Weighted-Sum-Rate Maximization in Certain Half-Duplex Cooperative Systems

Wessam Mesbah, *Member, IEEE*, and Timothy N. Davidson, *Member, IEEE*

***Abstract*—In half-duplex cooperative systems the problem of finding the jointly optimal power and channel resource allocation that maximizes the weighted sum rate (WSR) can be difficult to solve. Algorithms to solve that problem are provided herein for two classes of orthogonal half-duplex systems. For a class of systems in which the set of rates that can be achieved without time sharing is convex, the WSR problem is decomposed into a direct search in which a target rate problem is solved at each step. The target rate problem involves maximizing one of the rates subject to target values for the others, and can often be efficiently solved. For a subclass of those systems, the notion of perspective is used to transform the WSR problem into an efficiently-solvable convex optimization problem. Numerical examples involving a multiple access relay system and a pairwise cooperation scheme show that joint optimization can result in significantly larger weighted sum rates than optimization over the powers alone with a fixed channel resource allocation.**

***Index Terms*—Resource allocation; cooperative and relay systems; weighted sum rate; convex optimization.**

I. INTRODUCTION

BY providing the opportunity for the nodes in a network to cooperate with each other, or with dedicated relay nodes, cooperative communication systems have the potential to improve the quality of service that can be offered; e.g., [1]–[4]. A convenient framework for cooperation is that in which the nodes operate in a half-duplex fashion and the messages from each user are transmitted over orthogonal channels. However, the extent of the quality of service gains that can be obtained from such a framework is dependent on the appropriate allocation of power (e.g., [5]), and of the resources provided by the channel, such as time and bandwidth; e.g., [6]. In this paper, we consider a scenario in which the data traffic is tolerant of rate variations. Our goal is to find the jointly optimal power and channel resource allocations that maximize a weighted sum of the achievable rates. For most cooperative systems, the natural formulation of that problem is not convex and can be difficult to solve. We derive algorithms for solving this problem for two classes of orthogonal half-duplex systems.

First, we consider the class in which the region of rates that can be achieved by power and resource allocation, without time sharing, is convex. For these systems, the weighted sum rate (WSR) problem is decomposed into a convex outer problem and an inner "target-rate" problem, in which one seeks the joint allocation that maximizes the achievable rate of one node subject to specified target rates for the other nodes being achievable. We show that the outer problem is convex, and the inner problem can be efficiently solved in a number of cases, e.g., [7]–[9]. Hence, the globally optimal power and resource allocation can be found using a provably-convergent direct search method [10], in which the inner problem is solved at each step. In the two user case, one can use a golden-section search [11, p. 90]. The target rate approach has previously proven to be an effective way to select the operating point of some orthogonal half-duplex systems in scenarios in which the data traffic from some of the nodes has minimum rate requirements [7]–[9], but the approaches therein do not yield effective algorithms for maximizing the weighted sum rate.

A subclass of the above class of systems is that in which the achievable rates are concave functions of the powers. For such systems, if the channel resource allocation is fixed, the power allocation that maximizes the weighted sum rate can be efficiently found; e.g., [5]. However, our goal is to obtain an efficient algorithm for joint optimization over the power and channel resource allocations. Although the direct-search algorithm described above could be applied, we take a different approach and use the notion of perspective to show that the WSR problem can be transformed into a convex optimization problem that can be efficiently solved.

II. SYSTEM MODEL

In order to state the proposed algorithms in a broad context, we will consider a rather abstract system model. Examples of systems admitted by this model are provided in Section V. Consider a K-user multiple access system that employs half-duplex relaying with the messages from each user being transmitted on orthogonal subchannels. This framework enables straightforward per-user decoding, yet still allows for coherent combining at the receiver. The subchannels are synthesized either in time or in frequency, and a fraction r_i of the chosen resource is allocated to the transmission of the message from Node i, with $\sum_i r_i = 1$. Let $\mathbf{p}_i$ denote the vector that contains the operating power levels used in the transmission of the message of Node i, and let $\mathbf{p} = [\mathbf{p}_1^T,\ \mathbf{p}_2^T, \ldots, \mathbf{p}_K^T]^T$. The average power transmitted by Node k will be constrained to be less than $\bar{p}_k$. Since a given node may relay messages from other nodes, the average power for Node k may depend on elements of $\mathbf{p}_i$, $i \neq k$, as well as elements of $\mathbf{p}_k$. In the case of subchannels synthesized in time, the average power

Manuscript received November 30, 2011. The associate editor coordinating the review of this letter and approving it for publication was L. Le.

