# DESIGN OF ROBUST REDUNDANT PRECODING FILTER BANKS WITH ZERO-FORCING EQUALIZERS FOR UNKNOWN FREQUENCY-SELECTIVE CHANNELS

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## ABSTRACT

Redundant multirate filter bank transceivers have recently been proposed for block-based transmission over channels whose characteristics are known at both the receiver and the transmitter. In this paper we propose a criterion for the design of such transceivers for applications in which the channel is not known at the transmitter. The design objective is the minimization of the average mean square error of the data estimates produced by a zero-forcing equalizer over a statistically modelled class of channels. Two solution methods are proposed, one of which appears to be amenable to modification for the solution of certain other robust performance problems. It is shown that the optimal transmitter provides substantially improved performance over a scheme based on multi-carrier modulation.

## 1. INTRODUCTION

Block-based data transmission is an effective technique for digital communication over channels which are affected by intersymbol interference (ISI). The block structure provides a convenient framework with which controlled redundancy can be embedded within the transmitted data stream. The key is to embed it in such a way that the ISI can be efficiently mitigated at the receiver. Examples of such schemes include the Discrete Multitone Transmission (DMT) techniques [1,2] used in Digital Subscriber Line (DSL) applications, and Coded Orthogonal Frequency Division Multiplexing (OFDM) [1,3] which has been proposed for digital audio and video broadcasting in Europe.

Recently, redundant multirate filter bank precoding structures have been proposed as a large and easily implementable class of block-based transmission schemes [4,5]. In applications such as DSL, in which the channel environment is relatively constant, a channel model can be constructed at the receiver and made known to the transmitter. In that case the redundant multirate filter banks at the transmitter and receiver can be chosen to maximize the mutual information between the transmitter and the receiver [6]. However, in some wireless applications, the channel may undergo substantial variations, and it may not be possible for the transmitter to obtain an accurate model of the channel. In this paper we provide a criterion for the design of the redundant multirate filter bank transceiver for these applications.

The design criterion is the mean square error (MSE) of the symbol estimates produced by a zero-forcing equalizer at the receiver. The actual channel is not known by the transmitter, but it is assumed that a statistical model for the channel is available at the transmitter. A natural approach to the design of the transmitter filter bank is to minimize the average MSE over the statistical model. Using parameterizations based on the singular value decomposition, the design problem can be simplified and analytically solved. However, it is difficult to generalize that analytic solution to control other (robust) performance criteria. As an alternative, we show how the design problem can be formulated as a convex semidefinite programme [7] which can be efficiently solved using interior point methods. We will demonstrate the effectiveness of the design methods, by showing that a designed transmitter can provide substantially improved performance over a a multi-carrier modulation scheme in a frequency-selective Rayleigh fading environment. Finally, we will illustrate the flexibility of the semidefinite programming method by showing that it can be modified to minimize the worst-case mean square error over a class of deterministically bounded channels.

## 2. MULTIRATE FILTERBANK PRECODERS

The redundant multirate filter bank transceiver which we will design is illustrated in Figure 1. (This structure was proposed in [4–6, 8].) The message data $s[n]$ are grouped into blocks of length $M$, with the $m$-th element of each block being $s_m[n] = s[nM+m]$. Each sequence $s_m[n]$ is upsampled by $P$ and filtered by a filter $f_m[n]$ of length $\leq P$. The output blocks from the transmitter can be written as:

$$\boldsymbol{u}[n] = \boldsymbol{F}_0\boldsymbol{s}[n]. \tag{1}$$

Here the length $P$ vector $\boldsymbol{u}[n]$ has $\left[\boldsymbol{u}[n]\right]_i = u[nP+i], 0 \leq i \leq P-1$, the length $M$ vector $\boldsymbol{s}[n]$ has $\left[\boldsymbol{s}[n]\right]_m = s[nM+m], 0 \leq m \leq M-1$, and the $P \times M$ matrix $\boldsymbol{F}_0$ has $\left[\boldsymbol{F}_0\right]_{i+1,m+1} = f_m[i], i = 0,\ldots,P-1$, $m = 0,\ldots,M-1$. Similarly, the estimate of $\boldsymbol{s}[n]$ generated by the receiver can be described by

