondiagonal) matrices $\mathbf{U}_1^*$ and $\mathbf{U}_2^*$ were optimal. Hence, when $\mathbf{H}_1$, $\mathbf{H}_2$ and $\mathbf{R}$ are diagonal, the optimal $\mathbf{U}_1$ and $\mathbf{U}_2$ are diagonal. **Q.E.D.**

Theorem 3.1 should be good news to practitioners since it says that in "diagonal" scenarios, there is no need to implement full precoder matrices because diagonal precoders are optimal. Notice that diagonal precoders simply represent power loading/subcarrier allocation at the transmitters. Therefore, Theorem 3.1 implies that the MMSE transceivers for a multiuser OFDM system can be implemented by optimally assigning subcarriers and allocating power to them. In applications where the noise correlation matrix is not diagonal, one may wish to approximate the true noise correlation matrix with a diagonal one (by setting the off diagonal elements to zero) to enjoy the benefits of reduced implementation complexity, as predicted by Theorem 3.1. In particular, such an approximation will lead to simplified (i.e., diagonal) transmitter/receiver design at the expense of reduced performance (e.g., with increased overall MSE error). (An analogous approximation is typically used in the design of single-user DMT schemes in the presence of colored noise.)

Another important implication of Theorem 3.1 is the significant simplification in the computation of the optimal MMSE transceivers. In particular, Theorem 3.1 suggests that we only need to search among all the diagonal transmitters in order to achieve the minimum MSE. Therefore, if $\mathbf{H}_1$, $\mathbf{H}_2$, and $\mathbf{R}$ are diagonal, it is only necessary to solve (3.2) below rather than the SDP (2.15). Before we state this formally, we point out that when the channel matrices $\mathbf{H}_j$ have been diagonalized using the FFT and IFFT, the ith diagonal element is $H_j(i)$, where $H_j(i)$ is the frequency response of user j's channel at the ith point on the FFT grid $\omega_i = 2\pi i/n$. Define the diagonal entries of $\mathbf{U}_1$, $\mathbf{U}_2$ by $\mathbf{u}_1 = \mathrm{diag}(\mathbf{U}_1)$, $\mathbf{u}_2 = \mathrm{diag}(\mathbf{U}_2)$. Then, using $\mathbf{u}_1$, $\mathbf{u}_2$ as the new variables to be optimized, and letting $\mathbf{R} = \mathrm{diag}\{\rho_i^2\}$, the reduced optimization problem becomes

$$\begin{aligned}
\text{minimize}_{\mathbf{u}_1,\mathbf{u}_2} \quad & \sum_{i=1}^{n} \rho_i^2 \left(|H_1(i)|^2 \mathbf{u}_1(i) \right. \\
& \qquad \left. + |H_2(i)|^2 \mathbf{u}_2(i) + \rho_i^2 \right)^{-1} \\
\text{subject to} \quad & \sum_{i=1}^{n} \mathbf{u}_1(i) \le p_1, \quad \sum_{i=1}^{n} \mathbf{u}_2(i) \le p_2 \\
& \mathbf{u}_1(i) \ge 0, \quad \mathbf{u}_2(i) \ge 0 \\
& i = 1, 2, \ldots, n.
\end{aligned} \tag{3.2}$$

Introducing an auxiliary vector $\mathbf{w}$, we can transform (3.2) into the following (rotated) second-order cone program:

$$\begin{aligned}
\text{minimize}_{\mathbf{w},\mathbf{u}_1,\mathbf{u}_2} \quad & \sum_{i=1}^{n} \rho_i^2 \mathbf{w}(i) \\
\text{subject to} \quad & \sum_{i=1}^{n} \mathbf{u}_1(i) \le p_1, \quad \sum_{i=1}^{n} \mathbf{u}_2(i) \le p_2 \\
& \mathbf{w}(i) \left(|H_1(i)|^2 \mathbf{u}_1(i) \right. \\
& \qquad \left. + |H_2(i)|^2 \mathbf{u}_2(i) + \rho_i^2 \right) \ge 1 \\
& \mathbf{u}_1(i) \ge 0, \quad \mathbf{u}_2(i) \ge 0 \\
& i = 1, 2, \ldots, n.
\end{aligned} \tag{3.3}$$

There exist highly efficient (general purpose) interior point methods [11] to solve the above second-order cone program with total computational complexity of $O(n^{3.5}\log(1/\epsilon))$, where $\epsilon > 0$ is the solution accuracy. This is a significant improvement from the complexity of $O(n^{6.5}\log(1/\epsilon))$ if we solve the MMSE transceiver design problem as an SDP (2.15).

A. Interpretation of Our MMSE Design Criterion

Let $\mathbf{u}_1 = \mathrm{diag}(\mathbf{U}_1) \ge 0$, $\mathbf{u}_2 = \mathrm{diag}(\mathbf{U}_2) \ge 0$ be the diagonal transmitters designed for the two users. It is possible that some entries of $\mathbf{u}_1$, $\mathbf{u}_2$ are zero, indicating that no power are allocated to the corresponding subcarriers. For example, if $\mathbf{u}_1(i) = 0$ for some subcarrier i, then user 1 does not allocate power to this subcarrier i. If user 1 still sends information symbols along subcarrier i, then the receiver will not be able to detect any signal of user 1 on subcarrier i, resulting in loss of information symbols and a large symbol error rate.

To ensure a low symbol error rate, it is natural not to send any information symbols along a subcarrier where no power has been allocated. This results in a reduction of the symbol rate. Specifically, let I_1 (respectively, I_2) denote the set of indices i for which $\mathbf{u}_1(i) > 0$ (respectively, $\mathbf{u}_2(i) > 0$). Then, during each transmission slot, user k sends exactly one information symbol per each subcarrier indexed by I_k. Consequently, the symbol rate for user k becomes $|I_k|$ symbols/transmission, whereas the *symbol rate loss* is given by $\ell_k = n - |I_k|$. On the other hand, the total MSE is given by

$$\mathrm{MSE} = \sum_{i \in I_1^c} \mathbf{e}_1^2(i) + \sum_{i \in I_1} \mathbf{e}_1^2(i) + \sum_{i \in I_2^c} \mathbf{e}_2^2(i) + \sum_{i \in I_2} \mathbf{e}_2^2(i) \tag{3.4}$$

where I_k^c denotes the complement of I_k with respect to the set $\{1, 2, \ldots, n\}$. The second and fourth terms on the right-hand side of (3.4) represent the MSE in the transmitted symbols, and the first and third terms represent the additional MSE incurred by not transmitting on all subcarriers. Since we have assumed that $E(\mathbf{s}_i \mathbf{s}_i^H) = \mathbf{I}$

$$\begin{aligned}
\mathrm{MSE} &= (|I_1^c| + |I_2^c|) + \sum_{i \in I_1} \mathbf{e}_1^2(i) + \sum_{i \in I_2} \mathbf{e}_2^2(i) \\
&= (\ell_1 + \ell_2) + \sum_{i \in I_1} \mathbf{e}_1^2(i) + \sum_{i \in I_2} \mathbf{e}_2^2(i).
\end{aligned} \tag{3.5}$$

In this way, the total MSE can be interpreted as a sum of the symbol rate loss for each user and the MSE of the symbols that are actually transmitted. As a result, when we design a system to minimize the total MSE, we are, in fact, optimizing the combined effects of high symbol rates and high fidelity (low MSE) for the symbols that are actually transmitted. To put it in another way, even though we appear to be optimizing a mixture between rates and MSE errors, we are actually minimizing the total MSE. This is because ℓ_1 and ℓ_2 in (3.5) are not artificially chosen *a priori*; they are a consequence of the MSE based design. If the terms ℓ_1 and ℓ_2 were not taken into account or chosen differently, (3.5) would no longer represent the total MSE, and hence the "=" sign in (3.5) would be invalid.

B. Structure of Optimal Power Loading Scheme

In this subsection, we will establish some important properties for the optimal transceiver design obtained from solving

(3.3). These properties suggest, among other things, that the optimal power loading [as stipulated by (3.2)] is always achieved by an appropriate allocation of subcarriers according to the relative subchannel gains for the two users.