W. Mesbah is with the Department of Electrical Engineering, King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia (e-mail: mesbahw@gmail.com).

T. N. Davidson is with the Department of Electrical and Computer Engineering, McMaster University, Hamilton, Ontario, Canada (e-mail: davidson@mcmaster.ca).

A preliminary version of this paper appeared in *Proc. 2010 Int. Conf. Acoust., Speech, Signal Process.*

Digital Object Identifier 10.1109/WCL.2012.031912.110216

constraint on Node k takes the form $\sum_i r_i \mathbf{S}_i(k,:)\mathbf{p}_i \leq \bar{p}_k$, where $\mathbf{S}_i(k,:)$ denotes the kth row of a matrix $\mathbf{S}_i$ whose elements are non-negative. If Node k plays no role in the transmission of the message from Node i, then all of the elements of $\mathbf{S}_i(k,:)$ are zero. In the case of subchannels synthesized in frequency, the average power constraint takes the form $\sum_i \mathbf{S}_i(k,:)\mathbf{p}_i \leq \bar{p}_k$. To simplify our notation, we will sometimes use the generic expression $\mathbf{G}(\mathbf{r})\mathbf{p} \preccurlyeq \bar{\mathbf{p}}$, where $\mathbf{r}$ is a vector containing the channel resource allocations, and the inequality is to be interpreted element-wise.

We will consider a quasi-static channel environment with coherent reception in which each (scalar) link can be modelled as a frequency-flat discrete-time channel of power gain $|h|^2$ with additive white Gaussian noise of variance σ^2 at the receiver. Since we are considering systems with orthogonal transmission, the achievable rate for Node i depends on the allocation of the channel resource, r_i, and the powers allocated to that node's message, $\mathbf{p}_i$. Indeed, for a given $(\mathbf{r},\mathbf{p})$ pair, the achievable rate region takes the form

$$\mathcal{R}(\mathbf{r},\mathbf{p}) = \big\{(R_1, R_2, \ldots, R_K) \mid R_i \leqslant \bar{R}_i(r_i,\mathbf{p}_i)\ \forall i\big\}. \quad (1)$$

For systems with subchannels synthesized in time, $\bar{R}_i(r_i,\mathbf{p}_i)$ takes the form $r_i f_i(\mathbf{p}_i)$, e.g., [5], [8], and for systems with subchannels synthesized in frequency, $\bar{R}_i(r_i,\mathbf{p}_i)$ takes the form $r_i f_i\big(\mathbf{p}_i/r_i\big)$, e.g., [7]. Some examples of these functions are provided in Section V. The region of rates that can be achieved using joint power and channel resource allocation, without time sharing between different operating points is

$$\mathcal{R}(\bar{\mathbf{p}}) = \bigcup_{\substack{r_i\in[0,1],\, \sum_i r_i = 1\\ \{\mathbf{p}\mid\mathbf{p}\succcurlyeq\mathbf{0},\, \mathbf{G}(\mathbf{r})\mathbf{p}\preccurlyeq\bar{\mathbf{p}}\}}} \mathcal{R}(\mathbf{r},\mathbf{p}). \quad (2)$$

