Joint Power and Channel Resource Allocation for Two-User Orthogonal Amplify-and-Forward Cooperation

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***Abstract*—We consider the jointly optimal allocation of the radio resources for a two-user orthogonal amplify-and-forward (AF) cooperation scheme. In particular, we derive a simple efficient algorithm for determining the power and channel resource allocations required to operate at any point on the boundary of the achievable rate region. The algorithm is based on two results derived herein: a closed-form solution for the optimal power allocation for a given channel resource allocation; and the fact that the channel resource allocation problem is quasi-convex. The structure of the optimal power allocation reveals that at optimality at most one user acts as a relay, and hence a fraction of the channel resource will be idle. We propose a modified orthogonal AF cooperation scheme that uses the channel resources more efficiently and hence provides a larger achievable rate region.**

***Index Terms*—Amplify-and-forward relaying, cooperative multiple access, achievable rate region, quasi-convexity.**

I. INTRODUCTION

THE growing demand for reliable spectrally-efficient wireless communication has led to a resurgence of interest in systems in which nodes cooperate in the transmission of messages to a destination node; e.g., [1]. (See Fig. 1.) An achievable rate region for a full-duplex two-user cooperative multiple access system was obtained in [1], based on earlier work in [2], and this achievable rate region was shown to be larger than the capacity region for conventional multiple access without cooperation between the source nodes. However, full-duplex cooperation requires sufficient electrical isolation between the transmitting and receiving circuits at each node in order to mitigate near-end cross-talk (e.g., [3], [4], [5], [6]), and this is often difficult to achieve in practice. In order to avoid the need for stringent electrical isolation, the cooperation scheme can be constrained so that the source nodes do not simultaneously transmit and receive over the same channel, and such schemes are often said to be half-duplex; e.g., [3], [7]. The subclass of half-duplex schemes with orthogonal components (e.g., [3]) further constrains the source nodes to use orthogonal subchannels. (These subchannels can be synthesized by time division, e.g., [3], or by frequency division, e.g., [6].) This enables "per-user" decoding at the destination node, rather than joint decoding, and hence simplifies the receiver at the destination node. Motivated by this simplicity, we will focus on orthogonal (half-duplex) cooperation schemes in this paper.

A feature of orthogonal cooperation schemes is that they can be decomposed into parallel relay channels, each with orthogonal components [5], [8], [6]. Therefore, the remaining design issues reduce to the choice of the relaying strategy, and the allocation of the radio resources to the parallel relay channels. For the relaying strategy, a number of choices are available (e.g., [9], [10], [3], [11], [12], [13], [14]), and we will focus on the amplify-and-forward (AF) strategy, because it is the simplest in terms of the hardware requirements of the cooperating nodes. As such, the cooperative scheme that we will consider is a generalization of the orthogonal AF scheme in [3]. One of our contributions will be the development of a simple efficient algorithm for joint power and channel resource allocation for this scheme, for scenarios in which full channel state information (CSI) is available. (That is, the design is based on knowledge of the (effective) channel gain on each of the four links in Fig. 1.)

As mentioned above, the design of an orthogonal AF cooperation scheme requires the appropriate allocation of powers and the channel resource (typically time or bandwidth) to the components of each of the underlying parallel relay channels. Unfortunately, the problem of joint power and resource allocation so as to enable operation on the boundary of the achievable rate region is not convex; a fact that might suggest that this is a rather difficult problem to solve.

Some progress has been made by considering power allocation alone [15].[1] In particular, it was shown in [15] that for a given resource allocation, the problem of finding the power allocation that maximizes a weighted-sum of achievable rates can be written in a quasi-convex (e.g., [24]) form. In this paper, we will consider the problem of jointly allocating the power and the channel resource. In particular, we will show that for a given target rate of one node, the maximum

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[1]See [16] for some related work on a non-orthogonal AF cooperation scheme, [17] for some related work with an outage objective, and [18], [19] for some related work on half-duplex cooperation with decode-and-forward relaying. There has also been a considerable amount of work on power and resource allocation for a variety of relaying schemes with achievable rate or outage objectives; e.g., [5], [6], [20], [14], [21], [22], [23].

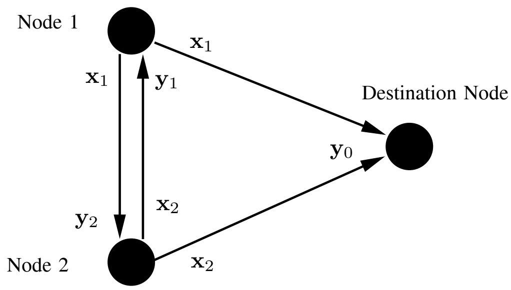

Fig. 1. Transmitted and received signals of the cooperative channel.

$x_1(1) = \sqrt{P_{11}}B_{11}$ $x_2(1) = 0$	$x_1(2) = 0$ $x_2(2) = A_2y_2(1)$	$x_1(3) = 0$ $x_2(3) = \sqrt{P_{22}}B_{21}$	$x_1(4) = A_1y_1(3)$ $x_2(4) = 0$
r		$\hat{r} = 1 - r$	

Fig. 2. A frame of the orthogonal half-duplex amplify-and-forward cooperation scheme under consideration.

achievable rate of the other node can be written as a convex function of the transmission powers (see Section III-A) and a quasi-convex function of the resource allocation parameter (see Section IV). Furthermore, using the Karush-Kuhn-Tucker (KKT) optimality conditions (e.g., [24]), we will derive a closed-form solution for the optimal power allocation for a given resource allocation (see Section III-C). By combining this closed-form solution with the quasi-convexity of the maximum achievable rate in the resource allocation parameter, a simple efficient algorithm for the jointly optimal power and channel resource allocation will be obtained (see Section IV). In addition to the computational efficiencies that this approach provides, the ability to directly control the rate of one of the nodes can be convenient in the case of heterogeneous traffic at the cooperative nodes, especially if one node has a constant rate requirement and the other is dominated by "best effort" traffic. The structure of the closed-form solution to the optimal power allocation problem for a fixed channel resource allocation (see Section III-B) suggests that under our assumption that channel state information is available, the cooperative communication scheme that we have adopted (which is based on that proposed in [3]) does not use all the available channel resources. (A related observation was made in [16] for a non-orthogonal half-duplex AF cooperation scheme.) Hence, the cooperative scheme itself incurs a reduction in the achievable rate region. In order to mitigate this rate reduction, in Section V we propose a modified cooperative scheme that retains the orthogonal half-duplex property of the original scheme (and that in [3]), yet can achieve a significantly larger achievable rate region. Similar to original cooperative scheme, we obtain a closed-form solution to the problem of optimal power allocation (for fixed resource allocation) for this modified scheme, and we show that the problem of optimal channel resource allocation is quasi-convex (see Section V). Therefore, the jointly optimal power and channel resource allocation for this modified scheme can also be obtained using a simple efficient algorithm.

Although the focus of this paper will be on scenarios in which perfect channel state information (CSI) is available, in Section VI we will provide an example of a simple modification of the proposed approach that encompasses scenarios with imperfect CSI.

II. System Model and Direct Formulation

We will consider a system in which two users (Nodes 1 and 2) wish to cooperate in the transmission of messages to a destination node (Node 0); cf. Fig. 1. In order to enable simple implementation, we will adopt the orthogonal half-duplex amplify-and-forward cooperation scheme illustrated in Fig. 2. In this scheme, the orthogonal sub-channels are synthesized by time division, and although we will focus on that case, our work can be modified in a straightforward way to address the case of frequency division. The scheme in Fig. 2 is a mild generalization of that in [3], in the sense that channel resource allocation is implemented by allocating a fraction r of each frame for the message from Node 1, and a fraction $\hat{r} = 1 - r$ of the frame for the message of Node 2. In the scheme in [3], the channel resource allocation parameter is fixed to $r = 1/2$.

In the scheme in Fig. 2, each frame consists of four time blocks, with the first two blocks being of fractional length $r/2$ and the second two blocks having fractional length $\hat{r}/2$. The first and second blocks have the same length because the adoption of amplify-and-forward relaying means that the length of the signals to be transmitted in these two blocks is the same. For that reason the third and fourth blocks are also of the same length. In the first block, Node 1 transmits its message while Node 2 listens. In the second block, Node 2 works as a relay for Node 1; it amplifies the signal received in the first block by a factor A_2 and re-transmits that signal to the master node. In the third and fourth blocks the roles of Nodes 1 and 2 are reversed, so that Node 1 works as a relay for Node 2.

We will consider a block fading channel model with a coherence time that is long enough for us to focus on the case in which full channel state information (CSI) can be acquired without expending a significant fraction of the available power and channel resources. (We will consider a scenario with uncertain CSI in Section VI.) If we define $\mathbf{y}_n(\ell)$ to be the signal block received by Node n during block ℓ, then the received signals of interest are $\mathbf{y}_1(\ell)$ for $\ell \bmod 4 = 3$, $\mathbf{y}_2(\ell)$ for $\ell \bmod 4 = 1$, and $\mathbf{y}_0(\ell)$ for all ℓ. (We will use $\mathbf{0}$ to represent blocks in which the receiver is turned off.) If we define K_{mn} to be the complex channel gain between Nodes $m \in \{1, 2\}$ and $n \in \{0, 1, 2\}$, and $\mathbf{z}_n(\ell)$ to be the zero-mean additive white circular complex Gaussian noise with variance σ_n^2 at Node n, then the received signal blocks can be written as

$$\mathbf{y}_1(\ell) = \begin{cases} K_{21}\mathbf{x}_2(\ell) + \mathbf{z}_1(\ell) & \ell \bmod 4 = 3, \\ \mathbf{0} & \ell \bmod 4 \neq 3, \end{cases} \tag{1}$$

$$\mathbf{y}_2(\ell) = \begin{cases} K_{12}\mathbf{x}_1(\ell) + \mathbf{z}_2(\ell) & \ell \bmod 4 = 1, \\ \mathbf{0} & \ell \bmod 4 \neq 1, \end{cases} \tag{2}$$

$$\mathbf{y}_0(\ell) = \begin{cases} K_{10}\mathbf{x}_1(\ell) + \mathbf{z}_0(\ell) & \ell \bmod 4 = 1, \\ K_{20}A_2\mathbf{y}_2(\ell - 1) + \mathbf{z}_0(\ell) & \ell \bmod 4 = 2, \\ K_{20}\mathbf{x}_2(\ell) + \mathbf{z}_0(\ell) & \ell \bmod 4 = 3, \\ K_{10}A_1\mathbf{y}_1(\ell - 1) + \mathbf{z}_0(\ell) & \ell \bmod 4 = 0, \end{cases} \tag{3}$$

where A_1 and A_2 represent the amplification factors of Nodes 1 and 2, respectively, when they act as a relay. Let us define P_{ij} to be the power allocated by Node i to the transmission of the message from Node j. With that definition, the powers of the (non-zero) transmitted signals in the blocks in Fig. 2 are P_{11}, P_{21}, P_{22}, and P_{12}, respectively. Furthermore, the amplification factors A_1 and A_2 that ensure that all the available relaying power is used are [3]

$$A_1 = \sqrt{\frac{P_{12}}{|K_{21}|^2 P_{22} + \sigma_1^2}},\ A_2 = \sqrt{\frac{P_{21}}{|K_{12}|^2 P_{11} + \sigma_2^2}}. \quad (4)$$

We will impose average transmission power constraints on each node, namely, the power components should satisfy the average power constraints $\frac{r}{2}P_{i1} + \frac{\hat{r}}{2}P_{i2} \leqslant \bar{P}_i$, where $\bar{P}_i$ is the maximum average power for Node i. For notational simplicity, we will define the effective channel gain to be $\gamma_{mn} = |K_{mn}|^2/\sigma_n^2$.

