On the Design of Linear Transceivers for Multiuser Systems with Channel Uncertainty

Michael Botros Shenouda, *Student Member, IEEE,* and Timothy N. Davidson, *Member, IEEE,*

***Abstract*—We consider the design of linear transceivers for multiuser communication systems in the presence of uncertain channel state information (CSI), with an emphasis on downlink systems with a single antenna at each receiver. For systems with uplink-downlink reciprocity, we consider a stochastic model for the channel uncertainty, and we propose an efficient algorithm for the joint design of the linear precoding matrix at the base station and the equalizing gains at the receivers so as to minimize the average mean-square-error (MSE) over the channel uncertainty. The design is based on a generalization, derived herein, of the MSE duality between the broadcast and multiple access channels (MAC) to scenarios with uncertain CSI, and on a convex formulation for the design of robust transceivers for the dual MAC. For systems in which quantized channel feedback is employed, we consider a deterministically-bounded model for the channel uncertainty, and we study the design of robust downlink transceivers that minimize the worst-case MSE over all admissible channels. While we show that the design problem is NP-hard, we also propose an iterative local optimization algorithm that is based on efficiently-solvable convex conic formulations. Our framework is quite flexible, and can incorporate different bounded uncertainty models as well as a variety of power constraints. In particular, we study a "system-wide" uncertainty model, and although the resulting design problem is still NP hard, it does result in a significantly simpler iterative local design algorithm than the "per-user" uncertainty model. Our approaches to the minimax design for the downlink can be extended to the uplink, and we provide explicit formulations for the resulting uplink designs. Simulation results indicate that the proposed approaches to robust linear transceiver design can significantly reduce the sensitivity of the downlink to uncertain CSI, and can provide improved performance over that of existing robust designs.**

***Index Terms*—multiuser transceiver design; broadcast channel; multiple access channel; channel uncertainty; MSE duality; statistical robustness; minimax robustness; bilinear matrix inequality.**

I. Introduction

THE PROVISION OF multiple antennas at the base station facilitates the transmission of independent messages to different users on the downlink of a multiuser system; e.g., [1]. For these broadcast channels, the availability of accurate channel state information (CSI) at the transmitter is required in order to spatially multiplex the messages for different users by precoding them in a way that mitigates the effects of multiuser interference. Assuming that perfect CSI is available, several precoding techniques have been proposed, including the class of schemes that apply linear precoding at the transmitter jointly with linear equalization at each receiver. Those schemes offer a desirable trade-off between performance and transmitter complexity, and examples include zero-forcing techniques for channel inversion [2], [3], regularized channel inversion [4], minimum mean square error (MMSE) techniques [5], [6], and beamforming with a prespecified signal to interference plus noise ratio (SINR) at the receivers [1], [7].

Many precoding schemes assume that the transmitter has perfect channel knowledge of all the users' channels, but in practice the CSI at the transmitter suffers from inaccuracies caused by errors in channel estimation and/or limited, delayed or erroneous feedback, and the performance of downlink linear precoding systems is rather sensitive to these channel uncertainties. For example, it was recently shown [8] that imperfect channel knowledge at the transmitter can result in the downlink becoming interference limited; i.e., the growth of SINR of each user with the transmitted power saturates.

Due to the inevitability of imperfect channel information, robust communication schemes that take into account the channel uncertainty are of interest in practice; e.g., [9], [10]. The goal of the work herein is to propose robust linear transceivers for the downlink that explicitly take into account the uncertainties in the channel model, with an emphasis on systems with a single antenna at each receiver. In systems with reciprocity between the uplink and the downlink (e.g., time division duplex systems), the base station can estimate the channel and the channel uncertainty is mainly due to channel estimation errors. In that case, a stochastic model for the uncertainty in the channel model is appropriate, and possible design approaches include those based on average performance measures, and those based on notions of outage. For these systems, we consider the joint design of the linear precoder matrix and the users' equalizing gains so as to minimize the average, over the channel uncertainty, of the sum of the MSEs of each user. Since this design objective is not a jointly convex function of the precoding matrix and the equalizing gains, previous robust approaches considered a simpler design problem that restricts the equalizing gains to be equal (e.g., [11] [12]), or used a simpler detection model [13], [14]. The proposed approach for solving the general design problem (without restricting the equalizing gains), involves the generalization of the MSE duality between the broadcast channel and multiuser access channel (MAC) [15], [16] to scenarios with uncertain CSI. Using this duality,

