Probabilistically-Constrained Approaches to the Design of the Multiple Antenna Downlink

M. Botros Shenouda and T. N. Davidson
Department of Electrical and Computer Engineering
McMaster University, Hamilton, Ontario, L8S4K1, Canada

***Abstract*— We consider the downlink of a cellular system in which the base station is equipped with multiple antennas and each user has a single antenna. We study the design of linear precoders with probabilistically-constrained Quality of Service (QoS) requirements for each user, in scenarios with uncertain channel state information (CSI) at the transmitter. Our goal is to design the precoder so as to minimize the total transmitted power subject to the satisfaction of the QoS constraints with a maximum allowed outage probability. We consider two stochastic models for the uncertainty in the channel coefficients of each user. The first is a Gaussian model that is appropriate for uncertainty that results from estimation errors. The second one is uniform model that is appropriate for the quantization errors in systems with quantized feedback of channel state information. We formulate the design problem as a chance constrained optimization problem, in which each chance constraint involves randomly perturbed second order cone constraints. We adopt a conservative approach that yields (deterministic) convex and efficiently-solvable design formulations that guarantee the satisfaction of the probabilistic QoS constraints. Furthermore, based on these convex formulations, we propose computationally-efficient algorithms that can reduce the level of conservatism in the initial formulations. Our simulations indicate that the proposed methods can significantly expands the range of QoS requirements that can be satisfied in the presence of uncertainty in the CSI.**

I. Introduction

The desire to provide service to several classes of users within a single cellular system, and the desire for that system to support interactive services with low latency requirements, such as video conferencing, has led to development of design techniques that guarantee that a certain quality of service (QoS) constraint is satisfied for each user. Of interest in this paper are the design techniques for the physical layer of downlink systems in which the base station is equipped with multiple antennas. The availability of multiple antennas enables spatial precoding of the data to be transmitted to each user in a way that mitigates the multiuser interference at the (non-cooperating) receivers. This technique offers the potential for significant improvement in the fidelity of the received signals, and hence the quality of service that can be offered to each user. Assuming perfect channel state information (CSI) at the transmitter, the problem of designing a linear precoder that minimizes the transmitted power required to satisfy a set of QoS constraints specified by the users, usually in terms of the received signal-to-interference-plus-noise-ratio (SINR), has received considerable attention, e.g., [1], [2].

In practical downlink systems, the CSI that is available at the transmitter is subject to a variety of sources of imperfection. For downlink systems with uplink-downlink reciprocity, the base station can estimate the users' channels on the uplink, and the mismatch in the CSI at the transmitter is typically dominated by estimation errors. In systems in which the receivers feed back quantized estimates of their channels to the transmitter (e.g., [3]), the uncertainty in the CSI at the transmitter is typically dominated by quantization errors. Downlink precoder design methods that assume perfect CSI at the transmitter are particularly sensitive to these mismatches. For example, it has been shown [3], [4] that imperfect channel knowledge at the transmitter can result in the downlink becoming interference limited; i.e., the growth of the SINR of each user with the transmitted power saturates.

One approach to incorporating robustness against channel uncertainty into the precoder design is to consider a bounded model for the error in the transmitter's estimate of the channels and to constrain the design of the precoder so that the users' QoS requirements are satisfied for all channels admitted by this model [5], [6], [1]. In this paper, we consider an alternative approach that assumes a stochastic model for the uncertainty in the transmitter's estimate of the channel coefficients of each user, and we design the precoder so as to minimize the total transmitted power subject to the satisfaction of the SINR-based QoS constraints with a maximum allowed outage probability. We consider two stochastic models for the uncertainty in the the channel coefficients of each user. The first is a Gaussian model that is appropriate for uncertainty that results from estimation errors, such as those that arise in systems with uplink-downlink reciprocity. The second one is uniform model that is appropriate for the quantization errors in systems with quantized feedback of CSI. We formulate the design problem as a chance constrained optimization problem, in which each chance constraint involves randomly perturbed second order cone (SOC) constraints. Chance constraints that involve this type of cone are generally intractable [7], and (conservative) approaches that guarantee that the chance constraints are satisfied are usually considered. We adopt a conservative approach that yields (deterministic) convex and efficiently-solvable design formulations that guarantee the satisfaction of the probabilistic QoS constraints. Furthermore, based on these convex formulations, we propose computationally-efficient algorithms that can reduce the level of conservatism in the initial formulations. Our simulations indicate that the proposed methods can significantly expands the range of QoS requirements that can be satisfied in the presence of uncertainty in the CSI.