For a given allocation for the power components, $\mathcal{P} = (P_{11}, P_{12}, P_{21}, P_{22})$, and a given value for r, the achievable rate region of the system described above is the set of all rate pairs (R_1, R_2) that satisfy [3]

$$\begin{aligned} R_1 &\leqslant \bar{R}_1(\mathcal{P}, r) \\ &= \frac{r}{2}\log\left(1 + \gamma_{10}P_{11} + \frac{\gamma_{20}\gamma_{12}P_{11}P_{21}}{1 + \gamma_{20}P_{21} + \gamma_{12}P_{11}}\right), \quad (5a) \\ R_2 &\leqslant \bar{R}_2(\mathcal{P}, r) \\ &= \frac{\hat{r}}{2}\log\left(1 + \gamma_{20}P_{22} + \frac{\gamma_{10}\gamma_{21}P_{12}P_{22}}{1 + \gamma_{21}P_{22} + \gamma_{10}P_{12}}\right). \quad (5b) \end{aligned}$$

Since we are considering scenarios in which full channel state information (CSI) is available (i.e., γ_{10}, γ_{20}, γ_{12}, and γ_{21} are known), one way in which the power and channel resource allocation required to approach a specified point on the boundary of the achievable rate region can be found is by maximizing a weighted sum of $\bar{R}_1$ and $\bar{R}_2$ subject to the bound on the transmitted powers; i.e.,

$$\begin{aligned} \max_{P_{ij}\geqslant 0,\, r\in[0,1]} \quad & \mu\bar{R}_1(\mathcal{P}, r) + (1-\mu)\bar{R}_2(\mathcal{P}, r) && (6a) \\ \text{subject to} \quad & \tfrac{r}{2}P_{i1} + \tfrac{\hat{r}}{2}P_{i2} \leqslant \bar{P}_i \qquad i = 1, 2, && (6b) \end{aligned}$$

where $\mu \in [0,1]$ is the weight. An alternative approach to finding the required power and channel resource allocation is to maximize $\bar{R}_i$ for a given target value of $\bar{R}_j$, subject to the bound on the transmitted powers; i.e.,

$$\begin{aligned} \max_{P_{ij}\geqslant 0,\, r\in[0,1]} \quad & \bar{R}_1(\mathcal{P}, r) && (7a) \\ \text{subject to} \quad & \bar{R}_2(\mathcal{P}, r) \geqslant R_{2,\mathrm{tar}}, && (7b) \\ & \tfrac{r}{2}P_{i1} + \tfrac{\hat{r}}{2}P_{i2} \leqslant \bar{P}_i \qquad i = 1, 2. && (7c) \end{aligned}$$

Unfortunately, neither (6) nor (7) is jointly convex P_{ij} and r, and this makes the development of a reliable efficient allocation algorithm rather difficult. However, we will show below that by adopting the approach in (7), the direct formulation can be transformed into the composition of a convex optimization problem and a quasi-convex problem. Furthermore, we will derive a closed-form solution for the (inner) convex problem (see Section III), and we will show that this enables the solution of (7) using a simple efficient search over the resource allocation parameter, r; see Section IV.

III. Optimal Power Allocation

In this section we obtain a closed-form expression for the optimal power allocation for a given channel resource allocation r. The derivation of this closed-form expression involves three main steps: the derivation of a convex problem that is equivalent to the problem in (7) with a fixed value for r; an analysis of KKT optimality conditions for that problem; and analytic solutions to a pair of scalar optimization problems. To simplify our development, we will let $R_{2,\max}(r)$ denote the maximum achievable value for R_2 for a given value of r; i.e., the value of $\bar{R}_2(\mathcal{P}, r)$ in (5b) with $\mathcal{P} = (0, 2\bar{P}_1, 0, 2\bar{P}_2)$.

A. A convex equivalent to (7) with a fixed value for r

For a given value for r, the problem in (7) involves optimization over P_{ij} only. If we define $\tilde{P}_{i1} = rP_{i1}$ and $\tilde{P}_{i2} = \hat{r}P_{i2}$, then for $r \in (0,1)$ and $R_{2,\mathrm{tar}} \in (0, R_{2,\max}(r))$ we can rewrite (7) as[2]

$$\begin{aligned} \max_{\tilde{P}_{ij}\geqslant 0} \quad & \frac{r}{2}\log\left(1 + \frac{\gamma_{10}\tilde{P}_{11}}{r} + \frac{\gamma_{20}\gamma_{12}\tilde{P}_{11}\tilde{P}_{21}}{r(r + \gamma_{20}\tilde{P}_{21} + \gamma_{12}\tilde{P}_{11})}\right) && (8a) \\ \text{subject to} \quad & \frac{\hat{r}}{2}\log\left(1 + \frac{\gamma_{20}\tilde{P}_{22}}{\hat{r}} + \frac{\gamma_{10}\gamma_{21}\tilde{P}_{12}\tilde{P}_{22}}{\hat{r}(\hat{r} + \gamma_{21}\tilde{P}_{22} + \gamma_{10}\tilde{P}_{12})}\right) \\ & \qquad \geqslant R_{2,\mathrm{tar}}, && (8b) \\ & \tilde{P}_{i1} + \tilde{P}_{i2} \leqslant 2\bar{P}_i \qquad i = 1, 2. && (8c) \end{aligned}$$

The formulation in (8) has the advantage that the power constraints in (8c) are linear in $\tilde{P}_{ij}$, whereas the corresponding constraints in (7) are bilinear in P_{ij} and r.

For a given positive value of r and non-negative constant values of a, b, c and d, the function $\log(1 + \frac{ax}{r} + \frac{bcxy}{r(r+bx+cy)})$ is not concave in x and y, and hence (8) is not a convex problem. However, by showing (analytically) that its Hessian is negative semi-definite, the function $h(x, y) = \sqrt{\frac{ax}{r} + \frac{bcxy}{r(r+bx+cy)}}$ can be shown to be concave in x and y (on the non-negative orthant). By taking the exponent of both sides, the constraint in (8b) can be rewritten as

$$\sqrt{\frac{\gamma_{20}\tilde{P}_{22}}{\hat{r}} + \frac{\gamma_{10}\gamma_{21}\tilde{P}_{12}\tilde{P}_{22}}{\hat{r}(\hat{r} + \gamma_{21}\tilde{P}_{22} + \gamma_{10}\tilde{P}_{12})}} \geqslant \sqrt{2^{\frac{2R_{2,\mathrm{tar}}}{\hat{r}}} - 1}. \quad (9)$$

Furthermore, since the logarithm and the square root functions are monotonically increasing functions for positive arguments, maximizing the objective function in (8a) is equivalent to maximizing $h(\tilde{P}_{11}, \tilde{P}_{21})$ with $a = \gamma_{10}$, $b = \gamma_{20}$ and $c = \gamma_{12}$. Therefore, the problem in (8) is equivalent to

$$\begin{aligned} \max_{\tilde{P}_{ij}\geqslant 0} \quad & \sqrt{\frac{\gamma_{10}\tilde{P}_{11}}{r} + \frac{\gamma_{20}\gamma_{12}\tilde{P}_{11}\tilde{P}_{21}}{r(r + \gamma_{20}\tilde{P}_{21} + \gamma_{12}\tilde{P}_{11})}} && (10a) \\ \text{subject to} \quad & \sqrt{\frac{\gamma_{20}\tilde{P}_{22}}{\hat{r}} + \frac{\gamma_{10}\gamma_{21}\tilde{P}_{12}\tilde{P}_{22}}{\hat{r}(\hat{r} + \gamma_{21}\tilde{P}_{22} + \gamma_{10}\tilde{P}_{12})}} \\ & \qquad \geqslant \sqrt{2^{\frac{2R_{2,\mathrm{tar}}}{\hat{r}}} - 1}, && (10b) \\ & \tilde{P}_{i1} + \tilde{P}_{i2} \leqslant 2\bar{P}_i \qquad i = 1, 2. && (10c) \end{aligned}$$

[2]In (7), the resource allocation $r = 1$ is feasible only if $R_{2,\mathrm{tar}} = 0$, and the allocation $r = 0$ is optimal only if $R_{2,\mathrm{tar}} = R_{2,\max}(0)$. Also, if $R_{2,\mathrm{tar}} = 0$, the optimal $\mathcal{P} = (2\bar{P}_1, 0, 2\bar{P}_2, 0)$ and if $R_{2,\mathrm{tar}} = R_{2,\max}(r)$, the optimal $\mathcal{P} = (0, 2\bar{P}_1, 0, 2\bar{P}_2)$.

The concavity of $h(x,y)$ implies that (10) is a convex optimization problem. Furthermore, for all $R_{2,\text{tar}} \in (0, R_{2,\max}(r))$ the problem in (10) satisfies Slater's condition (e.g., [24]), and hence the KKT optimality conditions are necessary and sufficient. As we will show below, this observation is a key step in the derivation of our closed-form solution to (8).