$$\hat{\boldsymbol{s}}[n] = \boldsymbol{G}_0(\boldsymbol{x}[n]+\boldsymbol{v}[n]), \tag{2}$$

where signal and noise components of the received signal, $\boldsymbol{x}[n]$ and $\boldsymbol{v}[n]$ respectively, are length $P$ vectors with the same structure as $\boldsymbol{u}[n]$. Here, $\hat{\boldsymbol{s}}[n]$ has the same structure as $\boldsymbol{s}[n]$, and the $M \times P$ matrix $\boldsymbol{G}_0$ has $\left[\boldsymbol{G}_0\right]_{j+1,p+1} = g_p[j]$, $j = 0,\ldots,M-1$, $p = 0,\ldots,P-1$, and the $g_p[j]$ are filters of length $\leq M$. If we assume that the linear time invariant channel $h[\ell]$ is a finite impulse response (FIR) filter of length $L$ and that $M$ and $P$ are chosen to satisfy $M \geq L$ and $P = M+L-1$, then we can write [6]:

$$\hat{\boldsymbol{s}}[n] = \boldsymbol{G}_0\boldsymbol{H}_0\boldsymbol{F}_0\boldsymbol{s}[n] + \boldsymbol{G}_0\boldsymbol{H}_1\boldsymbol{F}_0\boldsymbol{s}[n-1] + \boldsymbol{G}_0\boldsymbol{v}[n], \tag{3}$$

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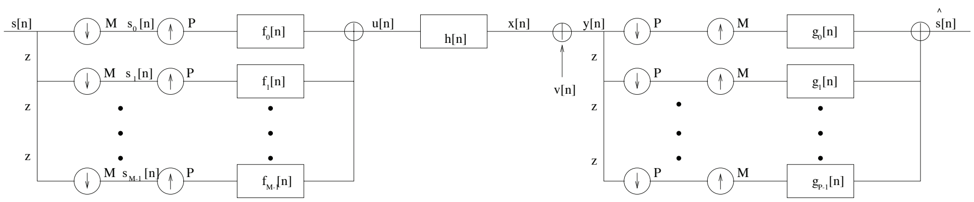


Figure 1: The baseband redundant multirate filter bank transceiver model

where $\boldsymbol{H}_0$ is a $P \times P$ Toeplitz lower triangular matrix with first column $\left[h[0], h[1], \ldots, h[L-1], 0, \ldots, 0\right]^T$, and $\boldsymbol{H}_1$ is a $P \times P$ Toeplitz upper triangular matrix with last column $\left[h[1], h[2], \ldots, h[L-1], 0, \ldots, 0\right]^T$. The first term in (3) represents the desired block, the second term represents interblock interference, and the third term represents noise and external interference. For simplicity, we will focus our attention on real-valued channels and transceivers.

The interblock interference term in (3) complicates the design of the receiver. However, by observing the upper triangular structure of $\boldsymbol{H}_1$, it is clear that interblock interference can be avoided by forcing the last $L-1$ taps of the transmitter filters to be zero; i.e. forcing

$$\boldsymbol{F}_0 = \left[\boldsymbol{F}^T \boldsymbol{0}\right]^T, \tag{4}$$

where $\boldsymbol{F}$ is a $M \times M$ matrix [6]. In that case, the second term of (3) is zero for all $h[\ell]$ of length $\leq L$.