The design of a linear downlink precoder based on probabilistic QoS constraints was also considered in [8], but in that work the uncertainty in the CSI at the transmitter is modelled using a Gaussian assumption on the entries of the estimates of each user's channel covariance matrix. Under that model, a conservative design was obtained. In our approach, the uncertainty is modelled directly using statistical models for the estimates of the channel coefficients themselves. Probabilistic constraints have been recently considered in other contexts too, such as in the design of linear receivers for space time coded multiple access schemes [9].

II. System Model

We consider narrow band broadcast channels with N_t antennas at the transmitter and K receivers, each with a single antenna. The use of multiple antennas at the transmitter will facilitate the transmission of independent data symbols to each receiver. Let $\mathbf{s} \in \mathbb{C}^K$ be the vector of independent data symbols, whose j^{th} element is the symbol to be transmitted to the j^{th} user. The transmitter generates a vector of transmitted signals, $\mathbf{x} \in \mathbb{C}^{N_t}$, by linearly precoding the vector $\mathbf{s}$:

$$\mathbf{x} = \mathbf{P}\mathbf{s} = \textstyle\sum_{j=1}^{K} \mathbf{p}_j s_j, \tag{1}$$

where $\mathbf{p}_j$ is the j^{th} column of the precoding matrix $\mathbf{P}$. Without loss of generality, we will assume that $\mathrm{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}$, and hence, the total transmitted power is given by:

$$\mathrm{tr}(\mathbf{P}^H\mathbf{P}) = \textstyle\sum_{k=1}^{K} \|\mathbf{p}_k\|^2. \tag{2}$$

At the k^{th} receiver, the received signal y_k is given by:

$$y_k = \mathbf{h}_k\mathbf{x} + n_k, \tag{3}$$

where $\mathbf{h}_k \in \mathbb{C}^{1\times N_t}$ is a row vector representing the channel gains from the transmitting antennas to the k^{th} receiver, and n_k is the zero-mean additive white noise at the k^{th} receiver whose variance is $\sigma_{n_k}^2$. We will find it convenient to concatenate the K instances of (3) as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{4}$$

where $\mathbf{H}$ is the broadcast channel matrix whose k^{th} row is $\mathbf{h}_k$, and the noise vector $\mathbf{n}$ has covariance matrix $\mathrm{E}\{\mathbf{n}\mathbf{n}^H\} = \mathrm{diag}(\sigma_{n_1}^2, \ldots, \sigma_{n_K}^2)$.

We consider broadcast scenarios in which each receiver has a quality of service requirement that is specified in terms of a lower bound, γ_k, on its signal to interference plus noise ratio:

$$\mathrm{SINR}_k = \frac{|\mathbf{h}_k\mathbf{p}_k|^2}{\sum_{j=1, j\neq k}^{K} |\mathbf{h}_k\mathbf{p}_j|^2 + \sigma_{n_k}^2} \geq \gamma_k. \tag{5}$$

This SINR constraint represents a rather general constraint on the minimum quality of service received by the k^{th} user. In particular, the SINR constraint can be translated into an equivalent constraint on the symbol error rate or the achievable data rate [10].