In this paper we will develop algorithms for finding the pair $(\mathbf{r},\mathbf{p})$ that maximizes a weighted sum of the rates in $\mathcal{R}(\bar{\mathbf{p}})$, for certain classes of systems. This WSR problem can be written as: Given weights $\mu_i \in [0,1]$ with $\sum_i \mu_i = 1$,

$$\max_{\mathbf{p}_i\succcurlyeq\mathbf{0},\, r_i\in[0,1]} \quad \sum_{i=1}^{K} \mu_i \bar{R}_i(r_i,\mathbf{p}_i) \quad (3a)$$

$$\text{subject to} \quad \mathbf{G}(\mathbf{r})\mathbf{p} \preccurlyeq \bar{\mathbf{p}} \quad \text{and} \quad \sum_{i=1}^{K} r_i = 1. \quad (3b)$$

Unfortunately, for most of the considered systems, this natural formulation of the WSR problem is not convex and can be difficult to solve. Even in the case in which the functions $f_i(\cdot)$ that are implicit in (1) are concave in the powers, the formulation in (3) is not necessarily convex.

III. Systems for which $\mathcal{R}(\bar{\mathbf{p}})$ is Convex

For systems for which $\mathcal{R}(\bar{\mathbf{p}})$ is convex, we will reformulate the WSR problem in a hierarchical form with inner and outer problems. First, we select one user, say user k, and introduce auxiliary design variables R_j, $j \neq k$ and define $\breve{R}_j(r_j)$ to be the maximum achievable rate of Node j when it is allocated the fraction r_j of the channel resources. (This value can be found by allocating all the available power to the transmission of the message of Node j.) The hierarchical reformulation is based on the observation that the problem in (3) is equivalent to the "outer" problem: $\max_{\{R_j\in[0,\breve{R}_j(1)]\}_{j\neq k}} \ W(\{R_j\}_{j\neq k})$, where

$$W(\{R_j\}_{j\neq k}) = \mu_k R_{k,\max}(\{R_j\}_{j\neq k}) + \sum_{j\neq k} \mu_j R_j \quad (4)$$

and $R_{k,\max}(\{R_j\}_{j\neq k})$ denotes the maximum achievable rate for Node k for given rates for the other nodes; that is, $R_{k,\max}(\{R_j\}_{j\neq k})$ is the optimal value of the "inner" target rate problem:

$$\max_{\mathbf{p}\succcurlyeq\mathbf{0},\, r_i\in[0,1]} \quad \bar{R}_k(r_k,\mathbf{p}_k) \quad (5a)$$

$$\text{subject to} \quad \bar{R}_j(r_j,\mathbf{p}_j) \geqslant R_j, \quad \forall j \neq k \quad (5b)$$

$$\mathbf{G}(\mathbf{r})\mathbf{p} \preccurlyeq \bar{\mathbf{p}} \quad \text{and} \quad \sum_{i=1}^{K} r_i = 1. \quad (5c)$$

Although it has been obtained informally, this hierarchical formulation is actually a simple decomposition; e.g., [12].

To show that the above reformulation can lead to an effective algorithm, we first observe that the jointly optimized achievable rate region in (2) is the intersection of the hypograph of the function $R_{k,\max}(\{R_j\}_{j\neq k})$ and the non-negative orthant. For the considered systems, that region is convex, and hence $R_{k,\max}(\{R_j\}_{j\neq k})$ is a concave function [13, p. 75], and so is $W(\{R_j\}_{j\neq k})$. However, we do not have a closed-form expression for $R_{k,\max}(\{R_j\}_{j\neq k})$, and conventional derivative-based algorithms cannot be applied. Instead, there are a number of direct search techniques that can be shown to converge to the (globally) optimal solution; e.g., [10]. One simple example is "compass search". At each iteration of that method, steps are taken, alternately, along the coordinate axes and the corresponding target rate problem is solved until an improved value for $W(\{R_j\}_{j\neq k})$ is obtained. If no improved value is found, the search is repeated with steps of half the size. Most convergent direct search algorithms generate a monotonically increasing sequence of values for $W(\{R_j\}_{j\neq k})$, and typically make good progress toward the optimal solution in the early iterations. As such, they provide an effective approach to finding a "good" solution. However, the rate of refinement of the good solution to the optimal solution may be slow.