## 3. MSE FOR THE ZERO-FORCING EQUALIZER

Under the constraint in (4), the estimated signal in (3) can be rewritten as

$$\hat{\boldsymbol{s}}[n] = \boldsymbol{G}_0 \bar{\boldsymbol{H}}_0 \boldsymbol{F} \boldsymbol{s}[n] + \boldsymbol{G}_0 \boldsymbol{v}[n],$$

where $\bar{\boldsymbol{H}}_0$ is the $P \times M$ matrix consisting of the first $M$ columns of $\boldsymbol{H}_0$. The matrix $\bar{\boldsymbol{H}}_0$ has full column rank (except in the extreme case where all of the channel taps are zero), and therefore, a zero-forcing equalizer exists if and only if $\boldsymbol{F}$ is non-singular. In that case, the 'minimum-norm' zero-forcing equalizer is [5], $\boldsymbol{G}_0 = \boldsymbol{F}^{-1} \bar{\boldsymbol{H}}_0^{\dagger}$, where $\bar{\boldsymbol{H}}_0^{\dagger} = \left(\bar{\boldsymbol{H}}_0^T \bar{\boldsymbol{H}}_0\right)^{-1} \bar{\boldsymbol{H}}_0^T$ denotes the 'minimum-norm' pseudo inverse of $\bar{\boldsymbol{H}}_0$. If $\boldsymbol{F}$ and $\bar{\boldsymbol{H}}_0$ are known at the receiver, then employing this equalizer yields symbol estimates

$$\hat{\boldsymbol{s}}[n] = \boldsymbol{s}[n] + \boldsymbol{F}^{-1} \bar{\boldsymbol{H}}_0^{\dagger} \boldsymbol{v}[n].$$

If we assume that the signal and noise are mutually uncorrelated, zero-mean, stationary and white, with covariance matrices $\boldsymbol{R}_{ss} = \sigma_{ss}^2 \boldsymbol{I}$ and $\boldsymbol{R}_{vv} = \sigma_{vv}^2 \boldsymbol{I}$ respectively, then the MSE of $\hat{\boldsymbol{s}}[n]$ in a given channel is

$$\begin{aligned} \mathrm{MSE} &= \mathrm{tr}\left[E\left\{(\hat{\boldsymbol{s}}[n] - \boldsymbol{s}[n])(\hat{\boldsymbol{s}}[n] - \boldsymbol{s}[n])^T\right\}\right] \\ &= \mathrm{tr}\left[\boldsymbol{G}_0 \boldsymbol{R}_{vv} \boldsymbol{G}_0^T\right] \\ &= \sigma_{vv}^2 \mathrm{tr}\left[\boldsymbol{F}^{-1} (\bar{\boldsymbol{H}}_0^T \bar{\boldsymbol{H}}_0)^{-1} \boldsymbol{F}^{-T}\right], \end{aligned} \tag{5}$$

where we have employed the commutative properties of the trace operator [9], and the fact that $\bar{\boldsymbol{H}}_0$ has full column rank. If the transmitted symbol is to be deduced from the elements of $\hat{\boldsymbol{s}}[n]$ in a symbol-by-symbol fashion, a natural objective for the design of the transmitter is to minimize the MSE, subject to a bound on the transmitter power, $\mathrm{tr}[\boldsymbol{F}\boldsymbol{F}^T] \leq P_0$. If the channel matrix $\bar{\boldsymbol{H}}_0$ is known at the transmitter, and if $\boldsymbol{W} \boldsymbol{\Lambda} \boldsymbol{W}^T = (\bar{\boldsymbol{H}}_0^T \bar{\boldsymbol{H}}_0)^{-1}$ is an eigenvalue decomposition of the positive definite matrix $(\bar{\boldsymbol{H}}_0^T \bar{\boldsymbol{H}}_0)^{-1}$, then the optimal $\boldsymbol{F}$ is given by

$$\boldsymbol{F}_{\mathrm{opt}} = \sqrt{P_0 / \mathrm{tr}[\boldsymbol{\Lambda}^{1/2}]}\ \boldsymbol{W} \boldsymbol{\Lambda}^{1/4} \boldsymbol{U}^T, \tag{6}$$

where $\boldsymbol{U}$ is an arbitrary orthonormal matrix. (This result was stated in [5, Footnote 5]. Its proof is a special case of the proof in Section 4.1.) However, in some wireless applications it may be difficult for the transmitter to obtain an accurate model for $h[\ell]$, and an alternative approach is required.