Theorem 3.2: Let $\mathbf{u}_1^* \geq 0, \mathbf{u}_2^* \geq 0$ be the optimal solution of (3.2). Let us define the four index sets:

$$\begin{cases} I_1 = \{i | \mathbf{u}_1^*(i) > 0, \quad \mathbf{u}_2^*(i) = 0\} \\ I_2 = \{i | \mathbf{u}_1^*(i) = 0, \quad \mathbf{u}_2^*(i) > 0\} \\ I_s = \{i | \mathbf{u}_1^*(i) > 0, \quad \mathbf{u}_2^*(i) > 0\} \\ I_u = \{i | \mathbf{u}_1^*(i) = 0, \quad \mathbf{u}_2^*(i) = 0\} \end{cases}$$

where I_1 and I_2 denote the set of subcarriers allocated to user 1 and user 2, respectively, whereas I_s and I_u denote the set of subcarriers shared and unused by the two users, respectively. Then, we have the following:

1) The four index sets I_1, I_2, I_s, and I_u form a partition of $I = \{1, 2, \ldots, n\}$.
2) For each $i \in I_1$ and $j \in I_2$, we have
$$\frac{|H_1(i)|^2}{|H_2(i)|^2} \geq \frac{|H_1(j)|^2}{|H_2(j)|^2}.$$
3) For all $i, j \in I_s$, we have
$$\frac{|H_1(i)|^2}{|H_2(i)|^2} = \frac{|H_1(j)|^2}{|H_2(j)|^2}.$$
4) For any $i \in I_u$ and any $j \in I_1 \cup I_s$, we have $|H_1(i)|^2/\rho_i^2 < |H_1(j)|^2/\rho_j^2$. Similarly, for any $i \in I_u$ and any $j \in I_2 \cup I_s$, we have $|H_2(i)|^2/\rho_i^2 < |H_2(j)|^2/\rho_j^2$.

Proof: The fact that $I = I_1 \cup I_2 \cup I_s \cup I_u$ forms a partition is obvious. By the standard optimality condition [5, Th. 9.1.1, p. 200] for (3.2), there exist two Lagrangian multipliers μ_1, μ_2 such that for all $i = 1, 2, \ldots, n$, the first equation at the bottom of the page holds. Using the definitions of the index sets I_1, I_2, I_s, and I_u, we can further refine the above optimality condition as (3.6), shown at the bottom of the page. From the first pair of relations in (3.6) we obtain

$$\frac{|H_1(i)|^2}{|H_2(i)|^2} \geq \frac{\mu_1}{\mu_2}, \quad \forall i \in I_1 \tag{3.7}$$

whereas the second pair of relations in (3.6) implies

$$\frac{|H_1(j)|^2}{|H_2(j)|^2} \leq \frac{\mu_1}{\mu_2}, \quad \forall j \in I_2. \tag{3.8}$$

Combining (3.7) with (3.8) gives

$$\frac{|H_1(i)|^2}{|H_2(i)|^2} \geq \frac{|H_1(j)|^2}{|H_2(j)|^2}, \quad \text{for all } i \in I_1 \text{ and } j \in I_2$$

which proves part 2 of the theorem. In addition, the third pair of relations in (3.6) shows that

$$\frac{|H_1(i)|^2}{|H_2(i)|^2} = \frac{\mu_1}{\mu_2}, \quad \forall i \in I_s$$

so the ratio $|H_1(i)|^2/|H_2(i)|^2$ is independent of $i \in I_s$. This proves part 3 of the theorem.

Finally, for any $i \in I_u$ and any $j \in I_1 \cup I_s$, we have from (3.6) that

$$\frac{|H_1(i)|^2}{\rho_i^2} \leq \mu_1 = \frac{|H_1(j)|^2 \rho_j^2}{\left(|H_1(j)|^2 \mathbf{u}_1^*(j) + \rho_j^2\right)^2} < \frac{|H_1(j)|^2}{\rho_j^2}$$

where in the last step, we have used the fact that $\mathbf{u}_1^*(j) > 0$. Thus, we have $|H_1(i)|^2/\rho_i^2 < |H_1(j)|^2/\rho_j^2$, as desired. Similarly, we can show $|H_2(i)|^2/\rho_i^2 < |H_2(j)|^2/\rho_j^2$ for all $i \in I_u$ and $j \in I_2 \cup I_s$. This completes the proof of the theorem. **Q.E.D.**

It is important to note from Theorem 3.2 that the optimal power loading is dependent on the *magnitude* of the subchannel gains only. This is good news for practitioners since the phases of the subchannel gains are usually more difficult to estimate. In addition, Theorem 3.2 has an intuitively appealing interpretation. From the MMSE transceiver design standpoint, we should allocate a subcarrier i to user 1 and a subcarrier j to user 2 only if

$$\frac{|H_1(i)|^2}{|H_2(i)|^2} \geq \frac{|H_1(j)|^2}{|H_2(j)|^2}.$$

In other words, the subcarriers are allocated to the two users according to the relative ratios of the subchannel gains. In

$$\begin{cases} \frac{|H_1(i)|^2 \rho_i^2}{\left(|H_1(i)|^2 \mathbf{u}_1^*(i) + |H_2(i)|^2 \mathbf{u}_2^*(i) + \rho_i^2\right)^2} \leq \mu_1, \quad \mathbf{u}_1^*(i) \left(\frac{|H_1(i)|^2 \rho_i^2}{\left(|H_1(i)|^2 \mathbf{u}_1^*(i) + |H_2(i)|^2 \mathbf{u}_2^*(i) + \rho_i^2\right)^2} - \mu_1 \right) = 0 \\ \frac{|H_2(i)|^2 \rho_i^2}{\left(|H_1(i)|^2 \mathbf{u}_1^*(i) + |H_2(i)|^2 \mathbf{u}_2^*(i) + \rho_i^2\right)^2} \leq \mu_2, \quad \mathbf{u}_2^*(i) \left(\frac{|H_2(i)|^2 \rho_i^2}{\left(|H_1(i)|^2 \mathbf{u}_1^*(i) + |H_2(i)|^2 \mathbf{u}_2^*(i) + \rho_i^2\right)^2} - \mu_1 \right) = 0 \\ \mu_1 \geq 0, \quad \mu_2 \geq 0. \end{cases}$$

$$\begin{cases} \frac{|H_1(i)|^2 \rho_i^2}{\left(|H_1(i)|^2 \mathbf{u}_1^*(i) + \rho_i^2\right)^2} = \mu_1, & \frac{|H_2(i)|^2 \rho_i^2}{\left(|H_1(i)|^2 \mathbf{u}_1^*(i) + \rho_i^2\right)^2} \leq \mu_2, & \forall i \in I_1 \\ \frac{|H_1(i)|^2 \rho_i^2}{\left(|H_2(i)|^2 \mathbf{u}_1^*(i) + \rho_i^2\right)^2} \leq \mu_1, & \frac{|H_2(i)|^2 \rho_i^2}{\left(|H_2(i)|^2 \mathbf{u}_1^*(i) + \rho_i^2\right)^2} = \mu_2, & \forall i \in I_2 \\ \frac{|H_1(i)|^2 \rho_i^2}{\left(|H_1(i)|^2 \mathbf{u}_1^*(i) + |H_2(i)|^2 \mathbf{u}_2^*(i) + \rho_i^2\right)^2} = \mu_1, & \frac{|H_2(i)|^2 \rho_i^2}{\left(|H_1(i)|^2 \mathbf{u}_1^*(i) + |H_2(i)|^2 \mathbf{u}_2^*(i) + \rho_i^2\right)^2} = \mu_2, & \forall i \in I_s \\ \frac{|H_1(i)|^2}{\rho_i^2} \leq \mu_1, & \frac{|H_2(i)|^2}{\rho_i^2} \leq \mu_2, & \forall i \in I_u. \end{cases} \tag{3.6}$$