A. Precoding with QoS Constraints: Perfect CSI Case

Given perfect CSI at the transmitter, the design of a precoder that minimizes the total transmitted power required to satisfy the users' QoS constraints can be formulated as:

$$\min_{\mathbf{P}} \quad \textstyle\sum_{k=1}^{K} \|\mathbf{p}_k\|^2 \tag{6a}$$

$$\text{subject to} \quad \mathrm{SINR}_k = \frac{|\mathbf{h}_k\mathbf{p}_k|^2}{\sum_{j=1, j\neq k}^{K} |\mathbf{h}_k\mathbf{p}_j|^2 + \sigma_{n_k}^2} \geq \gamma_k. \tag{6b}$$

This problem can be transformed into a convex optimization problem in the precoding matrix $\mathbf{P}$ that can be efficiently solved [2]. Indeed, if we make the following definitions:

$$\underline{\mathbf{h}}_k = \begin{bmatrix} \mathrm{Re}\{\mathbf{h}_k\} & \mathrm{Im}\{\mathbf{h}_k\} \end{bmatrix}, \tag{7}$$

$$\underline{\mathbf{P}} = \begin{bmatrix} \mathrm{Re}\{\mathbf{P}\} & \mathrm{Im}\{\mathbf{P}\} \\ -\mathrm{Im}\{\mathbf{P}\} & \mathrm{Re}\{\mathbf{P}\} \end{bmatrix}, \tag{8}$$

$$\underline{\mathbf{p}}_k = \begin{bmatrix} \mathrm{Re}\{\mathbf{p}_k\} \\ -\mathrm{Im}\{\mathbf{p}_k\} \end{bmatrix}, \tag{9}$$

the problem in (6) can be formulated [2] as the following (convex) second order cone program (SOCP) with real variables:

$$\min_{\underline{\mathbf{P}},\, t} \quad t \tag{10a}$$

$$\text{subject to} \quad \big\|\mathrm{vec}\big([\underline{\mathbf{p}}_1, \ldots, \underline{\mathbf{p}}_K]\big)\big\| \leq t, \tag{10b}$$

$$\big\|[\underline{\mathbf{h}}_k\underline{\mathbf{P}}, \quad \sigma_{n_k}]\big\| \leq \beta_k \underline{\mathbf{h}}_k \underline{\mathbf{p}}_k, \quad 1 \leq k \leq K, \tag{10c}$$

where $\beta_k = \sqrt{1 + 1/\gamma_k}$. Our goal is to develop robust counterparts of the design problem in (10) that are based on probabilistic QoS guarantees for communication scenarios with imperfect CSI.

III. Channel Uncertainty Model

We consider an additive model of the uncertainty of each user's channel

$$\underline{\mathbf{h}}_k = \hat{\underline{\mathbf{h}}}_k + \underline{\mathbf{e}}_k, \tag{11}$$

where $\hat{\underline{\mathbf{h}}}_k$ is the transmitter's knowledge of the k^{th} user's actual channel, $\underline{\mathbf{h}}_k$, and $\underline{\mathbf{e}}_k$ is the corresponding mismatch. We will consider two statistical models for the elements of $\underline{\mathbf{e}}_k$:

- **Model-G**: In this model, the elements of $\underline{\mathbf{e}}_k$, $e_{k,\ell}$, are modeled as independent Gaussian distributed random variables with zero-mean and variance $\sigma_{e_{k,\ell}}^2$. This model is particularly suitable for communication schemes with uplink-downlink reciprocity, which allows transmitter to estimate the users' channels on the uplink.
- **Model-U**: In the second model, the elements $e_{k,\ell}$ are modeled as independent uniform distributed random variables on the interval $[-u_{k,\ell}, u_{k,\ell}]$. This model is suitable for communication schemes in which the users employ a (scalar) quantizer to quantize their channel state information and feed it back to the transmitter.

IV. Precoding with Probabilistic QoS constraints

Given the stochastic channel uncertainty models in Section III, our goal is to design a robust precoding matrix that minimizes the transmitted power necessary to ensure that the QoS constraint of the k^{th} user is satisfied with a probability of outage that is less than ϵ_k. Using the SOCP formulation in (10), the following (conservative) optimization problem guarantees the satisfaction of the probabilistic QoS constraints:

$$\min_{\underline{\mathbf{P}},\, t} \quad t \tag{12a}$$

$$\text{s. t.} \quad \left\|\text{vec}\left([\underline{\mathbf{p}}_1,\ \ldots,\ \underline{\mathbf{p}}_K]\right)\right\| \le t, \tag{12b}$$

$$\Pr\left\{\left\|[\underline{\mathbf{h}}\underline{\mathbf{P}},\quad \sigma_{n_k}]\right\| \le \beta_k \underline{\mathbf{h}}_k \underline{\mathbf{p}}_k\right\} \ge 1-\epsilon_k, \quad 1 \le k \le K. \tag{12c}$$