## 4. DESIGN FOR STATISTICALLY MODELLED CHANNEL UNCERTAINTY

For a class of channels described by a statistical model, the expected MSE is

$$\mathrm{MSE}_{\mathrm{av}} = \sigma_{vv}^2 \mathrm{tr}\left[\boldsymbol{F}^{-1} E\left\{(\bar{\boldsymbol{H}}_0^T \bar{\boldsymbol{H}}_0)^{-1}\right\} \boldsymbol{F}^{-T}\right]. \tag{7}$$

Therefore, if the matrix $\boldsymbol{Q} = E\left\{(\bar{\boldsymbol{H}}_0^T \bar{\boldsymbol{H}}_0)^{-1}\right\}$ is known at the transmitter, a natural design objective would be to minimize $\mathrm{MSE}_{\mathrm{av}}$, subject to a constraint on the transmitted power:

**Problem 1** *Given the statistical model of the channel, $\boldsymbol{Q}$, and a bound, $P_0$, on the transmitted power, find a matrix $\boldsymbol{F} \in \mathbb{R}^{M \times M}$ achieving*

$$\min\ \mathrm{tr}\left[\boldsymbol{F}^{-1} \boldsymbol{Q} \boldsymbol{F}^{-T}\right],$$

*subject to* $\mathrm{tr}\left[\boldsymbol{F}\boldsymbol{F}^T\right] \leq P_0$.

We now explore two approaches to the solution of Problem 1.

### 4.1. Analytic Solution

The optimal solution to Problem 1 has the same structure as $\boldsymbol{F}_{\mathrm{opt}}$ in (6), except that $\boldsymbol{W} \boldsymbol{\Lambda} \boldsymbol{W}^T = \boldsymbol{Q}$ rather than $(\bar{\boldsymbol{H}}_0^T \bar{\boldsymbol{H}}_0)^{-1}$. To prove that claim, let $\boldsymbol{F} = \boldsymbol{V} \boldsymbol{\Phi} \boldsymbol{U}^T$, where $\boldsymbol{V}$ and $\boldsymbol{U}$ are orthonormal matrices and $\boldsymbol{\Phi}$ is diagonal with real positive entries (since $\boldsymbol{F}$ must be non-singular), arranged in decreasing order. In addition, arrange $\boldsymbol{W}$ so that the elements on the diagonal of $\boldsymbol{\Lambda}$ are arranged in decreasing order. Using the commutativity properties of the trace operator, we can simplify the design problem to

$$\min_{\boldsymbol{\Phi}, \boldsymbol{V}_1}\ \mathrm{tr}[\boldsymbol{\Lambda} \boldsymbol{V}_1 \boldsymbol{\Phi}^{-2} \boldsymbol{V}_1^T] \tag{8}$$

subject to $\mathrm{tr}[\boldsymbol{\Phi}^2] \leq P_0$ and $\boldsymbol{V}_1$ being an orthonormal matrix. If $\phi_j$ and $\lambda_i$ denote the diagonal elements of $\boldsymbol{\Phi}$ and $\boldsymbol{\Lambda}$, respectively, then

$$\mathrm{tr}[\boldsymbol{\Lambda}\boldsymbol{V}_1\boldsymbol{\Phi}^{-2}\boldsymbol{V}_1^T] = \sum_{i=1}^{M} \lambda_i \sum_{j=1}^{M} |v_{ij}|^2 \phi_j^{-2}, \tag{9}$$

where $v_{ij}$ is the $(i,j)$-th element of $\boldsymbol{V}_1$. It can be shown that the minimum of the right hand side of (9) can be achieved by a permutation matrix $\boldsymbol{V}_1$ which places the $\phi_j$ in the same order as the $\lambda_i$ (see the Appendix of [5] for some related work). Since $\lambda_i$ and $\phi_j$ are both arranged in descending order, that permutation matrix is diagonal. Hence the design problem reduces to

$$\min_{\phi_i \geq \phi_{i+1} > 0} \sum_{i=1}^{M} \lambda_i \phi_i^{-2},$$

subject to $\sum_{i=1}^{M} \phi_i^2 \leq P_0$. Taking the derivatives of the Lagrangian function and equating them to zero we have that