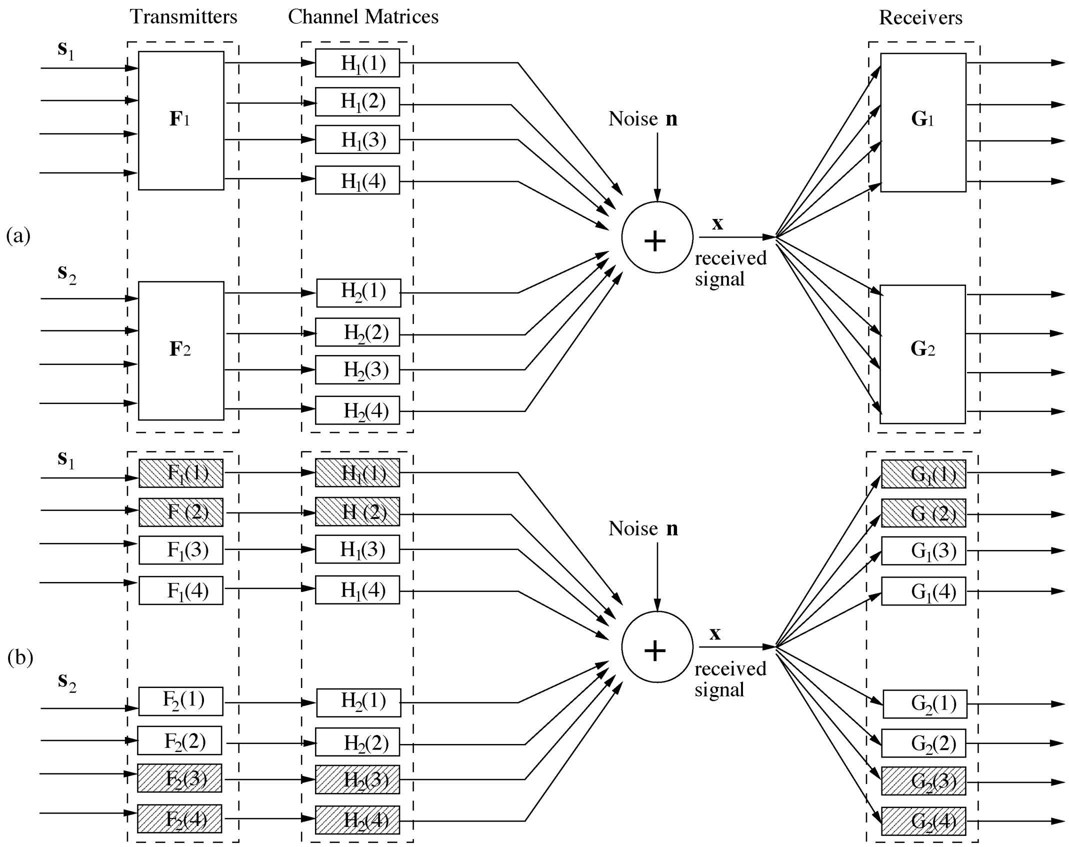

Fig. 3. (a) Structure of general linear transceiver for a multiuser cyclic-prefixed multicarrier scheme. (b) Structure of the optimal MMSE transceiver.

particular, the subcarriers for which $|H_1(i)|^2/|H_2(i)|^2$ is high should be assigned to user 1, whereas the subcarriers with small values of $|H_1(i)|^2/|H_2(i)|^2$ (or, equivalently, large values of $|H_2(i)|^2/|H_1(i)|^2$) should be assigned to user 2. For all the subcarriers that are shared by both users (i.e., for all $i \in I_s$), the subchannel gain ratio $|H_2(i)|^2/|H_1(i)|^2$ must be the same. In a fading environment, the subchannel gains $|H_1(i)|^2$, $|H_2(i)|^2$ are random (for example, Rayleigh or Rice distributed); therefore, the probability of having two equal subchannel gains is zero. This implies that from the MMSE transceiver design standpoint, *at most one subcarrier should be shared by the two users*. Of course, there may also be subcarriers in the index set I_u that are not used by either user. These subcarriers have small subchannel gain to (subcarrier) noise ratios for both users (i.e., both $|H_1(i)|^2/\rho_i^2$ and $|H_2(i)|^2/\rho_i^2$ are small), and according to Theorem 3.2, they should not be used by either user. In other words, they are useless subcarriers. Fig. 3 shows the implication of Theorem 3.2 in schematic form. In Fig. 3(a), we have the general transceiver structure for a diagonalized system, which involves full matrix precoders and equalizers. In Fig. 3(b), we have the optimized structure that consists of subcarrier allocation and power loading. The shaded boxes indicate the carriers allocated to that user. They emphasize the dramatic reduction in implementation complexity of the optimized system over the general system.

Notice that the rank of the optimally designed (diagonal) precoder matrices are given by the cardinalities of I_1 and I_2, denoted $|I_1|$ and $|I_2|$ respectively, when there is no commonly shared subcarrier ($I_s = \emptyset$). As a result, the optimized code rates for the two users are $|I_1|/n$ and $|I_2|/n$, respectively. When $|I_s| = 1$, then the code rates become $(|I_1|+1)/n$ and $(|I_2|+1)/n$.

C. Strongly Polynomial Time Algorithm for Optimal Power Loading

Theorem 3.2 completely characterizes the structure and the properties of the optimal MMSE multiple access transceiver design for an OFDM (or DMT) type system. In what follows, we will use these properties to devise an efficient algorithm to determine the optimal MMSE solution. For simplicity, we will assume that the subchannel gains as well as their ratios for the two users are distinct:

$$|H_1(i)|^2 \neq |H_1(j)|^2, \quad |H_2(i)|^2 \neq |H_2(j)|^2$$
$$\frac{|H_2(i)|^2}{|H_1(i)|^2} \neq \frac{|H_2(j)|^2}{|H_1(j)|^2}, \quad \forall i \neq j. \tag{3.9}$$

By the random nature of the subchannel gains, the above assumption is (almost) without loss of generality since it merely represents the *generic* state of the subchannels and is expected to hold with probability 1. After all, it is straightforward to install some simple control mechanism in any practical subcarrier allocation algorithm so that if the assumption (3.9) fails, the algorithm will simply find a reasonable suboptimal solution. We will also assume, again for simplicity, that the noise is white, i.e., $\mathbf{R} = \rho^2\mathbf{I}$. The extension to the case where $\mathbf{R} = \mathrm{diag}\{\rho_i^2\}$ is straightforward.

Before we proceed, we need to set up some extra notation. For convenience, let us assume that the ratios of subchannel gains are sorted:

$$\frac{|H_1(1)|^2}{|H_2(1)|^2} > \frac{|H_1(2)|^2}{|H_2(2)|^2} > \cdots > \frac{|H_1(n-1)|^2}{|H_2(n-1)|^2} > \frac{|H_1(n)|^2}{|H_2(n)|^2}. \tag{3.10}$$

Since the subchannel gain ratios are distinct (3.9), there is no loss of generality in the above ordering. In addition, we introduce two index mappings σ and τ such that

$$\sigma(j;i) = \text{the ranking of } |H_1(j)|^2 \text{ in } \left\{|H_1(1)|^2, |H_1(2)|^2 \ldots, |H_1(i)|^2\right\}, \text{ for } 1 \le j \le i \text{ and } 1 \le i \le n \tag{3.11}$$

and

$$\tau(j;i) = \text{the ranking of } |H_2(j)|^2 \text{ in } \left\{|H_2(i)|^2, |H_2(i+1)|^2 \ldots, |H_2(n)|^2\right\}, \text{ for } i \le j \le n \text{ and } 1 \le i \le n. \tag{3.12}$$

In addition, for any $1 \le j \le i \le n$, we define the index set

$$I_1(j;i) = \{\ell | \sigma(\ell;i) \le j,\ 1 \le \ell \le i\} \tag{3.13}$$

and for any $1 \le i \le j \le n$, we define the index set

$$I_2(j;i) = \{\ell | \tau(\ell;i) \le j,\ i \le \ell \le n\}. \tag{3.14}$$

In other words, $I_1(j;i)$ represents the set of the j subcarriers in $\{1, 2, \ldots, i\}$ with the largest subchannel gains for user 1, whereas $I_2(j;i)$ consists of the set of the j subcarriers in $\{i, i+1, \ldots, n\}$ with the largest subchannel gains for user 2. Notice that the cardinality of $I_1(j;i)$ and $I_2(j;i)$ is exactly equal to j. In addition, $I_1(i;i) = \{1, 2, \ldots, i\}$ and $I_2(i;i) = \{i, i+1, \ldots, n\}$. We will also need some notation and expressions for the Lagrangian multipliers when the index sets I_1, I_2 are fixed. These expressions are developed from the optimality condition (3.6). We will need to consider two cases. First, when $I_s = \emptyset$ (empty set), then the system (3.6) decouples, and the multipliers can be computed explicitly. In particular, for each index set $I \subseteq \{1, 2, \ldots, n\}$, we can solve the following system of optimality equations [obtained from (3.6)] in the variables $\mu_1(I)$ and $\{\mathbf{u}_1(\ell), \ell \in I\}$, subject to the power normalization constraints in (3.2):

$$\sum_{\ell \in I} \mathbf{u}_1(\ell) = p_1, \quad \frac{|H_1(\ell)|^2}{\left(|H_1(\ell)|^2 \mathbf{u}_1(\ell) + \rho^2\right)^2} = \mu_1(I), \quad \ell \in I.$$