In the two-user case, the golden-section direct search technique (e.g., [11, p. 90]) can be used. At each iteration of this bracketting technique, the optimal value for R_j is known to lie within a given interval, say $[R_j^a, R_j^b]$. The target rate problem is then solved for a particular $R_j \in [R_j^a, R_j^b]$, and based on $W(R_j)$ a portion of the interval is removed. At each iteration, the length of the interval is reduced by a factor $\tau = 2/(1+\sqrt{5}) \approx 0.618$, and hence we obtain the optimal value of R_j to within accuracy of ϵ in $\big\lceil \log\big(\breve{R}_j(0)/\epsilon\big)/|\log(\tau)|\big\rceil$ iterations. Since the number of iterations is only logarithmic in $1/\epsilon$, when the target rate problem can be efficiently solved, e.g., [7]–[9], the golden-section search technique yields an efficient algorithm for solving the two-user WSR problem.

IV. Systems for which each $f_i(\cdot)$ is Concave

If the functions $f_i(\cdot)$ that are implicit in (1) are concave in the powers, then the rate region in (2) is convex [14], and the WSR problem can be solved using the method in Section III.

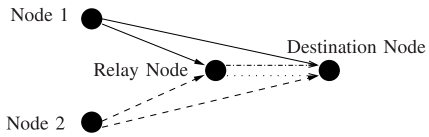

Fig. 1. A simple multiple access relay system with two source nodes.

In this section, we take a different approach and show how concavity of $f_i(\cdot)$ in the powers can be used to reformulate the WSR problem as a convex optimization problem that can be solved using conventional interior point methods [13]. An advantage of this approach over the more generally-applicable direct search approach in Section III is that conventional interior point methods exploit slope and curvature information.

For time-domain subchannels, the WSR problem takes the form of the problem in (3), with rates $\bar{R}_i(r_i, \mathbf{p}_i) = r_i f_i(\mathbf{p}_i)$ and power constraint $\mathbf{G}(\mathbf{r})\mathbf{p} = \sum_{i=1}^{K} r_i \mathbf{S}_i(k,:)\mathbf{p}_i \preccurlyeq \bar{\mathbf{p}}$. This natural formulation appears to be difficult to solve, due to the bilinear nature of the power constraint and the products $r_i f_i(\mathbf{p}_i)$ in the objective. However, if we restrict attention to the case of $r_i \in (0,1)$,[1] and if we define $\tilde{\mathbf{p}}_i = r_i \mathbf{p}_i$, which is the vector of average power components used in the transmission of Node i's message, then the WSR problem can be reformulated as

$$\max_{\tilde{\mathbf{p}}_i \succcurlyeq \mathbf{0},\, r_i \in (0,1)} \quad \sum_{i=1}^{K} \mu_i r_i f_i\big(\tilde{\mathbf{p}}_i / r_i\big) \tag{6a}$$

$$\text{subject to} \quad \sum_{i=1}^{K} \mathbf{S}_i \tilde{\mathbf{p}}_i \preccurlyeq \bar{\mathbf{p}} \quad \text{and} \quad \sum_{i=1}^{K} r_i = 1. \tag{6b}$$

In (6), the constraints are linear and hence convex. The objective is the weighted sum of functions of the form $r_i f_i\big(\tilde{\mathbf{p}}_i / r_i\big)$. Functions of this form are said to be the perspective of $f_i(\cdot)$, and the perspective operator preserves the convexity properties of $f_i(\cdot)$, [13, p. 89]. Since we are considering systems in which $f_i(\cdot)$ is concave, the objective in (6a) is concave and hence the problem in (6) can be efficiently solved.

In the case of frequency-domain subchannels, for $r_i \in (0,1)$, the direct formulation of the WSR problem takes the form in (6), with $\mathbf{p}_i$ replacing $\tilde{\mathbf{p}}_i$, and hence it can be efficiently solved. (A related observation was made in [15].)