$$\phi_{i,\mathrm{opt}} = \lambda_i^{1/4} \sqrt{\frac{P_0}{\sum_{i=1}^{M} \sqrt{\lambda_i}}},$$

and hence the optimal transmission filter bank is

$$\boldsymbol{F}_{\mathrm{opt}} = \sqrt{P_0/\mathrm{tr}[\boldsymbol{\Lambda}^{1/2}]}\ \boldsymbol{W}\boldsymbol{\Lambda}^{1/4}\boldsymbol{U}^T, \tag{10}$$

where $\boldsymbol{U}$ is an orthonormal matrix. Since it is assumed that the receiver knows the actual channel over which the data was transmitted, rather than just a statistical model, $\boldsymbol{G}_{\mathrm{opt}} = \boldsymbol{F}_{\mathrm{opt}}^{-1}\bar{\boldsymbol{H}}_0^{\dagger}$.

### 4.2. Semidefinite Programming Solution

Although the analytic solution in (10) offers considerable insight into the design problem, it may be difficult to generalize it to include other relevant constraints. Therefore, it is of interest to consider efficient computational solutions to Problem 1, with the expectation that they might be more easily generalized. Unfortunately, both the objective and the constraints in Problem 1 are non-convex functions of the elements of $\boldsymbol{F}$. Therefore any algorithm which attempts to solve Problem 1 directly is complicated by the intricacies of dealing with potential local minima. However, we will now show that Problem 1 can be precisely transformed to a convex semidefinite programme [7] which can be efficiently solved using interior point methods.

If we let $\boldsymbol{R} = \boldsymbol{F}\boldsymbol{F}^T$, then Problem 1 is equivalent to $\min_{\boldsymbol{R}} \mathrm{tr}\big[\boldsymbol{Q}\boldsymbol{R}^{-1}\big]$, subject to $\mathrm{tr}\big[\boldsymbol{R}\big] \leq P_0$, and $\boldsymbol{R} > \boldsymbol{0}$. Here, the relations $\boldsymbol{A} > \boldsymbol{B}$ and $\boldsymbol{A} \geq \boldsymbol{B}$ for symmetric matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ denote that $(\boldsymbol{A} - \boldsymbol{B})$ is positive definite and positive semidefinite, respectively. The additional constraint $(\boldsymbol{R} > 0)$ is a sufficient condition for $\boldsymbol{R}$ to be factorizable as $\boldsymbol{R} = \boldsymbol{F}\boldsymbol{F}^T$. It is also necessary in our case, because a singular $\boldsymbol{R}$ will lead to an unbounded objective. Once an optimal $\boldsymbol{R}$ has been found, an optimal $\boldsymbol{F}$ can be found by performing a Cholesky factorization of $\boldsymbol{R}$.

Let us write $\boldsymbol{Q}$ in terms of its Cholesky factors, $\boldsymbol{Q} = \boldsymbol{L}\boldsymbol{L}^T$, so that $\mathrm{tr}[\boldsymbol{Q}\boldsymbol{R}^{-1}] = \mathrm{tr}[\boldsymbol{L}^T\boldsymbol{R}^{-1}\boldsymbol{L}]$. For any (positive definite) symmetric matrix satisfying $\boldsymbol{S} \geq \boldsymbol{L}^T\boldsymbol{R}^{-1}\boldsymbol{L}$, we have that

$$\mathrm{tr}[\boldsymbol{S}] \geq \mathrm{tr}[\boldsymbol{L}^T\boldsymbol{R}^{-1}\boldsymbol{L}], \tag{11}$$