The resulting solution is given by

$$\mu_1(I) = \frac{\left(\sum_{\ell \in I} |H_1(\ell)|^{-1}\right)^2}{\left(p_1 + \rho^2 \left(\sum_{\ell \in I} |H_1(\ell)|^{-2}\right)\right)^2}$$
$$\mathbf{u}_1(\ell) = \frac{1}{\sqrt{\mu_1(I)}\,|H_1(\ell)|} - \frac{\rho^2}{|H_1(\ell)|^2}, \quad \ell \in I. \tag{3.15}$$

Similarly, for each index set $J \subseteq \{1, 2, \ldots, n\}$, we can solve the following system of optimality conditions in $\mu_2(J)$ and $\{\mathbf{u}_2(\ell), \ell \in J\}$, subject to the power normalization constraints in (3.2):

$$\sum_{\ell \in J} \mathbf{u}_2(\ell) = p_2, \quad \frac{|H_2(\ell)|^2}{\left(|H_2(\ell)|^2 \mathbf{u}_2(\ell) + \rho^2\right)^2} = \mu_2(J), \quad \ell \in J.$$

The resulting solution is given by

$$\mu_2(J) = \frac{\left(\sum_{\ell \in J} |H_2(\ell)|^{-1}\right)^2}{\left(p_2 + \rho^2 \left(\sum_{\ell \in J} |H_2(\ell)|^{-2}\right)\right)^2}$$
$$\mathbf{u}_2(\ell) = \frac{1}{\sqrt{\mu_2(J)}\,|H_2(\ell)|} - \frac{\rho^2}{|H_2(\ell)|^2}, \quad \ell \in J. \tag{3.16}$$

The above expressions of $\mu_1(I)$ and $\mu_2(J)$ are the Lagrangian multipliers when $I_1 = I$, $I_2 = J$, and $I_s = \emptyset$. When I_s is a singleton, say, $\{i\}$ for some $1 \le i \le n$, then we can obtain in a similar way the expressions of multipliers denoted by $\mu_1(I, J; i)$, $\mu_2(I, J; i)$. [Notice that the multipliers now depend on both index sets I and J as well as i, since the optimality conditions (3.6) are no longer decoupled due to the subcarrier i shared by the two users.] In particular, for each pair of disjoint index sets $I, J \subseteq \{1, 2, \ldots, n\} \setminus I_s$, we have (3.17)–(3.19), shown at the bottom of the page. Here, we adopt the convention that $0^{-1} = 0$ in case $H_j(\ell) = 0$ for user j and subcarrier ℓ.

$$\begin{cases} \mu_1(I, J; i) = \dfrac{|H_1(i)|^2\left(|H_1(i)|\left(\sum_{\ell \in I \cup I_s} |H_1(\ell)|^{-1}\right) + |H_2(i)|\left(\sum_{\ell \in J \cup I_s} |H_2(\ell)|^{-1}\right) - 1\right)^2}{\left(|H_1(i)|^2 p_1 + |H_2(i)|^2 p_2 + \left(|H_1(i)|^2\left(\sum_{\ell \in I \cup I_s} |H_1(\ell)|^{-2}\right) - 1 + |H_2(i)|^2\left(\sum_{\ell \in J \cup I_s} |H_2(\ell)|^{-2}\right)\right)\rho^2\right)^2} \\ \mathbf{u}_1(\ell) = \dfrac{1}{\sqrt{\mu_1(I, J; i)}|H_1(\ell)|} - \dfrac{\rho^2}{|H_1(\ell)|^2}, \quad \ell \in I \end{cases} \tag{3.17}$$

and

$$\begin{cases} \mu_2(I, J; i) = \dfrac{H_2^2(i)\left(|H_1(i)|\left(\sum_{\ell \in I \cup I_s} |H_1(\ell)|^{-1}\right) + |H_2(i)|\left(\sum_{\ell \in J \cup I_s} |H_2(\ell)|^{-1}\right) - 1\right)^2}{\left(|H_1(i)|^2 p_1 + |H_2(i)|^2 p_2 + \left(|H_1(i)|^2\left(\sum_{\ell \in I \cup I_s} |H_1(\ell)|^{-2}\right) - 1 + |H_2(i)|^2\left(\sum_{\ell \in J \cup I_s} |H_2(\ell)|^{-2}\right)\right)\rho^2\right)^2} \\ \mathbf{u}_2(\ell) = \dfrac{1}{\sqrt{\mu_2(I, J; i)}|H_2(\ell)|} - \dfrac{\rho^2}{|H_2(\ell)|^2}, \quad \ell \in J \end{cases} \tag{3.18}$$

and

$$\mathbf{u}_1(i) = p_1 - \sum_{\ell \in I} \mathbf{u}_1(\ell), \quad \mathbf{u}_2(i) = p_2 - \sum_{\ell \in J} \mathbf{u}_2(\ell). \tag{3.19}$$

Now, we are ready to describe the new algorithm for MMSE optimal transceiver design. By Theorem 3.2, the optimal MMSE transceiver design is characterized by the four index sets I_1, I_2, I_s, and I_u. By the assumption (3.9) and the preceding discussion, the index set I_s can have at most one element. Thus, there are two cases.

Case 1: $I_s = \emptyset$. In this case, we have from Theorem 3.2 that $I_1 \subseteq \{1, \ldots, i\}$ and $I_2 \subseteq \{i+1, \ldots, n\}$. We can search for the index sets I_1 and I_2 iteratively. In particular, notice that the subcarriers in $\{1, \ldots, i\}$ are either in I_1 or in I_u. Moreover, by part 4 of Theorem 3.2, with $\mathbf{R} = \rho^2 \mathbf{I}$, $|H_1(i)|^2 > |H_1(j)|^2$ whenever $i \in I_1$ and $j \in I_u$. Thus, if there are j $(1 \le j \le i)$ subcarriers in $\{1, 2, \ldots, i\}$ being allocated and used by user 1 (i.e., I_1 has j subcarriers), then these j subcarriers must have the j largest subchannel gains (for user 1) among the subcarriers in $\{1, 2, \ldots, i\}$. Consequently, $I_1 = I_1(j; i)$ [see the definition (3.13) above]. This implies that to search for the index set I_1 in $\{1, \ldots, i\}$, we only need to consider the following i possibilities:

$$I_1(1;i),\ I_1(2;i), \ldots,\ I_1(i;i). \tag{3.20}$$

Similarly, there are only $n-i-1$ possibilities when searching for I_2 in $\{i+1, \ldots, n\}$, namely

$$I_2(1; n-i-1), I_2(2; n-i-1), \ldots, I_2(n-i-1; n-i-1). \tag{3.21}$$

Now, for each I given by (3.20) and each J given by (3.21), we can compute $\mu_1(I)$, $\{\mathbf{u}_1(\ell), l \in I\}$, and $\mu_2(J)$, $\{\mathbf{u}_2(\ell), l \in J\}$ according to (3.15) and (3.16), respectively. Once these values are computed, we can check if the conditions

$$\mathbf{u}_1(\ell) \ge 0, \quad \ell \in I; \quad \mathbf{u}_2(\ell) \ge 0, \quad \ell \in J \tag{3.22}$$

and

$$\begin{aligned} \frac{|H_2(\ell)|^2}{\left(|H_1(\ell)|^2 \mathbf{u}_1(\ell) + \rho^2\right)^2} &\le \mu_2(J), \quad \forall \ell \in I \\ \frac{|H_1(\ell)|^2}{\left(|H_2(\ell)|^2 \mathbf{u}_2(\ell) + \rho^2\right)^2} &\le \mu_1(I), \quad \forall \ell \in J \end{aligned} \tag{3.23}$$

are satisfied. If they are, then we have found the index sets $I_1 = I$ and $I_2 = J$ and $I_s = \emptyset, I_u = \{1, 2, \ldots, n\}/(I_1 \cup I_2 \cup I_s)$ together with a set of multipliers $\mu_1(I)$, $\mu_2(J)$ and power levels $\mathbf{u}_1(\ell), \ell \in I$, and $\mathbf{u}_2(\ell), \ell \in J$ that satisfy the optimality condition (3.6), and the search terminates. If (3.22) and (3.23) are not satisfied, we search for a different pair of I and J, and the algorithm continues until either the search terminates successfully with a set of optimal index sets I_1, I_2, I_s, and I_u, or all possible pairs of I and J, as specified by (3.20) and (3.21), have been exhausted. In the latter case, we will increment i, and the procedure will be repeated. Notice from (3.15) that (3.22) is equivalent to

$$\min_{\ell \in I} |H_1(\ell)| \ge \rho^2 \sqrt{\mu_1(I)}, \quad \min_{\ell \in J} |H_2(\ell)| \ge \rho^2 \sqrt{\mu_2(J)} \tag{3.24}$$

and by the definition of $\mu_1(I)$, $\mu_2(J)$, (3.23) is equivalent to

$$\max_{\ell \in I} \frac{|H_2(\ell)|^2}{|H_1(\ell)|^2} \le \frac{\mu_2(J)}{\mu_1(I)}, \quad \max_{\ell \in J} \frac{|H_1(\ell)|^2}{|H_2(\ell)|^2} \le \frac{\mu_1(I)}{\mu_2(J)}. \tag{3.25}$$