V. Examples

A. Orthogonal Multiple Access Relay (OMAR) System

The OMAR system (cf. [4]) consists of a number of users (Nodes i, $i \in \{1, 2, \ldots N\}$) that independently transmit their messages over orthogonal subchannels to a common destination (Node 0) with the help of a relay node (Node R); cf. Fig. 1. We will consider the case of time-domain subchannels, in which the available time slot is partitioned into N fractions, r_i. In the first half of the ith fraction, Node i transmits and in the second half the relay assists the transmission of Node i's message according to the chosen relaying strategy. The maximum achievable rates for Node i under the regenerative and non-regenerative decode-and-forward strategies (RDF and NDF) and the amplify-and-forward (AF) and compress-and-forward (CF) strategies take the form $r_i f_i(\mathbf{p}_i)$, where $\mathbf{p}_i$ contains P_i, the power level at which Node i transmits, and P_{Ri}, the relay power level for the transmission of the message of Node i; e.g., [8, (2)]. For example, if we let $\gamma_{ij} = |h_{ij}|^2/\sigma_j^2$ denote the effective gain of the channel from Node i to Node j, then the rates for RDF and AF relaying are bounded by

$$\bar{R}_{i,\mathrm{RDF}}(r_i, \mathbf{p}_i) = \frac{r_i}{2} \min\big\{\log(1 + \gamma_{iR} P_i), \log(1 + \gamma_{i0} P_i + \gamma_{R0} P_{Ri})\big\}, \tag{7a}$$

$$\bar{R}_{i,\mathrm{AF}}(r_i, \mathbf{p}_i) = \frac{r_i}{2} \log\Big(1 + \gamma_{i0} P_i + \frac{\gamma_{iR}\gamma_{R0} P_i P_{Ri}}{1 + \gamma_{iR} P_i + \gamma_{R0} P_{Ri}}\Big) \tag{7b}$$

respectively. For the OMAR system, the average power constraint at each source node is $r_k P_k / 2 \le \bar{p}_k$ and the average power constraint for the relay is $\sum_k r_k P_{Rk}/2 \le \bar{p}_R$.

For the RDF and NDF strategies, some basic properties can be used to show that the functions $f_i(\cdot)$ are concave in $\mathbf{p}_i$, [5], and hence that the jointly optimal allocations can be found using either of the proposed algorithms. Actually, since the transmissions are orthogonal, the optimal source operating powers are $P_k^\star = 2\bar{p}_k / r_k$, and this can be used to simplify the algorithm. For the AF and CF strategies, this expression for the optimal source powers can be used to show that the achievable rates are convex in the remaining design variables when $\gamma_{k0}\bar{p}_k \ge 1/2$; cf. [8]. In that (common) case, a jointly optimal allocation can be found using either algorithm.

Now that we have an efficient algorithm, we can evaluate the advantage of joint optimization over power allocation alone with a fixed resource allocation [5]. Consider a three-user system with $\mu_1 = 0.3$ and $\mu_2 = 0.2$ in (3). For the power-allocation-only case, we consider equal resource allocation, $r_i = 1/3$, and a resource allocation that is matched to the weighting, $r_1 = 0.3$, $r_2 = 0.2$. In Fig. 2 we have plotted the average weighted sum rate against the relay's power budget, $\bar{p}_R$, for the RDF and AF relaying strategies under the source power constraint $\bar{p}_k = 2$. (The results for the NDF and CF strategies have similar characteristics; cf. [14].) The average was taken over 10,000 realizations of a Rician channel model with a K-factor of 5 and average power gains of the specular paths of $\alpha K/(K+1)$, with $\alpha_{1R} = 1.2$, $\alpha_{2R} = 0.8$, $\alpha_{3R} = 1$, $\alpha_{10} = 0.3$, $\alpha_{20} = 0.6$, $\alpha_{30} = 0.4$, $\alpha_{R0} = 0.4$. (The noise variances were normalized to one.) It can be seen from the figure that joint optimization can provide a significant gain in the weighted sum rate. In particular, for the RDF case the gain over equal resource allocation is around 17% for small relay power budgets and around 30% for large budgets. In this example, the source node power constraints are $\bar{p}_k = 2$, and hence as the relay's power budget increases, the curves in Fig. 2 saturate in the RDF case and exhibit diminishing returns in the AF case.