B. Structure of the optimal solution

We will now use the KKT optimality conditions for (10) to show that at optimality one (or both) of $\tilde{P}_{12}$ and $\tilde{P}_{21}$ is zero.[3] Therefore, at optimality, at least one of the nodes has its relay mode turned off. We begin by observing that for all feasible $R_{2,\text{tar}}$ the optimal power allocation satisfies power constraints in (10c) with equality; i.e., $\tilde{P}_{i2}^* = 2\bar{P}_i - \tilde{P}_{i1}^*$, where the asterisk indicates the optimal value. Therefore, the problem in (10) can be rewritten as the following (convex) optimization problem in $\tilde{P}_{11}$ and $\tilde{P}_{21}$:

$$\min_{\tilde{P}_{11},\tilde{P}_{21}} \quad f_0(\tilde{P}_{11},\tilde{P}_{21}) \tag{11}$$
$$\text{subject to} \quad f_i(\tilde{P}_{11},\tilde{P}_{21}) \leqslant 0 \quad i = 1,\ldots,5,$$

where

$$f_0(\tilde{P}_{11},\tilde{P}_{21}) = -\left(\frac{\gamma_{10}\tilde{P}_{11}}{r} + \frac{\gamma_{20}\gamma_{12}\tilde{P}_{11}\tilde{P}_{21}}{r(r+\gamma_{20}\tilde{P}_{21}+\gamma_{12}\tilde{P}_{11})}\right)^{1/2},$$
$$f_1(\tilde{P}_{11},\tilde{P}_{21}) = \left(2^{\frac{2R_{2,\text{tar}}}{\hat{r}}}-1\right)^{1/2} - \left(\frac{\gamma_{20}(2\bar{P}_2-\tilde{P}_{21})}{\hat{r}} + \frac{\gamma_{10}\gamma_{21}(2\bar{P}_1-\tilde{P}_{11})(2\bar{P}_2-\tilde{P}_{21})}{\hat{r}(\hat{r}+\gamma_{21}(2\bar{P}_2-\tilde{P}_{21})+\gamma_{10}(2\bar{P}_1-\tilde{P}_{11}))}\right)^{1/2},$$
$$f_2(\tilde{P}_{11},\tilde{P}_{21}) = -\tilde{P}_{11}, \qquad f_4(\tilde{P}_{11},\tilde{P}_{21}) = \tilde{P}_{11} - 2\bar{P}_1,$$
$$f_3(\tilde{P}_{11},\tilde{P}_{21}) = -\tilde{P}_{21}, \qquad f_5(\tilde{P}_{11},\tilde{P}_{21}) = \tilde{P}_{21} - 2\bar{P}_2.$$

The KKT optimality conditions for this problem are

$$f_i(\tilde{P}_{11}^*,\tilde{P}_{21}^*) \leqslant 0, \tag{12a}$$
$$\lambda_i^* \geqslant 0, \tag{12b}$$
$$\begin{pmatrix} \frac{\partial f_0(\tilde{P}_{11}^*,\tilde{P}_{21}^*)}{\partial \tilde{P}_{11}} + \lambda_1^* \frac{\partial f_1(\tilde{P}_{11}^*,\tilde{P}_{21}^*)}{\partial \tilde{P}_{11}} + \lambda_4^* - \lambda_2^* \\ \frac{\partial f_0(\tilde{P}_{11}^*,\tilde{P}_{21}^*)}{\partial \tilde{P}_{21}} + \lambda_1^* \frac{\partial f_1(\tilde{P}_{11}^*,\tilde{P}_{21}^*)}{\partial \tilde{P}_{21}} + \lambda_5^* - \lambda_3^* \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{12c}$$
$$\lambda_i^* f_i(\tilde{P}_{11}^*,\tilde{P}_{21}^*) = 0, \tag{12d}$$

where λ_i is the ith dual variable. In Appendix A we will show that in order for (12) to hold, either $\tilde{P}_{12}^* = 0$ or $\tilde{P}_{21}^* = 0$, or both, must be zero. An alternative, and possibly more intuitive, proof can be constructed via the bounding argument in Appendix B.

C. Closed-form solution to (8)

Since the problems in (8) and (10) are equivalent, the above KKT analysis has shown that the problem in (8) can be reduced to one of the following two one-dimensional problems:

$$\beta(r) = \max_{\tilde{P}_{21}\in[0,2\bar{P}_2]} \frac{r}{2}\log\left(1+\frac{2\gamma_{10}\bar{P}_1}{r} + \frac{2\gamma_{20}\gamma_{12}\bar{P}_1\tilde{P}_{21}}{r(r+\gamma_{20}\tilde{P}_{21}+2\gamma_{12}\bar{P}_1)}\right) \tag{13a}$$
$$\text{subject to} \quad \frac{\hat{r}}{2}\log\left(1+\frac{\gamma_{20}(2\bar{P}_2-\tilde{P}_{21})}{\hat{r}}\right) \geqslant R_{2,\text{tar}}, \tag{13b}$$

and

$$\alpha(r) = \max_{\tilde{P}_{11}\in[0,2\bar{P}_1]} \quad \frac{r}{2}\log\left(1+\frac{\gamma_{10}\tilde{P}_{11}}{r}\right) \tag{14a}$$
$$\text{subject to} \quad \frac{\hat{r}}{2}\log\left(1+\frac{2\gamma_{20}\bar{P}_2}{\hat{r}} + \frac{2\gamma_{10}\gamma_{21}(2\bar{P}_1-\tilde{P}_{11})\bar{P}_2}{\hat{r}(\hat{r}+2\gamma_{21}\bar{P}_2+\gamma_{10}(2\bar{P}_1-\tilde{P}_{11}))}\right) \geqslant R_{2,\text{tar}}, \tag{14b}$$

where (13) arises in the case that $\tilde{P}_{12}^* = 0$, and (14) arises in the case that $\tilde{P}_{21}^* = 0$. Using the properties of the logarithm, the transformation that led to (9), and the power constraints, it can be shown that the feasible set of each of these problems is a simple bounded interval. In both problems, the objective is monotonically increasing on that interval, and hence for all feasible $R_{2,\text{tar}}$, the optimal solutions to (13) and (14) occur when the constraints in (13b) and (14b), respectively, hold with equality. That is, the solutions to (13) and (14) are where

$$\tilde{P}_{21}^* = \tilde{Q}_\beta = 2\bar{P}_2 - \frac{\hat{r}}{\gamma_{20}}\left(2^{\frac{2R_{2,\text{tar}}}{\hat{r}}}-1\right), \tag{15}$$
$$\tilde{P}_{11}^* = \tilde{Q}_\alpha = 2\bar{P}_1 - \frac{(2\bar{P}_2\gamma_{21}+\hat{r})\left(\hat{r}(2^{\frac{2R_{2,\text{tar}}}{\hat{r}}}-1)-2\bar{P}_2\gamma_{20}\right)}{\gamma_{10}\left(2\bar{P}_2\gamma_{21}-\left(\hat{r}(2^{\frac{2R_{2,\text{tar}}}{\hat{r}}}-1)-2\bar{P}_2\gamma_{20}\right)\right)}, \tag{16}$$

respectively. The optimal solution to (8) is then the power allocation that corresponds to the larger of the values of $\beta(r)$ and $\alpha(r)$. However, since (13) corresponds to the case in which the target rate for Node 2 is met by direct transmission, then it will generate the larger value whenever $R_{2,\text{tar}}$ is less than $R_{2,\text{thresh}}(r) = (\hat{r}/2)\log(1+2\gamma_{20}\bar{P}_2/\hat{r})$. Therefore, if we let $\tilde{\mathcal{P}} = (\tilde{P}_{11},\tilde{P}_{12},\tilde{P}_{21},\tilde{P}_{22})$ denote a (scaled) power allocation, then for each $r \in (0,1)$ and each $R_{2,\text{tar}} \in (0, R_{2,\max}(r))$ the optimal solution to (8) is

$$\tilde{\mathcal{P}}^* = \begin{cases} (2\bar{P}_1,\,0,\,\tilde{Q}_\beta,\,2\bar{P}_2-\tilde{Q}_\beta) & \text{if } R_{2,\text{tar}} \le R_{2,\text{thresh}}(r), \\ (\tilde{Q}_\alpha,\,2\bar{P}_1-\tilde{Q}_\alpha,\,0,\,2\bar{P}_2) & \text{if } R_{2,\text{tar}} > R_{2,\text{thresh}}(r). \end{cases} \tag{17}$$

This expression clearly shows that at points on the boundary of the achievable rate region (for the given value of r), at most one node is acting as a relay; i.e., $\tilde{P}_{12} = 0$ or $\tilde{P}_{21} = 0$, or both.

IV. Optimal Power and Resource Allocation

The expression in (17) provides the optimal power allocation for a given value of r. However, different points on the boundary of the achievable rate region are not necessarily achieved with the same r, and our goal is to jointly optimize the power and resource allocations. Although the problem in

[3] A related observation was made in [16] for a non-orthogonal half-duplex amplify-and-forward cooperation scheme, although that observation arose from an analysis of the sum-rate optimization problem.

(7) is not jointly convex in r and the powers, the following result will enable us to develop a simple algorithm for finding the optimal value of r.

Proposition 1: If the direct channels of both source nodes satisfy $\gamma_{i0}\bar{P}_i \geqslant \frac{1}{2}$, then for a given target rate for Node j, $R_{j,\mathrm{tar}}$, the maximum achievable rate for Node i is a quasi-concave function of the channel resource allocation parameter r.