With the help of (3.10) and the index mapping σ and τ, the above two conditions can be checked easily in $O(1)$ operations (assuming $\mu_1(I)$ and $\mu_2(J)$ are known).

Case 2: $I_s = \{i\}$, for some $1 \le i \le n$. In this case, we have from Theorem 3.2 that $I_1 \subseteq \{1, \ldots, i-1\}$ and $I_2 \subseteq \{i+1, \ldots, n\}$. By a similar argument as in Case 1, we can conclude that I_1 has $i-1$ possibilities, which are given by (3.20), and I_2 has $n-i-1$ possibilities given by (3.21). For each I and J specified, respectively, by (3.20) and (3.21), we can compute $\mu_1(I, J; i)$, $\{\mathbf{u}_1(\ell), \ell \in I\}$, and $\mu_2(I, J; i)$, $\{\mathbf{u}_2(\ell), \ell \in J\}$, $\mathbf{u}_1(i)$, $\mathbf{u}_2(i)$ according to (3.17)–(3.19), respectively. Once these values are computed, we can check if the conditions

$$\mathbf{u}_1(\ell) \ge 0, \quad \ell \in I \cup \{i\}; \quad \mathbf{u}_2(\ell) \ge 0, \quad \ell \in J \cup \{i\} \tag{3.26}$$

and

$$\begin{aligned} \frac{|H_2(\ell)|^2}{\left(|H_1(\ell)|^2 \mathbf{u}_1(\ell) + \rho^2\right)^2} &\le \mu_2(I, J; i), \quad \forall \ell \in I \\ \frac{|H_1(\ell)|^2}{\left(|H_2(\ell)|^2 \mathbf{u}_2(\ell) + \rho^2\right)^2} &\le \mu_1(I, J; i), \quad \forall \ell \in J \end{aligned} \tag{3.27}$$

are satisfied. If they are, then we have found the index sets $I_1 = I$, $I_2 = J$, and $I_s = i$, $I_u = \{1, 2, \ldots, n\}/(I_1 \cup I_2 \cup I_s)$ together with a set of power levels $\mathbf{u}_1(\ell)$, $\ell \in I \cup \{i\}$, and $\mathbf{u}_2(\ell)$, $\ell \in J \cup \{i\}$ that satisfy the optimality condition (3.6), and the search terminates. If (3.26) and (3.27) are not satisfied, we search for a different pair of I and J, and the algorithm continues until either the search terminates successfully with a set of optimal index sets I_1, I_2, I_s, and I_u, or all possible pairs of I and J, as specified by (3.20) and (3.21) have been exhausted. In the latter case, we will increment i, and the procedure will be repeated. Similar to Case 1, (3.26) is equivalent to

$$\begin{aligned} \min_{\ell \in I} |H_1(\ell)| &\ge \rho^2 \sqrt{\mu_1(I, J; i)} \\ \min_{\ell \in J} |H_2(\ell)| &\ge \rho^2 \sqrt{\mu_2(I, J; i)} \end{aligned} \tag{3.28}$$

and by the definition of $\mu_1(I, J; i)$, $\mu_2(I, J; i)$, (3.27) is equivalent to

$$\max_{\ell \in I} \frac{|H_2(\ell)|^2}{|H_1(\ell)|^2} \le \frac{\mu_2(I, J; i)}{\mu_1(I, J; i)}, \quad \max_{\ell \in J} \frac{|H_1(\ell)|^2}{|H_2(\ell)|^2} \le \frac{\mu_1(I, J; i)}{\mu_2(I, J; i)}. \tag{3.29}$$

Again, with the help of (3.10) and the index mapping σ and τ, the above two conditions can be checked easily in $O(1)$ operations (assuming $\mu_1(I, J; i)$ and $\mu_2(I, J; i)$ are known).

Notice that the main computational steps in the above search algorithm are divided in three parts:

1) computing the index mappings σ and τ [cf. (3.11) and (3.12)];
2) computing the multipliers $\mu_1(I)$, $\mu_2(J)$ [or $\mu_1(I, J; i)$, $\mu_2(I, J; i)$] for each choice of I and J according to (3.15)–(3.19);
3) checking the validity of the conditions (3.22)–(3.27).

Part 1) can be carried out efficiently via any of the classical sorting algorithms and certainly takes no more than $O(n^2)$ arithmetic operations. Part 2) takes $O(n^3)$ operations since, as i varies, there can be in total at most $O(n^3)$ different pairs of candidate index sets I and J of the form (3.20) and (3.21), and for each fixed I and J, computing the multipliers takes at most $O(1)$ operations. The latter is because, as I changes from $I_1(k-1;i-1)$ to $I_1(k;i-1)$ [or as J changes from $I_2(k;n-i-1)$ to $I_2(k-1;n-i-1)$], we can recursively update the multipliers in $O(1)$ operations using (3.15)–(3.19). Part 3) takes $O(n^3)$ as well, since for each I and J pair checking the conditions (3.22)–(3.27), which is equivalent to checking (3.24), (3.25) and (3.28), (3.29), takes $O(1)$ operations, and there are at most $O(n^3)$ different I and J pairs.

In summary, the algorithm has a strongly polynomial (i.e., independent of solution accuracy ϵ) complexity of $O(n^3)$, and it terminates finitely with an exact optimal MMSE transceiver design. This is in contrast to the interior point algorithm for solving, say, the formulation (3.2), which is iterative and terminates with an approximate solution in $O(n^{3.5}\log(1/\epsilon))$ arithmetic operations, where ϵ is the solution accuracy.

The schematic description of the algorithm is given below.

An $O(n^3)$ Algorithm for Computing the MMSE Transceiver Design

Step 1) *Index mappings*. Use a sorting algorithm to compute the index mappings σ and τ according to (3.11) and (3.12).

Step 2) *Iteration* i $(1 \leq i \leq n)$. For each choice of I and J given by (3.20) and (3.21):

2.1. Consider the case $I_s = \emptyset$:
- Compute the multipliers $\mu_1(I)$ and $\mu_2(J)$ according to (3.15) and (3.16);
- Check if the conditions (3.24) and (3.25) are valid. If yes, set $I_1 = I$, $I_2 = J$, $I_s = \emptyset$ and $I_u = \{1,2,\ldots,n\}/(I_1 \cup I_2 \cup I_s \cup I_u)$, and terminate the algorithm. Else, continue to step 2.2.

2.2 Consider the case $I_s = \{i\}$:
- Compute the multipliers $\mu_1(I,J;i)$ and $\mu_2(I,J;i)$ according to (3.17) and (3.18);
- Check if the conditions (3.28), (3.29) are valid. If yes, set $I_1 = I$, $I_2 = J$, $I_s = \{i\}$ and $I_u = \{1,2,\ldots,n\}/(I_1 \cup I_2 \cup I_s \cup I_u)$, and terminate the algorithm. Else, continue to step 3.

Step 3 *Repeat*. Return to Step 2 with $i := i+1$.