B. Pairwise User Cooperation Systems

Whereas OMAR systems employ a dedicated relay node, in cooperative systems the nodes themselves work as relays for each other; e.g., Fig. 3. In this section we consider the orthogonal pairwise user cooperation system that was considered in [9], which is a block-based variants of a system

[1] The vertices can be considered separately, or as limiting cases.

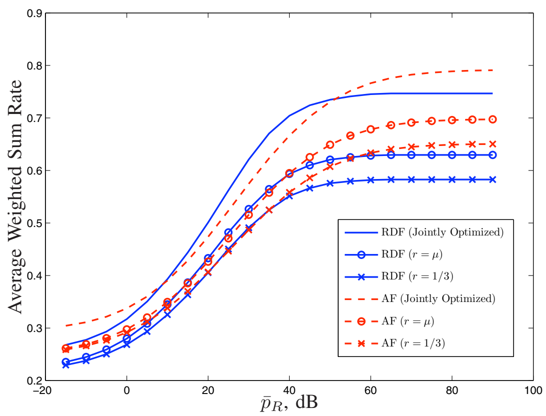

Fig. 2. Average weighted sum rate for three-user RDF and AF OMAR systems with $\mu_1 = 0.3, \mu_2 = 0.2$ in a Rician channel environment, against the relay's power budget, $\bar{p}_R$, with source power constraint $\bar{p}_k = 2$.

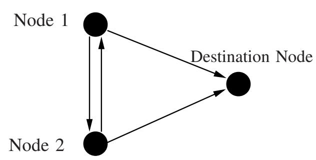

Fig. 3. The pairwise user cooperation system.

in [2]. In the system in [9], time division is used to synthesize the subchannels, and Node 1 transmits its own message for a fraction $r_1/2$ of the time slot and during a separate fraction $r_2/2 = (1-r_1)/2$ of the time slot it amplifies and forwards the signals it received from Node 2. Node 2 acts in the complementary way. The achievable rate region of this system can be characterized [9] using (1) and (2) with

$$\bar{R}_i(r_i, \mathbf{p}_i) = \frac{r_i}{2}\log\Big(1 + \gamma_{i0}P_{ii} + \frac{\gamma_{k0}\gamma_{ik}P_{ii}P_{ki}}{1+\gamma_{k0}P_{ki}+\gamma_{ik}P_{ii}}\Big), \tag{8}$$

where $i \neq k \in \{1,2\}$, P_{ki} is the operating power level allocated by Node k to the message of Node i, and $r_2 = (1-r_1)$. In this case, $\mathbf{p}_i = [P_{ii}, P_{ki}]^T$ and the average power constraint on Node k is $r_1P_{k1}/2 + (1-r_1)P_{k2}/2 \leq \bar{p}_k$. Although the expression in (8) resembles that in (7b) for the AF OMAR system, the power constraint is different, and this complicates the analysis. However, it can be shown, using [9], that even though the expression in (8) is not concave in the powers, for this system the achievable rate region in (2) is convex and that the target rate problem can be efficiently solved. Hence, the golden section search algorithm in Section III provides an efficient method for solving the WSR problem.