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The authors are with the Department of Electrical and Computer Engineering, McMaster University, Hamilton, Ontario, Canada (e-mail: botrosmw@mcmaster.ca; davidson@mcmaster.ca)

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we obtain a closed-form expression that relates the desired robust broadcast transceivers to the corresponding transceivers that optimize the same performance metric for the dual MAC. The solution to the robust transceiver design problem for the dual MAC results in a closed-form expression for the optimal equalizer, and a convex conic formulation for the dual MAC optimal transmitters. Hence, by exploiting the MSE duality between BC and MAC in scenarios with uncertain CSI, we are able to transform the non-convex design problem for the BC into a convex and efficiently-solvable equivalent design problem. (See, e.g., [17], [18] for an introduction to convex conic optimization.)

For systems in which the channel is estimated and quantized at the receiver and then fed back to the transmitter (e.g., [8], [19], [20], [21]), one has a bound on the (quantization) error and hence an appropriate approach to robust design would be to optimize the worst-case performance over errors of that size. For these systems, we study the design of robust downlink transceivers that minimize the worst-case MSE over a bounded uncertainty model of each user's channel. While we show that that design problem is NP-hard, we propose an iterative local optimization algorithm that is based on efficiently-solvable convex conic formulations. The problem formulation and proposed algorithms can incorporate different bounded uncertainty models, and they can be applied to systems with per-antenna, per cell, and spatial-shaping power constraints, as well as the standard constraint on the total transmitted power. In particular, we study a "system-wide" uncertainty model as an alternative to the "per-user" model that is suitable for large cells and for multi-cell designs. While the resulting design problem is still NP hard, it results in a significantly simpler iterative local design algorithm than the "per-user" uncertainty model. Our approaches to the minimax design for the downlink can be extended to the uplink, and we provide explicit formulations for the resulting uplink designs. Our simulation results demonstrate that the proposed approaches to robust linear transceiver design can significantly reduce the sensitivity of the downlink to uncertain CSI, and can provide improved performance over that of existing robust designs.

Our notation is as follows: Boldface type is used to denote matrices and vectors; $\mathbf{P}^H$ denotes the conjugate transpose of the matrix $\mathbf{P}$. The notation $\|\mathbf{p}\|$ refers to the Euclidean norm of vector $\mathbf{p}$, while $\|\mathbf{E}\|$ denotes the spectral norm (maximum singular value) of the matrix $\mathbf{E}$, [22]. The term $\mathrm{tr}(\mathbf{A})$ denotes the trace of matrix $\mathbf{A}$, and for symmetric matrices $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{A} \geq \mathbf{B}$ denotes the fact that $\mathbf{A} - \mathbf{B}$ is positive semidefinite. The notation $\mathrm{Diag}(\mathbf{x})$ denotes the diagonal matrix whose non-zero elements are the elements of $\mathbf{x}$.

II. System Model

We consider broadcast channels with N_t antennas at the transmitter and K receivers, each with a single antenna. Let $\mathbf{s} \in \mathbb{C}^K$ be the vector of data symbols intended for the receivers. The transmitter linearly precodes the vector $\mathbf{s}$ to form $\mathbf{x} \in \mathbb{C}^{N_t}$,

$$\mathbf{x} = \mathbf{P}\mathbf{s} = \sum_{j=1}^{K} \mathbf{p}_j s_j, \tag{1}$$

where $\mathbf{p}_j$ is the j^{th} column of the precoding matrix $\mathbf{P}$; i.e., the beamforming weights for the j^{th} user. Without loss of generality, we will assume that $\mathrm{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}$, and hence, the total transmitted power constraint $\mathrm{E}\{\mathbf{x}^H\mathbf{x}\} \leq P_{\text{total}}$ reduces to $\mathrm{tr}\{\mathbf{P}^H\mathbf{P}\} \leq P_{\text{total}}$.