In practical situations, we can expect the above subcarrier and power allocation algorithm to be much faster than $O(n^3)$, since it is possible that in the optimal design, i) no subcarrier is shared by the users ($I_s = \emptyset$), and ii) no subcarrier is wasted ($I_u = \emptyset$), or the set of "bad" subcarriers can be fixed in advance. If this is the case, then we will only need to search for the index sets I_1 and I_2, which forms a partition of $\{1,2,\ldots,n\}$. It follows from part 2) of Theorem 3.2 that $I_1 = I(i-1;i-1)$ and $I_2 = (n-i-1;n-i-1)$ for some $1 \leq i \leq n$. Thus, there are only n possible choices for I_1 and I_2. With this simplification, the resulting search procedure will take only $O(n)$ operations.

We remark that (single-user) DMT transmissions (such as DSL or digital cable TV) entail power loading and bit loading. Power loading is achieved by varying the amplitudes of different subcarriers. However, it is not known how one should optimally allocate power and subcarriers in a multiple access communication system. Some heuristic subcarrier/power allocation schemes (such as cyclic allocation) have been proposed in the literature; see, e.g., [21]. The work reported in this section provides means to achieve optimal power/subcarrier allocation in the MMSE sense for a two-user communication system.

IV. Multiple-User Case

So far, we have only presented results for the two-user case. It is possible to extend some of our results to the general m-user case. In particular, the formulations in Section II and the analysis therein as well as the diagonal designs can all be generalized to the general m-user case. In this section, we will state (mostly without proofs) the type of extensions that can be made in the general m-user case.

Consider the general m-user vector multiple access communication system (see Fig. 4) modeled by

$$\mathbf{x} = \mathbf{H}_1\mathbf{F}_1\mathbf{s}_1 + \mathbf{H}_2\mathbf{F}_2\mathbf{s}_2 + \cdots + \mathbf{H}_m\mathbf{F}_m\mathbf{s}_m + \mathbf{n} \tag{4.1}$$

where $\mathbf{s}_i$ and $\mathbf{H}_i$ are the ith user's signal and channel matrix, respectively, $\mathbf{F}_i$ is the ith precoder matrix to be designed, and $E(\mathbf{n}\mathbf{n}^H) = \mathbf{R}$. Let $\mathbf{G}_i$ be the linear MMSE matrix equalizer at the ith receiver, and generate the estimate of $\mathbf{s}_i$ by quantizing $\mathbf{G}_i\mathbf{x}$, according to the alphabet of $\mathbf{s}_i$, e.g., for BPSK

$$\hat{\mathbf{s}}_i = \mathrm{sign}(\mathbf{G}_i\mathbf{x}), \quad i = 1,2,\ldots,m.$$

Let $\mathbf{e}_i = \mathbf{s}_i - \mathbf{G}_i\mathbf{x}$ denote the error at the output of the ith equalizer. It can be shown (similar to the analysis in Section II) that the total MSE is given by

$$\mathrm{tr}\left(E\left(\mathbf{e}_1\mathbf{e}_1^H\right) + \cdots + E\left(\mathbf{e}_m\mathbf{e}_m^H\right)\right) = \mathrm{tr}\left(\left(\mathbf{H}_1\mathbf{F}_1\mathbf{F}_1^H\mathbf{H}_1^H + \cdots + \mathbf{H}_i\mathbf{F}_i\mathbf{F}_i^H\mathbf{H}_i^H + \mathbf{R}\right)^{-1}\mathbf{R}\right) + (m-1)n.$$

Let us introduce a set of new matrix variables $\mathbf{U}_i = \mathbf{F}_i\mathbf{F}_i^H$. Then, the power constrained optimal MMSE transmitter design problem can be described as

$$\begin{aligned} \text{minimize}_{\mathbf{U}_1,\ldots,\mathbf{U}_m} \quad & \mathrm{tr}\left(\left(\mathbf{H}_1\mathbf{U}_1\mathbf{H}_1^H + \cdots + \mathbf{H}_m\mathbf{U}_m\mathbf{H}_m^H + \mathbf{R}\right)^{-1}\mathbf{R}\right) \\ \text{subject to} \quad & \mathrm{tr}(\mathbf{U}_i) \leq p_i, \quad \mathbf{U}_i \succeq \mathbf{0} \\ & i = 1,\ldots,m. \end{aligned} \tag{4.2}$$

Using the auxiliary matrix variable $\mathbf{W} = (\mathbf{H}_1\mathbf{F}_1\mathbf{F}_1^H\mathbf{H}_1^H + \cdots + \mathbf{H}_i\mathbf{F}_i\mathbf{F}_i^H\mathbf{H}_i^H + \mathbf{R})^{-1}$ and a Schur complement argument analogous to that in Section II, we can rewrite (4.2) as the SDP formulation of the MMSE transceiver design problem in (4.3), shown at the bottom of the next page. This SDP formulation makes it possible to solve the optimal transmitter design problem using the highly efficient interior point methods for arbitrary channel matrices $\mathbf{H}_1, \mathbf{H}_2, \ldots, \mathbf{H}_m$ and noise correlation matrix $\mathbf{R}$. The total computational complexity of this approach is $O((mn)^{6.5}\log(1/\epsilon))$ [19]. Once the optimal solutions

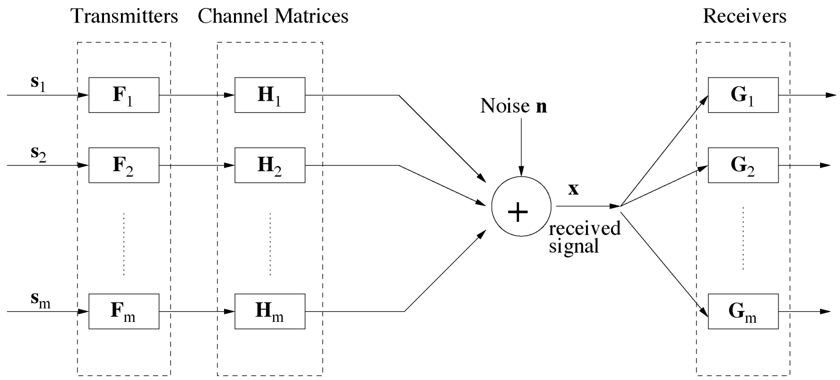

Fig. 4. Multiuser multiple access system.

$\mathbf{U}_1, \ldots, \mathbf{U}_m$ are determined from solving (4.3), we can first factorize (using, e.g., Cholesky factorization) these matrices as

$$\mathbf{U}_i = \mathbf{F}_i\mathbf{F}_i^H, \quad i = 1, 2, \ldots, m$$

for some transmitter matrices $\mathbf{F}_i$, and then, compute the corresponding optimal MMSE equalizers as

$$\mathbf{G}_i = \mathbf{F}_i^H\mathbf{H}_i^H\left(\mathbf{H}_1\mathbf{U}_1\mathbf{H}_1^H + \cdots + \mathbf{H}_m\mathbf{U}_m\mathbf{H}_m^H + \mathbf{R}\right)^{-1}.$$

In the case, where the channel matrices are diagonal (e.g., in the cyclic-prefixed GMC-CDMA scheme of [21]) and $\mathbf{R} = \text{diag}\{\rho_i^2\}$, it is possible to simplify (4.3) substantially. In particular, it can be shown that the optimal transmitter matrices are also diagonal. As a result, the SDP formulation (4.3) can be simplified to the following rotated SOCP formulation:

$$\begin{aligned}
\text{minimize}_{\mathbf{w},\mathbf{u}_1,\ldots,\mathbf{u}_m} \quad & \sum_{i=1}^{n}\rho_i^2\mathbf{w}(i)\\
\text{subject to} \quad & \sum_{i=1}^{n}\mathbf{u}_j(i) \le p_j, \quad j = 1, 2, \ldots, m\\
& \mathbf{w}(i)\left(|H_1(i)|^2\,\mathbf{u}_1(i) + \cdots\right.\\
& \qquad \left. + |H_m(i)|^2\,\mathbf{u}_m(i) + \rho_i^2\right) \ge 1\\
& \mathbf{u}_j(i) \ge 0, \quad i = 1, 2, \ldots, n\\
& \qquad\qquad j = 1, 2, \ldots, m.
\end{aligned} \tag{4.4}$$

There exist highly efficient (general-purpose) interior point methods (e.g., [11]) to solve the above SOCP with $O((mn)^{3.5}\log(1/\epsilon))$ complexity, where ϵ is the solution accuracy.

It is possible to characterize the optimal power loading [as stipulated by the formulation (4.4)] in much the same way as the two-user case. Indeed, it can be shown that the optimal power allocation is always achieved by an appropriate allocation of subcarriers according to the relative subchannel gains for the users.