Proof: For simplicity, we will consider the case in which $i = 1$ and $j = 2$. Our first step is to show (see Appendix D) that the condition $\gamma_{10}\bar{P}_1 \geqslant 1/2$ is sufficient for the function $\beta(r)$ in (13) to be quasi-concave in r, and hence the set of values of r for which $\beta(r)$ is greater than a given rate, say $R_{1,\mathrm{test}}$, is a convex set. Let $\mathcal{S}_\beta = \{r|\beta(r) \geq R_{1,\mathrm{test}}\}$ denote that set. Similarly, the condition $\gamma_{20}\bar{P}_2 \geqslant 1/2$ is sufficient for the set $\mathcal{S}_\alpha = \{r|\alpha(r) \geq R_{1,\mathrm{test}}\}$ to be a convex set. Therefore, the set of values for r for which the solution of (8) is greater than $R_{1,\mathrm{test}}$ is the union of $\mathcal{S}_\beta$ and $\mathcal{S}_\alpha$. To complete the proof, we must show that the union of these sets is, itself, convex. When only one of the problems can achieve a rate of at least $R_{1,\mathrm{test}}$, one of $\mathcal{S}_\beta$ and $\mathcal{S}_\alpha$ is empty, and hence the convexity of the union follows directly from the convexity of the non-empty set. For cases in which both $\mathcal{S}_\beta$ and $\mathcal{S}_\alpha$ are non-empty, the fact that r is a scalar means that it is sufficient to prove that the sets intersect. A proof that they do intersect is provided in Appendix E. Therefore, when $\gamma_{i0}\bar{P}_i \geqslant 1/2$, for a given target rate for Node 2, the set of values for r for which the maximum achievable value for the rate of Node 1 is greater than a given rate is a convex set. Hence, the maximum achievable rate for Node 1 is quasi-concave in r. ■

The result in Proposition 1 means that whenever the maximum achievable SNR of both direct channels is greater than -3 dB, as would typically be the case in practice, we can determine the optimal value for r using a standard efficient search method for quasi-convex problems; e.g., [24]. For the particular problem at hand, a simple approach that is closely related to bisection search is provided in Table I. At each step in that approach, we use the closed-form expression in (17) to determine the optimal power allocation for each of the current values of r. Since the quasi-convex search can be efficiently implemented and since it converges rapidly, the jointly optimal value for r and the (scaled) powers $\tilde{P}_{ij}$ can be efficiently obtained. Furthermore, since the condition $\gamma_{i0}\bar{P}_i > \frac{1}{2}$ depends only on the direct channel gains, the noise variance at the master node and the power constraints, this condition is testable before the design process commences.

V. Modified Orthogonal AF Cooperation Scheme

The analysis of the KKT optimality conditions for the problem in (8) showed that for the cooperation scheme in Fig. 2, the optimal power allocation results in at least one of $\tilde{P}_{12}$ and $\tilde{P}_{21}$ being equal to zero; see Section III-B. As a result, both nodes will be silent in at least one of the blocks in Fig. 2. This suggests that the cooperation scheme in Fig. 2 does not make efficient use of the channel resources. A question that then arises is whether there is an alternative orthogonal half-duplex amplify-and-forward cooperation scheme that retains the benefits of the original scheme, such as simplified transmitters and receivers, yet uses the channel resources more efficiently. In this section we will propose a modified version of the protocol in Fig. 2 that satisfies these requirements.

The proposed scheme is based on time sharing between the states shown in Fig. 3. In the first state, the message of Node 1 is transmitted using AF relaying in a fraction r_1 of the frame, with Node 2 working as the relay and allocating a power P_{21} to the relaying of the message of Node 1. In contrast, Node 2 employs direct transmission to transmit its message to the master node in a fraction $\hat{r}_1 = 1 - r_1$ of the frame using power P_{22}. In the second state, Nodes 1 and 2 exchange roles. In the first state, the received signal blocks can be expressed as

$$\mathbf{y}_2(\ell) = \begin{cases} K_{12}\mathbf{x}_1(\ell) + \mathbf{z}_2(\ell) & \ell \bmod 3 = 1, \\ \mathbf{0} & \ell \bmod 3 \neq 1, \end{cases} \tag{18}$$

$$\mathbf{y}_0(\ell) = \begin{cases} K_{10}\mathbf{x}_1(\ell) + \mathbf{z}_0(\ell) & \ell \bmod 3 = 1, \\ K_{20}A_2\mathbf{y}_2(\ell-1) + \mathbf{z}_0(\ell) & \ell \bmod 3 = 2, \\ K_{20}\mathbf{x}_2(\ell) + \mathbf{z}_0(\ell) & \ell \bmod 3 = 0, \end{cases} \tag{19}$$

where $A_2 = \sqrt{\frac{P_{21}}{|K_{12}|^2 P_{11} + \sigma_2^2}}$. In the second state, the received signal blocks are

$$\mathbf{y}_1(\ell) = \begin{cases} K_{21}\mathbf{x}_2(\ell) + \mathbf{z}_1(\ell) & \ell \bmod 3 = 2, \\ \mathbf{0} & \ell \bmod 3 \neq 2, \end{cases} \tag{20}$$

$$\mathbf{y}_0(\ell) = \begin{cases} K_{10}\mathbf{x}_1(\ell) + \mathbf{z}_0(\ell) & \ell \bmod 3 = 1, \\ K_{20}\mathbf{x}_2(\ell) + \mathbf{z}_0(\ell) & \ell \bmod 3 = 2, \\ K_{10}A_1\mathbf{y}_1(\ell-1) + \mathbf{z}_0(\ell) & \ell \bmod 3 = 0, \end{cases} \tag{21}$$

where $A_1 = \sqrt{\frac{P_{12}}{|K_{21}|^2 P_{22} + \sigma_1^2}}$.

TABLE I
A Simple Method for Finding r^*

Given $R_{2,\mathrm{tar}} \in (0, R_{2,\max}(0))$, for $r \in (0,1)$ define $\psi(r)$ denote the optimal value of (8) if $R_{2,\mathrm{tar}} \in (0, R_{2,\max}(r))$ and zero otherwise. Set $\psi(0) = 0$ and $\psi(1) = 0$. Set $t_0 = 0$, $t_4 = 1$, and $t_2 = 1/2$. Using the closed-form expression for the optimal power allocation in (17) compute $\psi(t_2)$. Given a tolerance ϵ,

1) Set $t_1 = (t_0 + t_2)/2$ and $t_3 = (t_2 + t_4)/2$.
2) Using the closed-form expression in (17), compute $\psi(t_1)$ and $\psi(t_3)$.
3) Find $k^* = \arg\max_{k\in\{0,1,\ldots,4\}} \psi(t_k)$.
4) Replace t_0 by $t_{\max\{k^*-1,0\}}$, replace t_4 by $t_{\min\{k^*+1,4\}}$, and save $\psi(t_0)$ and $\psi(t_4)$. If $k^* \notin \{0,4\}$ set $t_2 = t_{k^*}$ and save $\psi(t_2)$, else set $t_2 = (t_0 + t_4)/2$ and use (17) to calculate $\psi(t_2)$.
5) If $t_4 - t_0 \geq \epsilon$ return to 1), else set $r^* = t_{k^*}$.

$x_1(1) = \sqrt{P_{11}}B_{11}$	$x_1(2) = 0$	$x_1(3) = 0$
$x_2(1) = 0$	$x_2(2) = A_2 y_2(1)$	$x_2(3) = \sqrt{P_{22}}B_{21}$
r_1		$\hat{r}_1 = 1 - r_1$

(a) State 1

$x_1(1) = \sqrt{P_{11}}B_{11}$	$x_1(2) = 0$	$x_1(3) = A_1 y_1(2)$
$x_2(1) = 0$	$x_2(2) = \sqrt{P_{22}}B_{21}$	$x_2(3) = 0$
r_2	$\hat{r}_2 = 1 - r_2$	

(b) State 2

Fig. 3. One frame of each state of the proposed modified orthogonal amplify-and-forward cooperation scheme.

Using techniques analogous to those used in Sections III and IV, we can obtain a simple efficient algorithm for the joint optimization of the transmission powers and channel resource allocation for each state of the proposed cooperation scheme.

In particular, in the first state, for a given resource allocation parameter $r_1 \in (0,1)$ and a feasible $R_{2,\mathrm{tar}}$, it can be shown that $\tilde{P}_{12}^* = 0$, and hence the optimal power allocation problem can be reduced to

$$\phi(r_1) = \max_{\tilde{P}_{21}\in[0,2\bar{P}_2]} \frac{r_1}{2}\log\biggl(1+\frac{2\gamma_{10}\bar{P}_1}{r_1} + \frac{2\gamma_{20}\gamma_{12}\bar{P}_1\tilde{P}_{21}}{r_1(r_1+\gamma_{20}\tilde{P}_{21}+2\gamma_{12}\bar{P}_1)}\biggr) \tag{22a}$$

$$\text{subject to} \quad \hat{r}_1\log\biggl(1+\frac{\gamma_{20}(2\bar{P}_2-\tilde{P}_{21})}{2\hat{r}_1}\biggr) \geqslant R_{2,\mathrm{tar}}. \tag{22b}$$

Using a similar argument to that in Section III-C, the optimal solution to (22) can be shown to be $\tilde{P}_{21}^* = 2\bar{P}_2 - \breve{Q}_1$, where $\breve{Q}_1 = \frac{2\hat{r}_1}{\gamma_{20}}(2^{\frac{R_{2,\mathrm{tar}}}{\hat{r}_1}}-1)$, and hence the the optimal (scaled) power allocation is $\tilde{\mathcal{P}}_1^* = (2\bar{P}_1,\, 0,\, 2\bar{P}_2-\breve{Q}_1,\, \breve{Q}_1)$. In the second state, for a given $r_2 \in (0,1)$ and a feasible $R_{2,\mathrm{tar}}$, it can be shown that $\tilde{P}_{21}^* = 0$, and hence the optimal power allocation problem can be reduced to

$$\lambda(r_2) = \max_{\tilde{P}_{11}\in[0,2\bar{P}_1]} r_2\log\Bigl(1+\frac{\gamma_{10}\tilde{P}_{11}}{2r_2}\Bigr) \tag{23a}$$

$$\text{subject to} \quad \frac{\hat{r}_2}{2}\log\biggl(1+\frac{2\gamma_{20}\bar{P}_2}{\hat{r}_2} + \frac{2\gamma_{10}\gamma_{21}(2\bar{P}_1-\tilde{P}_{11})\bar{P}_2}{\hat{r}_2(\hat{r}_2+2\gamma_{21}\bar{P}_2+\gamma_{10}(2\bar{P}_1-\tilde{P}_{11}))}\biggr) \geqslant R_{2,\mathrm{tar}}. \tag{23b}$$

By adapting the argument in Section III-C, the optimal solution to (23) can be shown to be $\tilde{P}_{11}^* = 2\bar{P}_1 - \breve{Q}_2$, where

$$\breve{Q}_2 = \frac{(2\bar{P}_2\gamma_{21}+\hat{r}_2)\bigl(\hat{r}_2(2^{\frac{2R_{2,\mathrm{tar}}}{\hat{r}_2}}-1)-2\bar{P}_2\gamma_{20}\bigr)}{\gamma_{10}\bigl(2\bar{P}_2\gamma_{21}-\bigl(\hat{r}_2(2^{\frac{2R_{2,\mathrm{tar}}}{\hat{r}_2}}-1)-2\bar{P}_2\gamma_{20}\bigr)\bigr)}, \tag{24}$$

and hence the the optimal (scaled) power allocation is $\tilde{\mathcal{P}}_2 = (2\bar{P}_1-\breve{Q}_2,\, \breve{Q}_2,\, 0,\, 2\bar{P}_2)$.