Problems with probabilistic constraints, such as these in (12c), are often called chance constrained optimization problems, e.g., [11]. In the case of (12c), the constraint is on the probability that a randomly perturbed second order cone constraint holds. In general, chance constrained optimization problems are challenging optimization problems. The key step in the development of a computationally-tractable algorithm is to obtain an efficiently-computable representation of the chance constraints. In some simple problems, such a representation can be obtained in a straightforward manner (e.g., [9]), but when the chance constraints involve conic constraints, such as second order cone constraints or semidefinite constraints, they are generally intractable [12], [7]. Even chance constraints that involve simple linear inequalities can become intractable unless the probability distribution of the uncertainty parameters is assumed to be rotationally invariant, e.g., the i.i.d. Gaussian distribution. For example, chance constrains that involve linear inequalities are intractable when the uncertainty parameters are uniformly distributed [12, pp. 970].

One approach to solving a chance constrained optimization problem is to employ a scenario approximation. In this approach, each chance constraint is replaced by a sufficiently large number of deterministic versions of the original constraint, each for a different realization of the uncertain parameters. A drawback of this approach is that the number of constraints that is needed to approximate each chance constraint is dependent on the outage probability of the chance constraint, and is often prohibitively large. We will adopt an alternative approach that involves the development of deterministic convex design formulations that guarantee the satisfaction of the chance constraints. These formulations are efficiently solvable, and their size is independent of the outage probability of each chance constraint.

V. Design Formulation I

In this section, we will present an efficiently solvable design formulation for the robust precoder design problem with probabilistic SINR guarantees. This formulation is applicable to both of stochastic channel uncertainty models in Section III. To present this result, we will first define:

$$\mathbf{A}_{k,0} = \begin{bmatrix} \beta_k \hat{\underline{\mathbf{h}}}_k \underline{\mathbf{p}}_k & [\hat{\underline{\mathbf{h}}}_k \underline{\mathbf{P}},\quad \sigma_{n_k}] \\ [\hat{\underline{\mathbf{h}}}_k \underline{\mathbf{P}},\quad \sigma_{n_k}]^T & (\beta_k \hat{\underline{\mathbf{h}}}_k \underline{\mathbf{p}}_k)\mathbf{I} \end{bmatrix}, \tag{13}$$

$$\mathbf{A}_{k,\ell} = a_{k,\ell} \begin{bmatrix} \beta_k \underline{p}_{k,\ell} & [\mathbf{m}_k \underline{\mathbf{P}},\quad 0] \\ [\mathbf{m}_k \underline{\mathbf{P}},\quad 0]^T & \beta_k \underline{p}_{k,\ell} \mathbf{I} \end{bmatrix}, \tag{14}$$

where $\mathbf{m}_k$ is the k^{th} row of $\mathbf{I}$, $\underline{p}_{k,\ell}$ is the ℓ^{th} entry of $\underline{\mathbf{p}}_k$, and

$$a_{k,\ell} = \begin{cases} \sigma_{e_{k,\ell}} & \text{Model-G;} \\ u_{k,\ell} & \text{Model-U.} \end{cases} \tag{15}$$

We observe that each $\mathbf{A}_{k,\ell}$ has an "arrow" structure. We will also define the following functions, λ_k, of the outage probability, for each of the uncertainty models

$$\lambda_k = \begin{cases} \displaystyle\min_{0<z<0.5} \frac{\min\left(2\sqrt{2/z}, 10\sqrt{-\ln z}\right)}{\min\left(1, \frac{1-\phi(z)}{\sqrt{-2\ln 2\epsilon_k}}\right)} & \text{Model-G;} \\ \displaystyle\min_{0<z<0.5} \min\left(2\sqrt{2/z}, 4+4\sqrt{\ln(2/z)}\right) + 4\sqrt{-\ln 2\epsilon_k - \ln(1-z)} & \text{Model-U.} \end{cases} \tag{16}$$

where $\phi(\cdot)$ is the inverse of cumulative probability distribution (CDF) of the standard Gaussian random variable $N(0,1)$. Using these definitions, we can state the following result.