Theorem 4.1: Let $\mathbf{u}_i^* \ge 0, i = 1, 2, \ldots, m$ be the optimal solution of (4.4). Let us define the index sets, shown at the bottom of the page, where I_ℓ denotes the set of subcarriers allocated to user ℓ, whereas I_s denotes the set of subcarriers shared by at least two users, and I_u is the set of subcarriers not used by any user. Then, we have the following.

1) The index sets I_ℓ, I_s, and I_u form a partition of $I = \{1, 2, \ldots, n\}$.
2) For each $i \in I_\ell$ and $j \in I_k$ ($\ell \ne k$), we have
$$\frac{|H_\ell(i)|^2}{|H_k(i)|^2} \ge \frac{|H_\ell(j)|^2}{|H_k(j)|^2}.$$
3) Suppose $i, j \in I_s$, and they are shared by users ℓ and k; then
$$\frac{|H_\ell(i)|^2}{|H_k(i)|^2} = \frac{|H_\ell(j)|^2}{|H_k(j)|^2}.$$
4) For any $i \in I_u$ and any subcarrier j used by user ℓ, we have $|H_\ell(i)|^2/\rho_i^2 < |H_\ell(j)|^2/\rho_j^2$.

$$\begin{aligned}
\text{minimize}_{\mathbf{W},\mathbf{U}_1,\cdots,\mathbf{U}_m} \quad & \text{tr}\,(\mathbf{WR})\\
\text{subject to} \quad & \text{tr}(\mathbf{U}_i) \le p_i, \quad \mathbf{U}_i \succeq \mathbf{0}, \quad i = 1, 2, \ldots, m\\
& \begin{bmatrix} \mathbf{W} & \mathbf{I}\\ \mathbf{I} & \mathbf{H}_1\mathbf{U}_1\mathbf{H}_1^H + \cdots + \mathbf{H}_m\mathbf{U}_m\mathbf{H}_m^H + \mathbf{R}\end{bmatrix} \succeq 0.
\end{aligned} \tag{4.3}$$

$$\begin{cases}
I_\ell = \{i \mid \mathbf{u}_\ell^*(i) > 0, \quad \mathbf{u}_k^*(i) = 0, \quad \text{for all } k \ne \ell \text{ and } 1 \le k \le m\}, \qquad \ell = 1, \ldots, m\\
I_s = \{i \mid \mathbf{u}_\ell^*(i) > 0, \quad \mathbf{u}_k^*(i) > 0, \quad \text{for some } \ell \ne k \text{ and } 1 \le \ell, k \le m\}\\
I_u = \{i \mid \mathbf{u}_\ell^*(i) = 0, \quad \text{for all } 1 \le \ell \le m\}
\end{cases}$$

The proof of Theorem 4.1 can be modeled after that of Theorem 3.2. In practice, we (almost) always have

$$|H_\ell(i)|^2 \neq |H_\ell(j)|^2, \quad \frac{|H_\ell(i)|^2}{|H_k(i)|^2} \neq \frac{|H_\ell(j)|^2}{|H_k(j)|^2}$$
$$\forall\, i \neq j,\ \ell \neq k \text{ and } 1 \leq i,\ j \leq n,\ 1 \leq \ell,\ k \leq m. \quad (4.5)$$

Under (4.5), part 3) of Theorem 4.1 implies that *each pair of users can share at most one subcarrier*, even though a subcarrier can be shared by *any* number of users. As a result, we can bound the size of I_s by $m(m-1)/2$. It is not clear if one can use Theorem 4.1 to directly design a fast combinatorial algorithm for optimal subcarrier and power allocation. In the two-user case, this has been done in Section II. Without such a direct algorithm, we will need to use interior point algorithms to solve the second-order cone program (4.4) to determine the optimal subcarrier and power allocation.

V. EXAMPLES

In this section, we demonstrate the effectiveness of our method through three examples.

Example 1: To demonstrate the power loading performed by formulation (3.3) (or, equivalently, by the combinatorial algorithm in Section III-C), we consider a two-user scenario in which each user encounters a three-tap channel. The impulse response of the channel for user one is $[0.6026+\mathrm{j}0.4191, 0.2779+\mathrm{j}0.1987, 0.1578-\mathrm{j}0.1938]$, where $\mathrm{j}=\sqrt{-1}$, and the impulse response of the channel for user two is $[0.0390-\mathrm{j}0.2746, -0.1394+\mathrm{j}0.0643, 0.4956+\mathrm{j}0.1172]$. Each user employs multicarrier modulation with 32 subcarriers and a cyclic prefix of length 2, and the noise is white with $\mathbf{R}=\rho^2\mathbf{I}$. The users transmit the same power $p_1=p_2$, and the block SNR p_i/ρ^2 was chosen to be low (5 dB) in order to enhance the visual clarity of the figure. The results of the multiuser MMSE power loading algorithm are shown in Fig. 5 in a form reminiscent of "waterfilling." (The height of each stem denotes the power allocated to that subcarrier.) Note that in this scenario, 19 subcarriers have been allocated to user 1 alone, 12 to user 2 alone, and subcarrier 25 is shared. In addition, note that subcarriers 5–7 are allocated to user 2, even though $|H_1(i)| > |H_2(i)|$ for $i = 5$, 6, and 7. This illustrates the fact that the ratios of the subchannel gains determine the optimal MMSE subcarrier allocation and that intuitively reasonable, but *ad hoc*, subcarrier assignment schemes can be suboptimal in the MSE sense. We also point out that over certain groups of subcarriers that are allocated to one user, the subcarrier power allocation exhibits the "smile" shape observed for single-user MMSE power loading [16, p. 198]. □

Example 2: In this example, we compare the performance of the jointly optimal MMSE transceiver with that of an orthogonal frequency division multiple access (OFDMA) scheme that does not require channel state information (CSI) to design the transmitter and that of a scheme in which CSI is used to design MMSE transceivers on a user-by-user basis. The scenario is a multiple access scheme with 16 active users. Each user encounters a three-tap (frequency-selective) Rayleigh channel in which each tap is a zero-mean complex Gaussian random variable with variance 0.5 per dimension. The noise is white with $\mathbf{R}=\rho^2\mathbf{I}$.

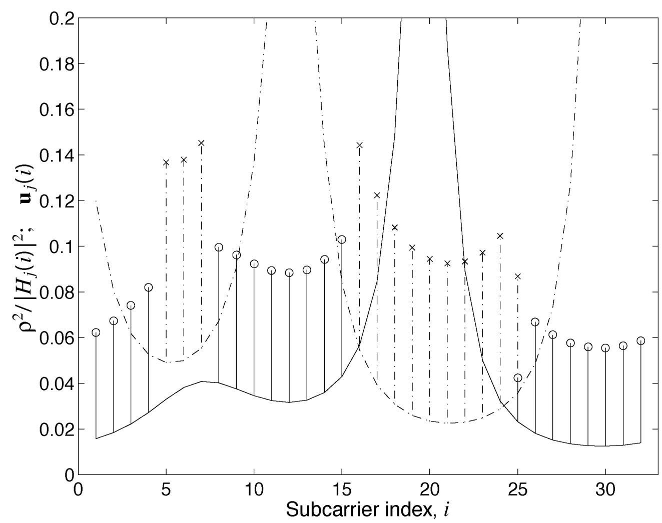

Fig. 5. Multiuser power allocation for Example 1. The curves are $\rho^2/|H_j(i)|^2$ for (solid) $j=1$ and (dash-dot) $j=2$. The stems are of length $\mathbf{u}_j(i)$ (where this is nonzero) for $j=1$ ("o") and $j=2$ ("×").

Each user employs a multicarrier modulation scheme with antipodal signalling. There are 128 available subcarriers that are to be allocated amongst the users and power loaded according to the following schemes.