Furthermore, using a proof analogous to that in Appendix D, it can be shown that $\phi(r_1)$ and $\lambda(r_2)$ are quasi-convex in r_1 and r_2, respectively. Hence, the jointly optimal power and channel resource allocation for each point on the boundary of the achievable rate region for each of the two states can be efficiently obtained. The achievable rate region for the proposed cooperation scheme is the convex hull of those two regions. In the simulation results section below, this region will be shown to subsume the region achieved by the cooperation scheme in Fig. 2.

VI. Simulation Results

The goal of this section is three fold. First, we compare the achievable rate regions of the original cooperative scheme in Fig. 2 with jointly optimal power and channel resource allocation to the achievable rate regions obtained by the same scheme with optimal power allocation but a fixed channel resource allocation. Second, we compare the achievable rate region of the (jointly optimized) modified cooperation scheme in Section V against that of the (jointly optimized) original scheme in Fig. 2. Finally, we investigate the performance of an approach to robust power and resource allocation that provides achievable rate guarantees in the presence of uncertain channel state information (CSI). For each investigation, we examine a (symmetric) scenario in which the gains of the direct channels of each user are the same, and an (asymmetric) scenario in which they are different.

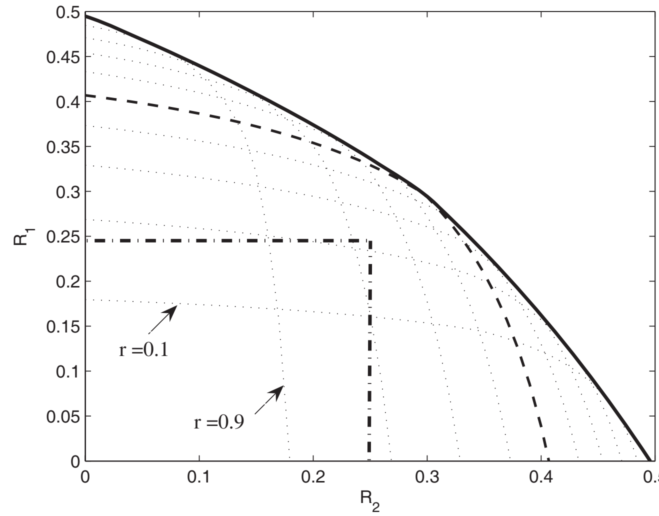

Fig. 4. Achievable rate region in a case of symmetric direct channels. $\bar{P}_1 = \bar{P}_2 = 2.0, \sigma_0^2 = \sigma_1^2 = \sigma_2^2 = 1, |K_{12}| = |K_{21}| = 0.7, |K_{10}| = |K_{20}| = 0.4$. The solid curve represents the case of joint optimization over $\tilde{P}_{ij}$ and r, the dotted curves represent the case of fixing $r = 0.1k$, $k = 1, 2, \ldots, 9$ and optimizing only over $\tilde{P}_{ij}$. The dashed curve represents the case of optimization over $\tilde{P}_{ij}$ for $r = 0.5$. The dash-dot curve represents the the case of equal power and resource allocation.

We first consider the original cooperation scheme in Fig. 2 in the case of symmetric direct channels. In Fig. 4, we have plotted with a solid line the achievable rate region that can be obtained by jointly optimizing both the power components and the time sharing parameter r using the efficient quasi-convex search method suggested in Section IV. In the same figure, we have plotted the rate regions that can be achieved with a fixed resource allocation parameter; i.e., fixed r and optimized power allocation. (We have plotted the rate region for equal power and resource allocation, as well.) Fig. 6 is analogous to Fig. 4, except that it considers a case of asymmetric direct channels. We note that in both Figs 4 and 6, the region bounded by the solid curve, which represents the achievable rate region when one jointly optimizes over both the transmission powers and r, subsumes the regions bounded by the dashed curve and all the dotted curves. In fact, the region bounded by the solid curve represents the convex hull of all the achievable rate regions for fixed resource allocation. Also, we point out that each of the dotted curves and the dashed curve touches the solid curve at only one point. This is the point at which this particular value of r is optimal.

The optimal value of the resource allocation parameter r and the optimized (scaled) power allocations $\tilde{P}_{11}$ and $\tilde{P}_{21}$ are plotted as a function of the target value $R_{2,\mathrm{tar}}$ in Figs 5 and 7.

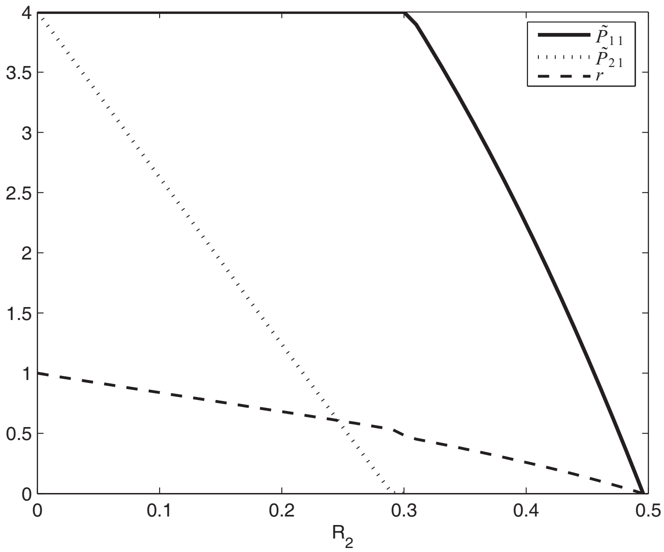

Fig. 5. Jointly optimized power and resource allocations in the case of symmetric direct channels considered in Fig. 4.

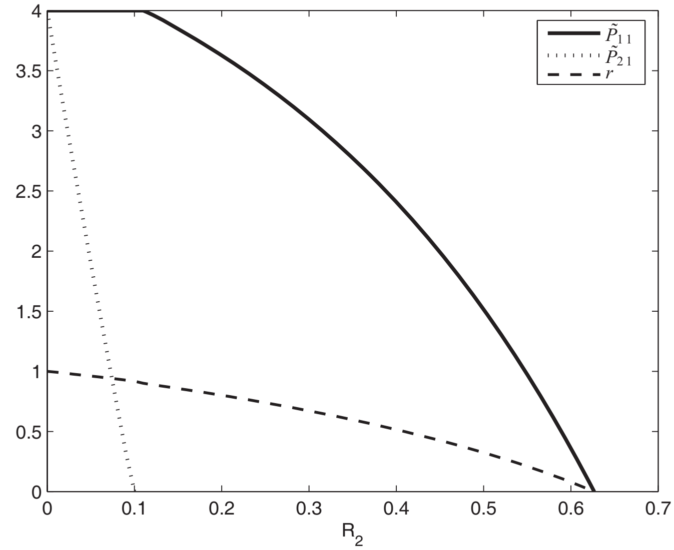

Fig. 7. Jointly optimized power and resource allocations in the case of asymmetric direct channels considered in Fig. 6.

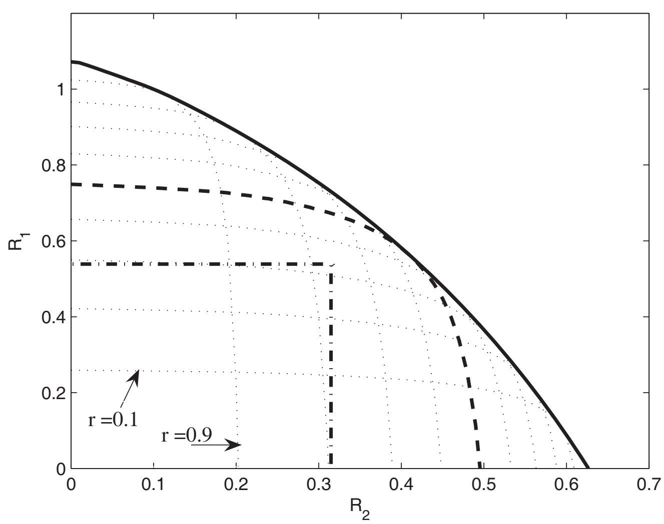

Fig. 6. Achievable rate region in a case of asymmetric direct channels. $\bar{P}_1 = \bar{P}_2 = 2.0, \sigma_0^2 = \sigma_1^2 = \sigma_2^2 = 1, |K_{12}| = |K_{21}| = 0.7, |K_{10}| = 0.9, |K_{20}| = 0.3$. The solid curve represents the case of joint optimization over $\tilde{P}_{ij}$ and r, the dotted curves represent the case of fixing $r = 0.1k$, $k = 1, 2, \ldots, 9$ and optimizing only over $\tilde{P}_{ij}$. The dashed curve represents the case of optimization over $\tilde{P}_{ij}$ for $r = 0.5$. The dash-dot curve represents the the case of equal power and resource allocation.

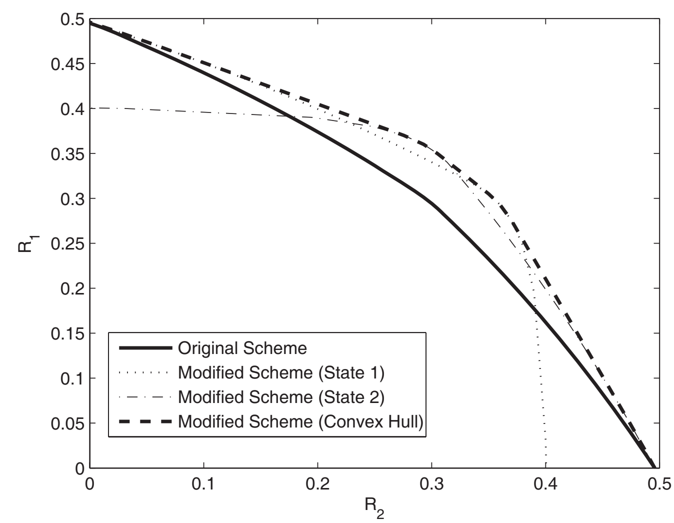

Fig. 8. Achievable rate region of the modified orthogonal AF scheme in the case of symmetric direct channels considered in Fig. 4.