To illustrate the gain that can be obtained using the proposed algorithm, consider a Rician channel model of the form used in the previous example, with $\alpha_{10} = 0.3$, $\alpha_{20} = 0.8$, and $\alpha_{12} = \alpha_{21} = 1.2$. For each of 10,000 channel realizations, we maximized the weighted sum rate with $\mu = 3/4$. In Fig. 4, we have plotted the resulting average weighted sum rates against the (equal) power budgets of the source nodes, $\bar{p}_1 = \bar{p}_2$. Fig. 4 demonstrates that the proposed joint optimization algorithm can increase the high-SNR slope of the weighted sum rate curve over that obtained using power allocation alone with

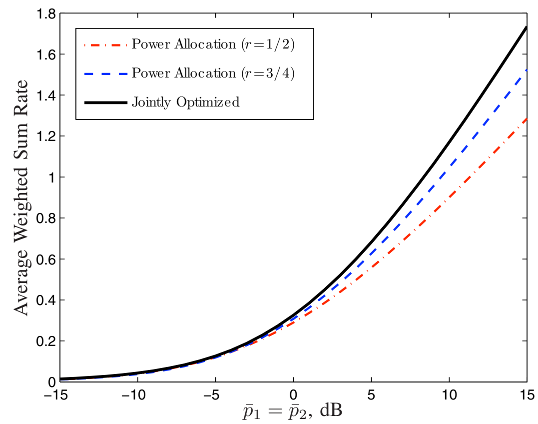

Fig. 4. Average weighted sum rate for the pairwise AF user cooperation system with $\mu = 3/4$, in a Rician channel environment, against the source nodes' power budget.

fixed resource allocation (with $r_1 = 1/2$ or $r_1 = 3/4$). As a result, with $\bar{p}_1 = \bar{p}_2 = 10$ dB the jointly optimized system provides an average weighted sum rate that is at least 30% higher than that of the power optimized system with $r = 1/2$.

References

[1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity—part I: system description," *IEEE Trans. Commun.*, vol. 51, pp. 1927–1938, Nov. 2003.

[2] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity—part II: implementation aspects and performance analysis," *IEEE Trans. Commun.*, vol. 51, pp. 1939–1948, Nov. 2003.

[3] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, pp. 3062–3080, Dec. 2004.

[4] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inf. Theory*, vol. 51, pp. 3037–3063, Sep. 2005.

[5] S. Serbetli and A. Yener, "Relay assisted F/TDMA ad hoc networks: node classification, power allocation and relaying strategies," *IEEE Trans. Commun.*, vol. 56, pp. 937–947, June 2008.

[6] Y. Liang and V. V. Veeravalli, "Gaussian orthogonal relay channels: optimal resource allocation and capacity," *IEEE Trans. Inf. Theory*, vol. 51, pp. 3284–3289, Sep. 2005.

[7] W. Mesbah and T. N. Davidson, "Optimized power allocation for pairwise cooperative multiple access," *IEEE Trans. Signal Process.*, vol. 56, pp. 2994–3008, July 2008.

[8] W. Mesbah and T. N. Davidson, "Power and resource allocation for orthogonal multiple access relay systems," *EURASIP J. Adv. Signal Process.*, article ID 476125, 15 pages, 2008, doi:10.1155/2008/476125.

[9] W. Mesbah and T. N. Davidson, "Joint power and resource allocation for orthogonal amplify-and-forward pairwise user cooperation," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 4681–4691, Nov. 2008.

[10] T. G. Kolda, R. M. Lewis, and V. Torczon, "Optimization by direct search: new perspectives on some classical and modern methods," *SIAM Rev.*, vol. 45, no. 3, pp. 385–482, 2003.

[11] P. E. Gill, W. Murray, and M. H. Wright, *Practical Optimization*. Academic Press, 1981.

[12] D. P. Palomar and M. Chiang, "A tutorial on decomposition methods for network utility maximization," *IEEE J. Sel. Areas Commun.*, vol. 24, pp. 1439–1451, Aug. 2006.

[13] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[14] W. Mesbah and T. N. Davidson, "Efficient weighted-sum-rate maximization for a class of half-duplex cooperative systems," in *Proc. 2010 IEEE Int. Conf. Acoustics, Speech, Signal Processing*, pp. 3134–3137.

[15] C. T. K. Ng and G. J. Foschini, "Transmit signal and bandwidth optimization in multiple-antenna relay channels," *IEEE Trans. Commun.*, vol. 59, pp. 2987–2992, Nov. 2011.