1) **OFDMA**: In the OFDMA scheme, the jth user is allocated subcarriers with frequencies $2\pi(j-1+16(i-1))/128$, $i=1,2,\ldots,8$, each of which is allocated the same power (as no CSI is used).
2) **Individually MMSE power-loaded OFDMA**: In this scheme, each user is allocated the same subcarriers as in the OFDMA scheme but knows the (magnitude) gain of each of its allocated subchannels. Since each user knows these gains, it can perform (single-user) optimal MMSE power loading over these subchannels. Although that can be done with a single-user version of the SOCP in (3.3), an analytic expression is available [13], [15].
3) **Multiuser MMSE power loaded OFDMA**: In this case, we use the SOCP formulation (3.3) to jointly optimize both the number and placement of the subcarriers allocated to each user and the power loading for each subcarrier. The SOCP was solved using the `SeDuMi` [17] toolbox for MATLAB. On average, this required around half a second of CPU time on an 800 MHz Pentium III workstation. For the subcarriers that are not shared, we simply allocate one bit for the subcarrier, as in schemes 1 and 2. However, a little more care is needed to deal with subcarriers that are shared, because (diagonal) linear detection can reliably detect at most one symbol per subcarrier. In order to avoid having to implement more sophisticated (and computationally expensive) detection for the shared subcarriers, each shared subcarrier was allocated to the user with the largest received signal power $|H_j(i)|^2\mathbf{u}_j(i)$, and that user alone. Again, one bit was allocated to each such subcarrier. The power loading for the other users that had previously shared the subcarrier can then be recalculated by applying the individual MMSE

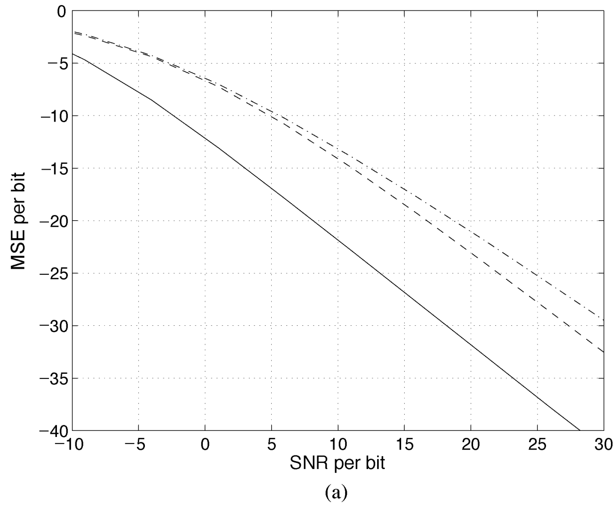

Fig. 6. Performance comparison between the three methods of Example 2. (Dash-dot) OFDMA. (Dashed) Individual power loading. (Solid) Multiuser power loading. (a) MSE per bit. (b) Bit error rate.

power loading method in scheme 2 to the subcarriers that are assigned to that user alone. An approximation of this reweighting that was implemented in the simulation is to simply rescale the power allocated to the unshared subcarriers so that the transmitted power bound is reached. (The performance of a scheme with this approximation is indistinguishable from one with the full recalculation at the scale of Fig. 6.) For a given block, the number of subchannels assigned to each user, and hence the data rate for that user, depends of the channel realization, but the average rate is 8 bits per block.

In our simulations, we computed the average MSE of the transmitted bits and the bit error rate (BER) for different SNRs for the three schemes above for a scenario in which each user was transmitting with the same power $p_j = p$. The averages were calculated over 100 independent channel realizations with 1000 blocks being transmitted per channel realization. The results are plotted in Fig. 6 against SNR per bit. For the OFDMA and individually power loaded OFDMA schemes, the SNR per bit is $p/(8\rho^2)$, and for the multiuser power loaded OFDMA scheme, the average SNR per bit is

$$\frac{p}{(16N_r)} \sum_{M=1}^{16} \sum_{k=1}^{N_r} \frac{1}{|I_{m,k}|} \approx \frac{p}{(8\rho^2)}$$

where $|I_{m,k}|$ is the number of subcarriers assigned to user m for the kth channel realization, and N_r is the number of channel realizations. It is clear from Fig. 6 that the multiuser MMSE scheme provides a significant reduction in MSE per bit over both OFDMA and individually power loaded OFDMA and a substantial "coding gain" (around 7 dB) over a broad range of BERs. □

Example 3: Our MMSE transceiver design technique has been developed under the assumption that the channel models used in the design were precise. In this example, we demonstrate that in the scenario of Example 2, our design technique is quite robust to mismatch in the design models. In order to focus on the effects of channel mismatch in the design, we

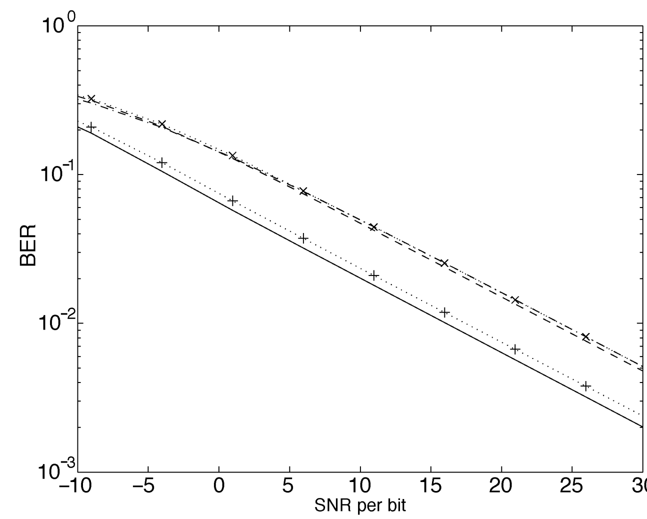

Fig. 7. Performance comparison of the three methods of Example 2 in the presence of the mismatched design models in Example 3. Legend—Solid and dotted with "+" multiuser MMSE power-loaded OFDMA with precise and mismatched design models, respectively. Dashed and dotted with "×." Individually MMSE power-loaded OFDMA with precise and mismatched design models, respectively. Dash-dot: OFDMA.

assume that when the data is detected, the receiver has a precise channel model. However, the transmitters are designed using the following set of mismatched impulse responses of the users' channels:

$$\tilde{h}_j[n] = (1+\alpha_j[n])\,\mathrm{Re}\,(h_j[n]) + \mathrm{j}\,(1+\beta_j[n])\,\mathrm{Im}\,(h_j[n]) \tag{5.1}$$

where $\mathrm{j} = \sqrt{-1}$, $h_j[n]$ is the actual impulse response of the jth user's channel, $\alpha_j[n]$ and $\beta_j[n]$ are independent zero-mean white Gaussian processes of standard deviation 0.5, and $\mathrm{Re}(\cdot)$ and $\mathrm{Im}(\cdot)$ denote the real and imaginary parts, respectively. That is, the channel models used in the design have a Gaussian relative error with a standard deviation of 50%. This represents quite a severe mismatch.

The BER curves for the individually MMSE power-loaded and multiuser MMSE power-loaded OFDMA schemes with these mismatched design models are compared with those for the precise design model in Fig. 7. The curves for the precise design model also appeared in Fig. 6(b). Of course, the OFDMA scheme is unaffected by the quality of the design model, as its design is channel independent. It is clear from the performance of the multiuser MMSE power-loaded OFDM scheme in Fig. 7 that our design scheme is quite robust to the rather large mismatch in the design models. □

VI. Concluding Remarks

We have presented several convex formulations and efficient algorithms for MMSE transceiver optimization for multiple access through ISI channels. The work reported in this paper clearly demonstrates the potential of applying convex optimization techniques in the design and management of modern communication systems. While the initial formulation of the transceiver design problems turns out to be nonconvex (thus difficult to solve), we have succeeded in transforming the problem into an equivalent convex second-order cone program that can be efficiently solved using general purpose interior point codes (e.g., `SeDuMi` [17]). Our initial computer simulations verify that our optimal power loading/subcarrier allocation scheme indeed offers superior performance over the standard (but *ad hoc*) subcarrier allocation schemes.

Throughout our development, we assumed that the exact channel state information is known. This assumption can be quite realistic in multiuser approaches to digital subscriber line (DSL) systems where the channel characteristics are essentially constant. It is also a realistic assumption in certain quasistatic wireless applications where reasonable channel state estimates can be obtained by use of training sequences. Furthermore, our simulations have shown that the system performance is rather insensitive to transceivers designed using inexact channel estimates. This is because the key component of the design is the subcarrier selection rather than the power allocated to the selected subcarriers, and our subcarrier selection scheme is robust to channel estimation error.

There are several possible extensions one can pursue. For example, our approach easily generalizes to the case where each user has multiple transmitting antennae and the base station has multiple receiving antennae. In this multi-input-multi-output case, each second-order cone constraint in (3.3) becomes an LMI of the size of the number of receive antennas with a matrix variable of the size of the number of transmit antennas. In addition, we are exploring other important system design issues such as quality of service in our formulation. We plan to report these generalizations in subsequent work.

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