In both these figures, we observe that the value of r decreases as $R_{2,\mathrm{tar}}$ increases. This is what one would expect, because for increasing values of $R_{2,\mathrm{tar}}$ the fraction of the channel resource allocated to Node 2 (i.e., $\hat{r} = 1 - r$) should be increased. Fig. 5 also verifies the analysis of the KKT conditions in Section III-B, which revealed that at optimality at least one of the nodes will turn off its relaying function. When $R_{2,\mathrm{tar}}$ is small, we observe that $\tilde{P}_{11} = 2\bar{P}_1$ and hence $\tilde{P}_{12} = 0$. This means that Node 1 does not allocate any power for relaying, and hence that Node 2 must transmit directly to the master node. At high target rates for Node 2, $\tilde{P}_{21} = 0$, which means that Node 2 does not relay the message of Node 1. For a small range of target rates around $R_{2,\mathrm{tar}} = 0.3$, both $\tilde{P}_{12} = 0$ and $\tilde{P}_{21} = 0$, and there is no cooperation between the two nodes. (Both nodes use direct transmission.) The increase in $R_{2,\mathrm{tar}}$ in this region is obtained by decreasing the resource allocation parameter r (i.e., increasing $\hat{r}$), and the change in the slope of the dashed curve that represents r in Fig. 5 can be clearly seen in this region.

In Figs 8 and 9 we compare the achievable rate region of the modified scheme proposed in Section V against that of the original AF scheme in Fig. 2. Fig. 8 considers the case of symmetric channels, and Fig. 9 considers the asymmetric case. It is clear from these figures that the (jointly optimized) modified scheme provides a significantly larger achievable rate region than the (jointly optimized) original scheme. In Fig. 8, we note that the maximum achievable rate for Node 1 for the modified scheme is the same as that of the original scheme. This is because in this scenario, the effective gain of the inter-user channel ($\gamma_{12} = \gamma_{21}$) is large enough, relative to the effective gain of the direct channel, so that the maximum value of R_1 for the modified scheme occurs in State 1, in

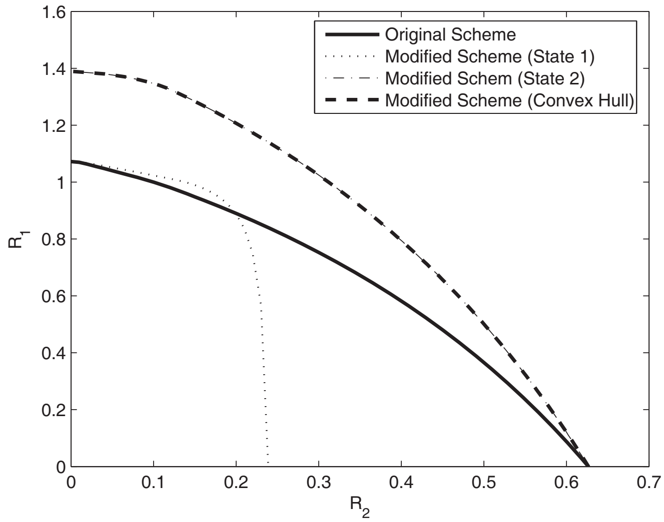

Fig. 9. Achievable rate region of the modified orthogonal AF scheme in the case of asymmetric direct channels considered in Fig. 6.

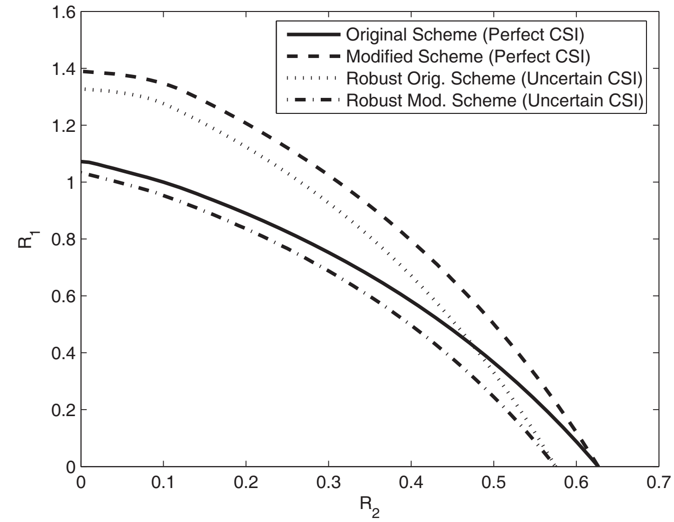

Fig. 11. Achievable rate regions of the original and modified schemes with perfect and uncertain channel state information in the case of asymmetric direct channels considered in Fig. 6.

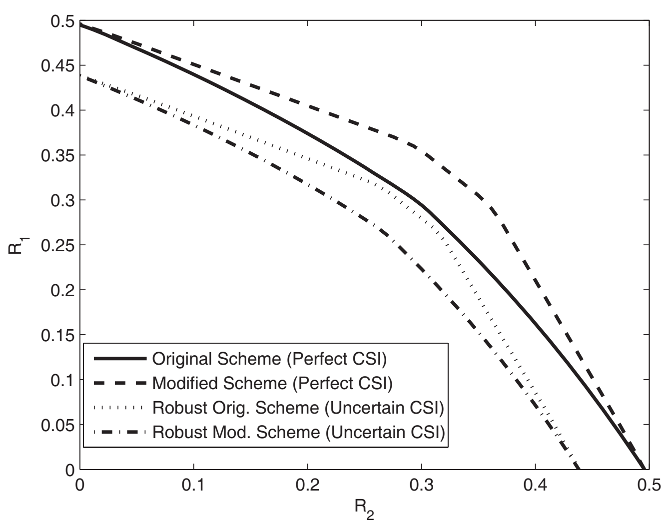

Fig. 10. Achievable rate regions of the original and modified schemes with perfect and uncertain channel state information in the case of symmetric direct channels considered in Fig. 4.

which the message of Node 1 is relayed by Node 2 and all the transmission powers and channel resources are allocated to the message of Node 1. In contrast, in the scenario plotted in Fig. 9 the gain of the direct channel of Node 1 is large enough, relative to the effective gain of the inter-user channel, so that the maximum value of R_1 occurs in State 2, in which the message of Node 1 is transmitted directly and all the channel resources are allocated to the message of Node 1. As can be seen in Fig. 9, this means that the maximum achievable rate for Node 1 is larger than that provided by the original scheme in Fig. 2.

In our final experiment, we consider a scenario in which the available CSI is uncertain, in the sense that the effective channel gains are only known to lie in the interval $\gamma_{ij} \in [\hat{\gamma}_{ij} - \delta_{ij}, \hat{\gamma}_{ij} + \delta_{ij}]$. This model is well matched to scenarios that involve the communication of quantized channel estimates of the channel gains via low-rate feedback. Our goal here is to obtain the power and resource allocations that provide the largest rate region that is guaranteed to be achievable under all admissible uncertainties. This can be achieved by applying the existing approaches to the scenario in which all the effective channel gains assume their lowest admissible value; i.e., $\gamma_{ij} = \hat{\gamma}_{ij} - \delta_{ij}$. In Figs 10 and 11 we compare the resulting robust achievable rate regions to the achievable rate regions for the case of perfect CSI, for a scenario in which $\delta_{ij} = 1/32$. (This corresponds to a quantization scheme for γ_{ij} with four bits and a dynamic range of $[0, 1]$.) As expected, channel uncertainty reduces the size of the achievable rate region, but this example demonstrates that robust performance in the presence of channel uncertainties can be obtained in a relatively straightforward manner.

VII. Conclusion

In this paper we addressed the problem of joint power and channel resource allocation for a two-user orthogonal amplify-and-forward cooperative scheme. We obtained a closed-form expression for the optimal power allocation problem for a given channel resource allocation, and we exploited the quasi-convexity of the power and channel resource allocation problem to obtain a simple efficient algorithm for the jointly optimal allocation. Analysis of some KKT optimality conditions showed that the original system under consideration does not use the channel resources efficiently. Therefore, we proposed a modified orthogonal AF cooperation scheme, and we demonstrated that with optimal power and channel resource allocation this scheme can provide a larger achievable rate region than that provided by the original scheme. Finally, we provided a simple strategy that enables efficient optimization of a guaranteed achievable rate region in the presence of bounded uncertainties in the available channel state information.

APPENDIX A
AT LEAST ONE OF $\tilde{P}_{12}$ AND $\tilde{P}_{21}$ WILL BE ZERO AT OPTIMALITY

Since all the partial derivatives will be evaluated at the optimal point $(\tilde{P}_{11}^*, \tilde{P}_{21}^*)$, for simplicity, we will use $\frac{\partial f_i}{\partial \tilde{P}_{ij}}$ to refer to $\frac{\partial f_i(\tilde{P}_{11}^*, \tilde{P}_{21}^*)}{\partial \tilde{P}_{ij}}$. We are interested in the case in which both nodes have information to transmit, and hence $\tilde{P}_{11}^* > 0$, and $\tilde{P}_{21}^* < 2\bar{P}_2$. In that case it can be (analytically) shown that $\frac{\partial f_0}{\partial \tilde{P}_{i1}} < 0$, and that $\frac{\partial f_1}{\partial \tilde{P}_{i1}} > 0$.