and since $\boldsymbol{L}^T\boldsymbol{R}^{-1}\boldsymbol{L}$ is symmetric, equality in (11) can be achieved by setting $\boldsymbol{S} = \boldsymbol{L}^T\boldsymbol{R}^{-1}\boldsymbol{L}$. Therefore, minimization of $\mathrm{tr}[\boldsymbol{Q}\boldsymbol{R}^{-1}]$ over $\boldsymbol{R} > \boldsymbol{0}$ is equivalent to minimization of $\mathrm{tr}[\boldsymbol{S}]$ over $\boldsymbol{S} > \boldsymbol{0}$ and $\boldsymbol{R} > \boldsymbol{0}$ subject to $\boldsymbol{S} \geq \boldsymbol{L}^T\boldsymbol{R}^{-1}\boldsymbol{L}$. This problem can be further simplified using the Schur Complement Theorem [9], which implies that if $\boldsymbol{R} > \boldsymbol{0}$ then

$$\boldsymbol{S} \geq \boldsymbol{L}^T\boldsymbol{R}^{-1}\boldsymbol{L} \quad \text{if and only if} \quad \begin{bmatrix} \boldsymbol{S} & \boldsymbol{L}^T \\ \boldsymbol{L} & \boldsymbol{R} \end{bmatrix} \geq \boldsymbol{0}. \tag{12}$$

Using these facts, Problem 1 can be re-cast as:

**Reformulation 1** *Given $\boldsymbol{Q} = \boldsymbol{L}\boldsymbol{L}^T$ and $P_0$, find symmetric matrices $\boldsymbol{R}, \boldsymbol{S} \in \mathbb{R}^{M\times M}$ achieving* $\min \mathrm{tr}[\boldsymbol{S}]$, *subject to*

$$P_0 - \mathrm{tr}[\boldsymbol{R}] \geq 0 \tag{13}$$

$$\begin{bmatrix} \boldsymbol{S} & \boldsymbol{L}^T \\ \boldsymbol{L} & \boldsymbol{R} \end{bmatrix} \geq \boldsymbol{0}. \tag{14}$$

Reformulation 1 consists of a linear objective, a linear constraint (13) and a linear matrix inequality (LMI) (14), and hence it is a convex semidefinite programme (SDP) [7]. Such programmes can be efficiently solved using interior point methods. (SeDuMi [10] is a particularly efficient Matlab-based tool.) Note that we have not explicitly enforced $\boldsymbol{R} > \boldsymbol{0}$ in Reformulation 1. This is because the LMI in (14) ensures that $\boldsymbol{R} \geq \boldsymbol{0}$ and because a singular $\boldsymbol{R}$ will lead to an unbounded objective.

An advantage of Reformulation 1 over the analytical solution in (10) is that constraints which are convex functions of $\boldsymbol{R}$ can be efficiently incorporated into the design. (Convex constraints have already been found to be useful in other transmitter design problems [11,12].) In addition, the approach which led to Reformulation 1 can be modified to obtain efficient design methods for transmitters which optimize certain other robust performance criteria, as we will illustrate in Section 5.

### 4.3. An example

To demonstrate the improved performance of redundant multirate filter bank transmitters designed using Problem 1, we consider the design of transmission schemes for communication over a slowly-varying frequency-selective Rayleigh fading channel. The fading channel has four real-valued taps, each of which is independent and has a Gaussian distribution with zero mean. The standard deviation of the first and last tap is 0.374, and the standard deviation of the two central taps is 0.6. Having calculated $\boldsymbol{Q} = E\big\{(\bar{\boldsymbol{H}}_0^T\bar{\boldsymbol{H}}_0)^{-1}\big\}$ for this fading channel, we designed an optimal transmitter using either (10) or Reformulation 1, with the transmitted power, $P_0$, normalized to one. (The efficiency of the convex SDP formulation is evident from the fact that Reformulation 1 was solved using SeDuMi [10], in just 0.34 seconds on a 400 MHz Pentium II workstation.) The resulting optimal transceiver generates an average mean square error of $\mathrm{MSE}_{\mathrm{av}} = 54.30$. For comparison, we consider a real-valued multi-carrier modulation scheme based on the Discrete Cosine Transform which also uses the zero-padding structure in Equation (4) to avoid interblock interference (and has the same transmitted power). That scheme has $\mathrm{MSE}_{\mathrm{av}} = 55.02$. Although the MSE improvement is only small (around 1.3%), this leads to a substantial reduction in bit error rate for simple threshold detection, as is shown in Figure 2. (The bit error rate plotted in that figure is the average bit error rate over 10,000 realizations of the fading channel described above.) At low signal to noise ratios, the bit error rate performance of the two schemes is indistinguishable, but at high signal to noise ratios the designed scheme has a substantially lower bit error rate.