Theorem 1: Consider the robust precoder design problem with probabilistic SINR guarantees in (12), and the definitions in (13), (14), and (16). For $\epsilon_k \in (0, 0.5)$, the optimal solution of the following (convex) semidefinite program (SDP)

$$\min_{\underline{\mathbf{P}},\, t} \quad t \tag{17a}$$

$$\text{s. t.} \quad \left\|\text{vec}\left([\underline{\mathbf{p}}_1,\ \ldots,\ \underline{\mathbf{p}}_K]\right)\right\| \le t, \tag{17b}$$

$$\begin{bmatrix} \frac{1}{\lambda_k}\mathbf{A}_{k,0} & \mathbf{A}_{k,1} & \ldots & \mathbf{A}_{k,2N_t} \\ \mathbf{A}_{k,1} & \frac{1}{\lambda_k}\mathbf{A}_{k,0} & & \\ \vdots & & \ddots & \\ \mathbf{A}_{k,2N_t} & & & \frac{1}{\lambda_k}\mathbf{A}_{k,0} \end{bmatrix} \ge \mathbf{0}, \quad 1 \le k \le K. \tag{17c}$$

is a conservative solution of (12) that guarantees that the probability of outage of SINR constraint of each user is at most ϵ_k. □

Proof: See [13]. ■

The optimization problem in (17) can be efficiently solved using general implementations of interior point methods, e.g., [14]. These implementations can exploit the block-arrow structure of the matrices in (17c) and the arrow structure of the constituent blocks. Furthermore, the values of λ_k in (16) can be computed and stored offline for different possible values of ϵ_k.

It can be verified from (16) that each λ_k is decreasing function of ϵ_k. As the desired outage probability ϵ_k decreases, the corresponding λ_k increases, and therefore the size of the feasible set described by the constraint in (17c) decreases. Hence, as one might expect, the transmitted power of the precoder design increases with decreasing outage probabilities.

A. Alternative design of λ

An advantage of the values of λ_k chosen in (16) is that they each guarantee that when the SDP in (17) is feasible, its optimal solution satisfies the corresponding SINR target at or below the specified outage probability, ϵ_k. Furthermore, this choice enables the values of λ_k to be computed and stored offline. In this section, we will describe an iterative algorithm that seeks values of λ_k that are smaller than those in (16), and hence require less transmission power to satisfy the specified SINR to the users at or below the specified outage probability, and expand the range of QoS targets for which the probabilistically-constrained precoder design problem is feasible.

For simplicity, we will consider the case in which the users may have different SINR requirements, γ_k, but they have the same outage probability, $\epsilon_k = \epsilon$. Since $\epsilon_k = \epsilon$ all values of λ_k are the same; i.e., $\lambda_k = \lambda$. We will denote the value of λ in (16) by λ_{max} and we study the possibility of existence of a value of $\lambda \in (0, \lambda_{\text{max}})$ such that the precoder design in (17) satisfies the constraints in (12c). Given the monotonicity property between λ and ϵ, we propose to search for such values of λ via a bisection search algorithm on $(0, \lambda_{\text{max}})$. In each iteration, we solve (17) for the given λ and then use a statistical validation procedure to determine whether that solution meets or exceeds the required outage probability. The outcome of the validation procedure determines the interval of λ that is to be bisected in the next iteration. The iterative algorithm finds the smallest such value of λ within an accuracy of μ_λ, or declares that there is no such λ, in a maximum number of iterations that is equal to $(\log_2 \frac{\lambda_{\text{max}}}{\mu_\lambda})$. Furthermore, the statistical validation procedure can be constructed (e.g., [7]) very efficiently compared to conventional Monte-Carlo techniques, and with arbitrary high reliability. In particular, let δ be a small number that denotes the probability that the statistical validation procedure makes an erroneous decision. The procedure proposed in [7] allows arbitrarily small δ, say 10^{-6}, that requires only a number of trials that is proportional to $(\log(1/\epsilon) + \log(1/\delta))$. As we will see in the simulation section, this computationally tractable procedure can provide a significant reduction in the transmission power that is used to satisfy the outage constraints of the users.