Let us first consider the case in which $\lambda_4^* = 0$. From (12c) we have that $\frac{\partial f_0}{\partial \tilde{P}_{11}} + \lambda_1^* \frac{\partial f_1}{\partial \tilde{P}_{11}} \geqslant 0$, and hence $\lambda_1^* \geqslant -\frac{\partial f_0}{\partial \tilde{P}_{11}} / \frac{\partial f_1}{\partial \tilde{P}_{11}}$. Therefore,

$$\frac{\partial f_0}{\partial \tilde{P}_{21}} + \lambda_1^* \frac{\partial f_1}{\partial \tilde{P}_{21}} \geqslant \left(\frac{\partial f_0}{\partial \tilde{P}_{21}} \frac{\partial f_1}{\partial \tilde{P}_{11}} - \frac{\partial f_0}{\partial \tilde{P}_{11}} \frac{\partial f_1}{\partial \tilde{P}_{21}} \right) / \frac{\partial f_1}{\partial \tilde{P}_{11}}. \tag{25}$$

By directly computing the partial derivatives and dropping some non-negative terms from the expression for $-\frac{\partial f_0}{\partial \tilde{P}_{11}} \frac{\partial f_1}{\partial \tilde{P}_{21}}$ it can be shown that

$$\begin{aligned}\frac{\partial f_0}{\partial \tilde{P}_{21}} \frac{\partial f_1}{\partial \tilde{P}_{11}} - \frac{\partial f_0}{\partial \tilde{P}_{11}} \frac{\partial f_1}{\partial \tilde{P}_{21}} >& (3r\hat{r}\tilde{P}_{11}^* + \gamma_{12}\hat{r}\tilde{P}_{11}^{*2})(2\bar{P}_2 - \tilde{P}_{21}^*) \\ &+ \gamma_{21} r \tilde{P}_{11}^* (2\bar{P}_2 - \tilde{P}_{21}^*)^2,\end{aligned} \tag{26}$$

where the right hand side is positive because $\tilde{P}_{21}^*$ is required to satisfy the power constraint $\tilde{P}_{21}^* \leqslant 2\bar{P}_2$. Using (25), and the positivity of (26) and $\frac{\partial f_1}{\partial \tilde{P}_{11}}$, we have that $\frac{\partial f_0}{\partial \tilde{P}_{21}} + \lambda_1^* \frac{\partial f_1}{\partial \tilde{P}_{21}} > 0$. Given this relation and (12b), in order for (12c) to be satisfied with $\lambda_4^* = 0$ we must have $\lambda_3^* > 0$. Using (12d), this implies that $\tilde{P}_{21} = 0$.

Using a similar strategy, we can show that if $\lambda_3^* = 0$, then $\frac{\partial f_0}{\partial \tilde{P}_{11}} + \lambda_1^* \frac{\partial f_1}{\partial \tilde{P}_{11}} < 0$. Given this relation and (12b), in order for (12c) to be satisfied with $\lambda_3^* = 0$ we must have $\lambda_4^* > 0$. Using (12d), this implies that $\tilde{P}_{11} = 2\bar{P}_1$ and hence that $\tilde{P}_{12} = 0$. Therefore, at optimality at least one of $\tilde{P}_{12}$ and $\tilde{P}_{21}$ equals zero.

APPENDIX B
A BOUNDING ARGUMENT

Consider a value for $r \in (0,1)$ and a set of feasible values for $\tilde{P}_{j1} = rP_{j1}$ and $\tilde{P}_{j2} = \hat{r}P_{j2}$ that satisfy (8c) with equality. Furthermore, assume that both nodes are relaying for each other; i.e., $\tilde{P}_{12} > 0$, and $\tilde{P}_{21} > 0$. The achievable rates $\bar{R}_1(\mathcal{P}, r)$ and $\bar{R}_2(\mathcal{P}, r)$ for this scenario are given by the expressions on the right hand side of (5). The key to the argument is to fix the rate of one of the nodes and show that re-allocating the powers so that this rate is achieved by direct transmission increases the achievable rate of the other node.

First, let us consider the case in which $\gamma_{10}\tilde{P}_{12} \geqslant \gamma_{20}\tilde{P}_{21}$, and let us fix the rate of Node 1. If we let $\Delta_1 = \frac{\tilde{\Delta}_1}{r}$ denote the extra power that would need to be added to P_{11} in order to achieve $\bar{R}_1(\mathcal{P}, r)$ by direct transmission, then we have $\bar{R}_1(\mathcal{P}, r) = \frac{r}{2}\log\left(1 + \frac{\gamma_{10}(\tilde{P}_{11} + \tilde{\Delta}_1)}{r}\right)$, from which we obtain

$$\gamma_{10}\tilde{\Delta}_1 = \frac{\gamma_{20}\gamma_{12}\tilde{P}_{11}\tilde{P}_{21}}{r + \gamma_{20}\tilde{P}_{21} + \gamma_{12}\tilde{P}_{11}}. \tag{27}$$

Now, let $\tilde{P}_{ij}^{'}$ denote the (scaled) power allocation for this scenario. First, $\tilde{P}_{11}^{'} = \tilde{P}_{11} + \tilde{\Delta}_1$, and in order for the power constraint to be satisfied, we must have $\tilde{P}_{12}^{'} \leqslant \tilde{P}_{12} - \tilde{\Delta}_1$. Furthermore, since Node 1 can achieve its desired rate by direct transmission, Node 2 need not allocate power to relay messages for Node 1; i.e., Node 2 can set $\tilde{P}_{21}^{'} = 0$ and hence can set $\tilde{P}_{22}^{'} = 2\bar{P}_2$. If Node 1 uses all of its remaining power to relay for Node 2, i.e., if $\tilde{P}_{12}^{'} = \tilde{P}_{12} - \tilde{\Delta}_1$, then the achievable rate for Node 2 is

$$\bar{R}_2(\mathcal{P}^{'}, r) = \frac{\hat{r}}{2}\log\left(1 + \frac{\gamma_{20}\tilde{P}_{22}^{'}}{\hat{r}} + \frac{\gamma_{10}\gamma_{21}\tilde{P}_{12}^{'}\tilde{P}_{22}^{'}}{\hat{r}(\hat{r} + \gamma_{21}\tilde{P}_{22}^{'} + \gamma_{10}(\tilde{P}_{12} - \tilde{\Delta}_1))}\right), \tag{28}$$

where $\mathcal{P}^{'}$ is defined in an analogous way to $\mathcal{P}$ in (5). Using the sequence of bounds in Appendix C, it can be shown that $\bar{R}_2(\mathcal{P}^{'}, r) > \bar{R}_2(\mathcal{P}, r)$. Therefore, for the same value for the rate of Node 1 a higher rate for Node 2 is achievable if Node 1 operates via direct transmission rather than operating cooperatively. In the case that $\gamma_{10}P_{12} \leqslant \gamma_{20}P_{21}$, an analogous argument applies, but with the rate of Node 2 being fixed and achievable by direct transmission.

APPENDIX C
PROOF OF $\bar{R}_2(\mathcal{P}^{'}, r) > \bar{R}_2(\mathcal{P}, r)$

The bounding argument below is based on the fact that for any non-negative and finite a, b, c and r

$$\frac{ab}{r+a+b} \leqslant a \qquad \text{and} \qquad \frac{\partial}{\partial a}\left(\frac{ab}{r+a+b}\right) \geqslant 0, \tag{29}$$

and that for $r > 0$

$$\frac{ab}{r+b+c} < a. \tag{30}$$

Now

$$\begin{aligned}
&\bar{R}_2(\mathcal{P}^{'}, r) \\
&= \frac{\hat{r}}{2}\log\left(1 + \frac{\gamma_{20}\tilde{P}_{22}^{'}}{\hat{r}} + \frac{\gamma_{10}\gamma_{21}\tilde{P}_{12}^{'}\tilde{P}_{22}^{'}}{\hat{r}(\hat{r} + \gamma_{21}\tilde{P}_{22}^{'} + \gamma_{10}(\tilde{P}_{12} - \tilde{\Delta}_1))}\right) \\
&= \frac{\hat{r}}{2}\log\left(1 + \frac{\gamma_{20}\tilde{P}_{22}}{\hat{r}} + \frac{\gamma_{20}\tilde{P}_{21}}{\hat{r}} + \frac{\gamma_{10}\gamma_{21}\tilde{P}_{12}\tilde{P}_{22}^{'}}{\hat{r}(\hat{r} + \gamma_{21}\tilde{P}_{22}^{'} + \gamma_{10}(\tilde{P}_{12} - \tilde{\Delta}_1))}\right. \\
&\qquad \left. - \frac{\gamma_{10}\gamma_{21}\tilde{\Delta}_1\tilde{P}_{22}^{'}}{\hat{r}(\hat{r} + \gamma_{21}\tilde{P}_{22}^{'} + \gamma_{10}(\tilde{P}_{12} - \tilde{\Delta}_1))}\right) \\
&>_a \frac{\hat{r}}{2}\log\left(1 + \frac{\gamma_{20}\tilde{P}_{22}}{\hat{r}} + \frac{\gamma_{20}\tilde{P}_{21}}{\hat{r}} + \frac{\gamma_{10}\gamma_{21}\tilde{P}_{12}\tilde{P}_{22}^{'}}{\hat{r}(\hat{r} + \gamma_{21}\tilde{P}_{22}^{'} + \gamma_{10}(\tilde{P}_{12} - \tilde{\Delta}_1))}\right. \\
&\qquad \left. - \frac{\gamma_{10}\tilde{\Delta}_1}{\hat{r}}\right) \\
&>_b \frac{\hat{r}}{2}\log\left(1 + \frac{\gamma_{20}\tilde{P}_{22}}{\hat{r}} + \frac{\gamma_{20}\tilde{P}_{21}}{\hat{r}} + \frac{\gamma_{10}\gamma_{21}\tilde{P}_{12}\tilde{P}_{22}^{'}}{\hat{r}(\hat{r} + \gamma_{21}\tilde{P}_{22}^{'} + \gamma_{10}(\tilde{P}_{12} - \tilde{\Delta}_1))}\right. \\
&\qquad \left. - \frac{\gamma_{20}\tilde{P}_{21}}{\hat{r}}\right) \\
&= \frac{\hat{r}}{2}\log\left(1 + \frac{\gamma_{20}\tilde{P}_{22}}{\hat{r}} + \frac{\gamma_{10}\gamma_{21}\tilde{P}_{12}\tilde{P}_{22}^{'}}{\hat{r}(\hat{r} + \gamma_{21}\tilde{P}_{22}^{'} + \gamma_{10}(\tilde{P}_{12} - \tilde{\Delta}_1))}\right) \\
&>_c \frac{\hat{r}}{2}\log\left(1 + \frac{\gamma_{20}\tilde{P}_{22}}{\hat{r}} + \frac{\gamma_{10}\gamma_{21}\tilde{P}_{12}\tilde{P}_{22}}{\hat{r}(\hat{r} + \gamma_{21}\tilde{P}_{22} + \gamma_{10}(\tilde{P}_{12} - \tilde{\Delta}_1))}\right) \\
&> \frac{\hat{r}}{2}\log\left(1 + \frac{\gamma_{20}\tilde{P}_{22}}{\hat{r}} + \frac{\gamma_{10}\gamma_{21}\tilde{P}_{12}\tilde{P}_{22}}{\hat{r}(\hat{r} + \gamma_{21}\tilde{P}_{22} + \gamma_{10}\tilde{P}_{12})}\right) > \bar{R}_2(\mathcal{P}, r),
\end{aligned}$$

where inequality a is a consequence of (30) (assuming $\hat{r} > 0$), inequality b is obtained by applying the first inequality in (29) to (27), which yields $\gamma_{10}\tilde{\Delta}_1 \leqslant \gamma_{20}\tilde{P}_{21}$, and inequality c comes from the second inequality in (29).