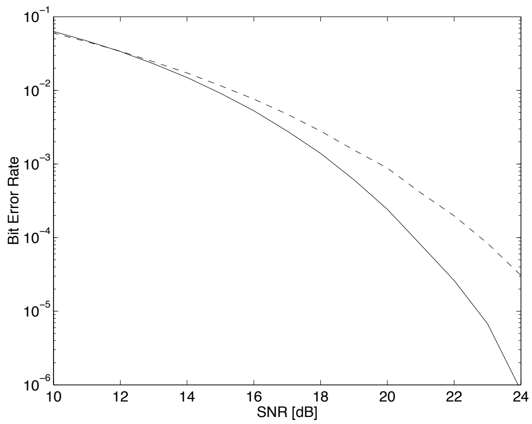


Figure 2: Bit error rate against signal to noise ratio for the designed (solid) and zero-padded OFDM (dashed) schemes in the frequency-selective Rayleigh fading channel in Section 4.3.

## 5. DESIGN FOR DETERMINISTICALLY MODELLED CHANNEL UNCERTAINTY

Solutions to Problem 1 provide robust performance in a statistically modelled class of channels, but they require that the matrix $\boldsymbol{Q}$ is known at the transmitter. When this matrix is not known at the transmitter, the following approach will provide robust performance to certain deterministically bounded channel errors: Equation (5) can be re-written as $\text{MSE} = \sigma_{vv}^2 \text{tr}\left[(\bar{\boldsymbol{H}}_0^\dagger)^T(\boldsymbol{F}\boldsymbol{F}^T)^{-1}\bar{\boldsymbol{H}}_0^\dagger\right]$. Therefore, if $\bar{\boldsymbol{H}}_0^\dagger$ is known at the transmitter, then a filter bank which minimizes the MSE is given by the solution to Reformulation 1 with $\boldsymbol{L} = \bar{\boldsymbol{H}}_0^\dagger$. (This can be shown using the Schur Complement Theorem.) Now suppose that the transmitter's estimate of $\bar{\boldsymbol{H}}_0^\dagger$ is inaccurate, and equals $\bar{\boldsymbol{H}}_0^\dagger + \Delta$, where $\|\Delta\|_2 \leq \rho$. For a given $\rho$, a transmitter which minimizes an upper bound on the worst-case MSE over all $\Delta$ such that $\|\Delta\|_2 \leq \rho$ can be efficiently found by applying robust semidefinite programming techniques [13] to Reformulation 1. In contrast, it appears to be difficult to modify the analytic solution in Section 4.1 to accommodate this scenario.

## 6. CONCLUSION

In this paper we have proposed a performance criterion for the design of robust redundant multirate filter bank precoders for unknown frequency-selective channels. We selected a zero-forcing equalizer at the receiver and chose the average mean square error over a statistically modelled class of channels as the design criterion. It was shown that the optimal transmitter filter bank can be found either via an analytic solution, or via an efficiently solvable convex optimization problem. Transceivers designed using either method were shown to perform substantially better than a scheme based on multi-carrier modulation in a frequency-selective Rayleigh fading channel. However, it appears that the convex optimization method is more amenable to modification to accommodate other (robust) performance criteria.

The design framework in Section 4 was based on perfect zero-forcing equalization at the receiver and knowledge of the statistical model of the channel at the transmitter. In continuing work in this area, we are comparing the performance of transmitters which are robust to statistically modelled uncertainties with those which are robust to deterministically modelled uncertainties (as in Section 5). We are also developing (efficient) design techniques which provide robustness to uncertainties in the transmitter's statistical model of the channel, and robustness to imperfect equalization. These ideas are also being extended to minimum mean square error equalizers. In each case, the convex optimization approach appears to be easier to modify than the analytic approach.

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