VI. Design Formulation II

In the previous section, we presented an efficiently-solvable convex formulation for robust precoder design based on probabilistic constraints on SINR. This formulation is applicable to both the stochastic uncertainty models in Section III. In this section, we will present an alternative design formulation that provides probabilistic SINR guarantees for the case of the Gaussian uncertainty model (Model-G).

Theorem 2: If λ_k is chosen such that $\epsilon_k = \sqrt{0.5e}\ \ \lambda_k\ \ \exp(-\lambda_k^2/4)$, then the optimal solution of the following (convex) second order cone program (SOCP)

$$\min_{\substack{\underline{\mathbf{P}}, t, \boldsymbol{\theta}, \\ \boldsymbol{\alpha}_1, \ldots, \boldsymbol{\alpha}_K}} \quad t \tag{18a}$$

$$\text{s. t.} \quad \left\|\operatorname{vec}\left([\underline{\mathbf{p}}_1, \ldots, \underline{\mathbf{p}}_K]\right)\right\| \leq t, \tag{18b}$$

$$\left\|[\underline{\mathbf{h}}_k \underline{\mathbf{P}}, \quad \sigma_{n_k}]\right\| \leq \beta_k \underline{\mathbf{h}}_k \underline{\mathbf{p}}_k - \lambda_k \theta_k, \;\; 1 \leq k \leq K, \tag{18c}$$

$$\left\|[\sigma_{e_{k,\ell}} \mathbf{m}_l \underline{\mathbf{P}}]\right\| \leq \alpha_{k,\ell} + \beta_k \sigma_{e_{k,\ell}} \underline{p}_{k,\ell}$$

$$1 \leq k \leq K, \quad 1 \leq \ell \leq 2N_t \tag{18d}$$

$$\left\|[\sigma_{e_{k,\ell}} \mathbf{m}_l \underline{\mathbf{P}}]\right\| \leq \alpha_{k,\ell} - \beta_k \sigma_{e_{k,\ell}} \underline{p}_{k,\ell}$$

$$1 \leq k \leq K, \quad 1 \leq \ell \leq 2N_t \tag{18e}$$

$$\left\|\boldsymbol{\alpha}_k\right\| \leq \theta_k, \tag{18f}$$

is a conservative solution of (12) under the assumptions of the Model-G. □

Proof: See [13]. ■

Similar to the previous section, the values of λ_k can be computed and stored offline. Furthermore, when all outage probabilities are equal $\epsilon_k = \epsilon$, an iterative algorithm that is similar to the one proposed in Section V-A can be used to obtain less conservative values of λ.

VII. Simulation Studies

In this section, we will compare the performance of the proposed design formulations, namely the first design formulation (Form. 1) in Section V, the first design formulation with bisection search over λ (Form. 1 Bisec. search) in Section V-A, and the second design formulation (Form. 2) and its counterpart with bisection search (Form. 2 Bisec. search) in Section VI. We will compare these formulations in an environment with $N_t = 3$ transmit antennas and $K = 3$ users. In our simulations we assume Rayleigh fading channels in which the coefficients of the fading channel are modeled as being independent proper complex Gaussian random variables with zero mean and unit variance. The uncertainty in the transmitter estimates of the channel is modelled using the Gaussian model in Section 3 with $\sigma^2_{e_{k,\ell}} = 0.001$. We consider a scenario in which the users' QoS requirements are the same. In particular, we will examine the case in which the QoS requirements are such that each user is provided with an SINR of $\gamma_k = \gamma$, with an outage probability of at most 10% ; i.e., $\epsilon_k = \epsilon = 0.1$.