APPENDIX D
QUASI-CONCAVITY OF THE RESOURCE ALLOCATION PROBLEM

Consider $r_1, r_2 \in \mathcal{S}_\beta$. Let x_1 and x_2 denote the maximizing values of $\tilde{P}_{21}$ corresponding to those values of r, respectively. Then the following pair of equations hold for $(r = r_1, \tilde{P}_{21} = x_1)$ and for $(r = r_2, \tilde{P}_{21} = x_2)$,

$$\frac{r}{2}\log\left(1 + \frac{2\gamma_{10}\bar{P}_1}{r} + \frac{2\gamma_{20}\gamma_{12}\bar{P}_1\tilde{P}_{21}}{r(r+\gamma_{20}\tilde{P}_{21}+2\gamma_{12}\bar{P}_1)}\right) \geqslant R_{1,\text{test}}, \tag{31a}$$

$$\frac{\hat{r}}{2}\log\bigl(1 + \frac{\gamma_{20}}{\hat{r}}(2\bar{P}_2 - \tilde{P}_{21})\bigr) \geqslant R_{2,\text{tar}}. \tag{31b}$$

The inequalities in (31) can be rewritten as

$$f_1(r_1, x_1) \geqslant g_1(r_1), \qquad f_1(r_2, x_2) \geqslant g_1(r_2), \tag{32a}$$

$$f_2(r_1, x_1) \geqslant g_2(r_1), \qquad f_2(r_2, x_2) \geqslant g_2(r_2), \tag{32b}$$

where

$$f_1(r,x) = \sqrt{r^2 + 2\gamma_{10}\bar{P}_1 r + \frac{2\gamma_{20}\gamma_{12}\bar{P}_1 x r}{(r+\gamma_{20}x+2\gamma_{12}\bar{P}_1)}},$$

$$f_2(r,x) = 2\bar{P}_2 - x,$$

$$g_1(r) = \sqrt{r^2 2^{\frac{2R_{1,\text{test}}}{r}}}, \quad g_2(r) = \hat{r}(2^{\frac{2R_{2,\text{tar}}}{\hat{r}}} - 1).$$

By evaluating their second-order derivatives, it can be shown that $g_1(r)$ and $g_2(r)$ are convex in r and that $f_2(r,x)$ is convex in r and x. Furthermore, by obtaining an analytic expression for the Hessian of $f_1(r,x)$, it can be shown that the condition $2\gamma_{10}\bar{P}_1 \geqslant 1$ is sufficient for the Hessian to be negative semi-definite, and hence is sufficient for $f_1(r,x)$ to be concave in r and x.

Let $r_3 = \theta r_1 + (1-\theta)r_2$, for some $\theta \in [0,1]$, and let $x_3 = \theta x_1 + (1-\theta)x_2$. Then we have

$$\begin{aligned} f_1(r_3, x_3) &\geqslant_a \theta f_1(r_1,x_1) + (1-\theta)f_1(r_2,x_2) \\ &\geqslant \theta g_1(r_1) + (1-\theta)g_1(r_2) \geqslant_b g_1(r_3), \end{aligned} \tag{33}$$

where inequality a follows from the concavity of $f_1(r,x)$ and inequality b follows from the convexity of $g_1(r)$. Similarly, we can show that $f_2(r_3,x_3) \geqslant g_2(r_3)$. Hence, for any two values of r (namely, r_1 and r_2), if there exist values of x (namely, x_1 and x_2), such that the conditions in (31) are satisfied, then for any value of r that lies between r_1 and r_2 (namely, r_3), there exists a value for x that lies between x_1 and x_2 (namely, x_3), such that the conditions in (31) are satisfied. Therefore, $\mathcal{S}_\beta$ is a convex set, and hence the objective in (13) is quasi-concave in r. Following similar steps, it can be shown that the objective in (14) is quasi-concave in r if $2\gamma_{20}\bar{P}_2 \geqslant 1$.

APPENDIX E
INTERSECTION OF THE SETS OF r THAT RESULT FROM (13) AND (14)

Here we will show that if $R_{1,\text{test}}$ is such that both $\mathcal{S}_\beta$ and $\mathcal{S}_\alpha$ are non-empty, then these two sets intersect. In doing so, we will show that if both sets are non-empty, then there exists a value of r, denoted r_3, such that a rate of at least $R_{1,\text{test}}$ can be achieved for Node 1 (and the target rate for Node 2 satisfied) using only direct transmission. Since direct transmission is a feasible solution for both (13) and (14), r_3 lies in both $\mathcal{S}_\beta$ and $\mathcal{S}_\alpha$, and hence these sets intersect.

Using the closed-form solution in (17) we have that

$$\beta(r) = \frac{r}{2}\log\left(1 + \frac{2\bar{P}_1\gamma_{10}}{r} + \frac{2\bar{P}_1\gamma_{12}(2\bar{P}_2\gamma_{20} - \hat{r}(2^{\frac{2R_{2,\text{tar}}}{\hat{r}}}-1))}{r(r + (2\bar{P}_2\gamma_{20} - \hat{r}(2^{\frac{2R_{2,\text{tar}}}{\hat{r}}}-1)) + 2\bar{P}_1\gamma_{12})}\right), \tag{34a}$$

$$\alpha(r) = \frac{r}{2}\log\left(1 + \frac{2\bar{P}_1\gamma_{10}}{r} - \frac{[2\bar{P}_2\gamma_{21} + \hat{r}][\hat{r}(2^{\frac{2R_{2,\text{tar}}}{\hat{r}}}-1) - 2\bar{P}_2\gamma_{20}]}{r(2\bar{P}_2\gamma_{21} - [\hat{r}(2^{\frac{2R_{2,\text{tar}}}{\hat{r}}}-1) - 2\bar{P}_2\gamma_{20}])}\right). \tag{34b}$$

Let $r_\beta \in \mathcal{S}_\beta$ and $r_\alpha \in \mathcal{S}_\alpha$. Since the constraints in (13b) and (14b) hold with equality at optimality, and since the power constraint must be satisfied, we have that

$$\hat{r}_\beta(2^{\frac{2R_{2,\text{tar}}}{\hat{r}_\beta}} - 1) \leqslant 2\gamma_{20}\bar{P}_2, \tag{35}$$

and that

$$2\gamma_{20}\bar{P}_2 \leqslant \hat{r}_\alpha(2^{\frac{2R_{2,\text{tar}}}{\hat{r}_\alpha}} - 1) \leqslant 2(\gamma_{20}+\gamma_{21})\bar{P}_2. \tag{36}$$

Since $r_\beta \in \mathcal{S}_\beta$ and $r_\alpha \in \mathcal{S}_\alpha$ by assumption, $\beta(r_\beta) \geqslant R_{1,\text{test}}$ and $\alpha(r_\alpha) \geqslant R_{1,\text{test}}$. By substituting the expressions in (34) into these bounds, and using the inequalities in (35) and (36) we have

$$r_\beta\bigl(2^{\frac{2R_{1,\text{test}}}{r_\beta}} - 1\bigr) \leqslant 2\bar{P}_1\gamma_{10} + 2\bar{P}_2\gamma_{20} - \hat{r}_\beta\bigl(2^{\frac{2R_{2,\text{tar}}}{\hat{r}_\beta}} - 1\bigr), \tag{37a}$$

$$r_\alpha\bigl(2^{\frac{2R_{1,\text{test}}}{r_\alpha}} - 1\bigr) \leqslant 2\bar{P}_1\gamma_{10} + 2\bar{P}_2\gamma_{20} - \hat{r}_\alpha\bigl(2^{\frac{2R_{2,\text{tar}}}{\hat{r}_\alpha}} - 1\bigr). \tag{37b}$$

It can be shown that $r(2^{\frac{2R_{1,\text{test}}}{r}} - 1)$ is convex decreasing in r, and hence for any $r_3 = \mu r_\beta + (1-\mu)r_\alpha$, we have

$$\begin{aligned} r_3(2^{\frac{2R_{1,\text{test}}}{r_3}} - 1) &\leqslant \mu r_\beta(2^{\frac{2R_{1,\text{test}}}{r_\beta}} - 1) + (1-\mu)r_\alpha(2^{\frac{2R_{1,\text{test}}}{r_\alpha}} - 1) \\ &\leqslant 2\bar{P}_1\gamma_{10} + 2\bar{P}_2\gamma_{20} - \hat{r}_3(2^{\frac{2R_{2,\text{tar}}}{\hat{r}_3}} - 1) \end{aligned} \tag{38}$$

If we choose r_3 such that $\hat{r}_3(2^{\frac{2R_{2,\text{tar}}}{\hat{r}_3}} - 1) = 2\bar{P}_2\gamma_{20}$, i.e., $R_{2,\text{tar}} = \frac{\hat{r}_3}{2}\log(1+\frac{2\bar{P}_2\gamma_{20}}{\hat{r}_3})$,[4] then we have $r_3(2^{\frac{2R_{1,\text{test}}}{r_3}} - 1) \leqslant 2\bar{P}_1\gamma_{10}$, and hence $R_{1,\text{test}} \leqslant \frac{r_3}{2}\log(1+\frac{2\bar{P}_1\gamma_{10}}{r_3})$. That is, for the choice $r = r_3$, direct transmission from both nodes (i.e., $\tilde{P}_{12} = 0$ and $\tilde{P}_{21} = 0$) yields a rate for Node 1 that is at least $R_{1,\text{test}}$. (Actually, direct transmission yields the largest achievable rate for Node 1 for this value of r.) Since direct transmission is a feasible solution to both (13) and (14), r_3 is an element of both $\mathcal{S}_\beta$ and $\mathcal{S}_\alpha$, and hence these sets intersect.

[4] The existence of an r_3 such that $\hat{r}_\beta \geqslant \hat{r}_3 \geqslant \hat{r}_\alpha$ follows directly from (35) and (36) and the decreasing nature of $\hat{r}(2^{\frac{2R_{2,\text{tar}}}{\hat{r}}} - 1)$ in $\hat{r}$.

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