In first experiment, we randomly generated 1000 realizations of the set of channel estimates $\{\hat{\mathbf{h}}_k\}_{k=1}^K$ and examined the performance of each design formulation as the minimum SINR requirement of all users, γ, increases from 0 to 30 dB. For each set of channel estimates and for each value of γ we determine whether each design formulation is able to generate a precoder (of finite power) that guarantees that the probabilistic SINR constraints are satisfied. In Fig. 1 we plot the percentage of channel realizations for which each design generated a precoder with finite power against the users' equal SINR requirement γ. From this figure, it is clear that the first design formulation (Form. 1) is less conservative than the second design (Form. 2), and provides probabilistic SINR guarantees to a larger percentage of the channel estimates and for larger range of SINR requirements γ. Formulation 1 has the additional advantage that it is applicable to systems with uniformly distributed uncertainties, as well as those with Gaussian uncertainties. However, the second design formulation (Form. 2) is a second order cone programming (SOCP) problem and can be solved at lower computational cost than Formulation 1 which is a semidefinite program (SDP). We

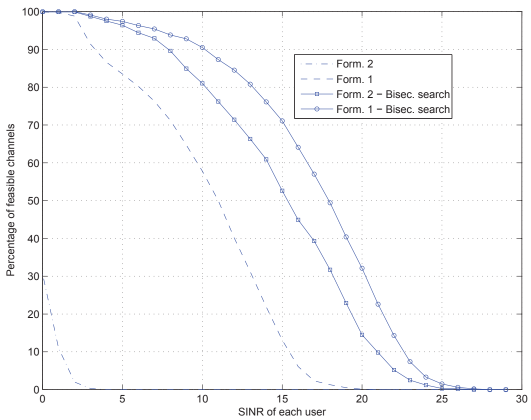

Fig. 1. Percentage of channel realizations for which the probabilistic SINR guarantee can be made, against the users' equal SINR requirements γ. The outage probability is $\epsilon_k = 0.1$.

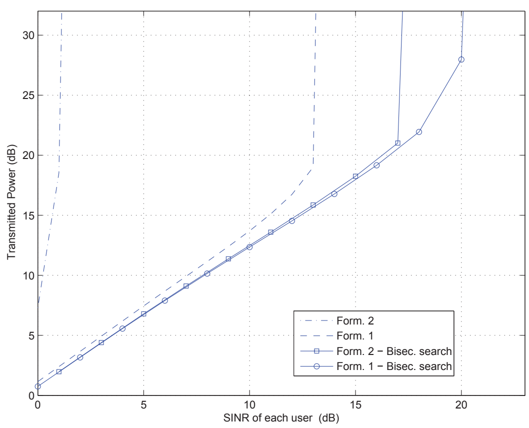

Fig. 2. Average of the transmitted power versus the users' equal SINR requirements γ. The outage probability is $\epsilon_k = 0.1$.

also observe the impact of the bisection search algorithm in improving the feasibility of each design formulation by searching for the smallest value of λ that provides the required outage.

In the second experiment, we selected all the channel estimates from the set of 1000 for which all four approaches are able to provide a probabilistic SINR guarantees at $\gamma = 1$ dB with 10% probability of outage. We calculated the average, over the 110 such channels, of the transmitted power required to achieve these probabilistic QoS guarantees and we have plotted the results for different values of γ in Fig. 2. The average transmitted power approaches infinity for a given value of γ when for one (or more) of the channel estimates the method under consideration cannot provide the probabilistic SINR guarantee with finite power. In Fig. 2, the fact that Formulation 1 employs a less conservative approach than Formulation 2 manifests itself in the slower growth of the required transmission power with growth in the SINR requirement, and also in the SINR requirement at which the problem becomes infeasible in at least one of the instances studied.

VIII. Conclusion

We considered the design of a precoder for the downlink of a multiple antenna systems with probabilistically-constrained QoS requirements in the presence of uncertain CSI at the transmitter. We considered two stochastic models for the uncertainty in the CSI, and we studied the design of a robust linear precoder that seeks to minimize the total transmitted power subject to the satisfaction of the QoS constraints with a maximum allowed outage probability. To overcome the absence of computationally-tractable solution for these type of problems, we presented two conservative design approaches that yield convex and computationally-efficient conservative designs of the original design problem. As illustrated by the simulations, the proposed methods can significantly expands the range of QoS requirements that can be satisfied in the presence of uncertainty in the CSI.

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