The signal y_k received by the k^{th} user is given by

$$y_k = \mathbf{h}_k\mathbf{x} + n_k, \tag{2}$$

where $\mathbf{h}_k \in \mathbb{C}^{1\times N_t}$ is a row vector[1] representing the channel gains from the transmitting antennas to the k^{th} receiver, and n_k is the additive zero-mean white noise at the k^{th} receiver whose variance is σ_n^2. Collecting the received signals in the vector $\mathbf{y}$, we will find it convenient to use the vector notation $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$, where $\mathbf{H}$ is the broadcast channel matrix whose k^{th} row is $\mathbf{h}_k$, and the covariance matrix of the noise vector $\mathbf{n}$ is $\mathrm{E}\{\mathbf{n}\mathbf{n}^H\} = \sigma_n^2\mathbf{I}$. Due to the decentralized nature of the receivers, joint processing of the received vector $\mathbf{y}$ is not possible. Instead, each receiver will process its received signal y_k independently using a single equalizing gain g_k to obtain an estimate of its intended symbol

$$\hat{s}_k = g_k y_k. \tag{3}$$

Using (3), the mean square error MSE_k associated with the k^{th} symbol can be written as:

$$\begin{aligned}\mathrm{MSE}_k &= \mathrm{E}\{|\hat{s}_k - s_k|^2\} \\ &= \sum_{j=1}^{K} |g_k|^2 \mathbf{p}_j^H(\mathbf{h}_k^H\mathbf{h}_k)\mathbf{p}_j + \sigma_n^2|g_k|^2 \\ &\qquad - g_k\mathbf{h}_k\mathbf{p}_k - g_k^H\mathbf{p}_k^H\mathbf{h}_k^H + 1 \\ &= \|g_k\mathbf{h}_k\mathbf{P} - \mathbf{m}_k\|^2 + \sigma_n^2|g_k|^2,\end{aligned} \tag{4}$$

where $\mathbf{m}_k$ is the i^{th} row of $\mathbf{I}$. Similarly, the total MSE can be written as:

$$\begin{aligned}\mathrm{MSE} = \mathrm{E}\{\|\hat{\mathbf{s}} - \mathbf{s}\|^2\} &= \sum_{k=1}^{K} \mathrm{MSE}_k \\ &= \mathrm{tr}\{(\mathbf{GHP} - \mathbf{I})^H(\mathbf{GHP} - \mathbf{I})\} + \sigma_n^2\|\mathbf{g}\|^2,\end{aligned} \tag{5}$$

where $\mathbf{g} = (g_1, \ldots, g_K)$ and $\mathbf{G} = \mathrm{Diag}(\mathbf{g})$.

The purpose of this paper is to determine efficient algorithms for the joint design of $\mathbf{P}$ and $\mathbf{g}$ with the goal of minimizing the MSE, in the presence of channel uncertainty. We will adopt the common implementation (e.g., [4], [11], [12], [14], [15], [16]) in which $\mathbf{P}$ and $\mathbf{g}$ are jointly designed at the base station (using the available CSI), and the base station informs each receiver of the equalizing gain, g_k, that it is to use. Actually, from (4) it can be seen that the phase component of each g_k can be absorbed into $\mathbf{p}_k$ without affecting MSE_k, and hence only the magnitude of g_k needs to be sent to receiver k; e.g., [15], [16]. We will point out below that this observation also applies to the robust transceiver designs for scenarios with uncertain CSI that we will develop herein.

[1] Although treating $\mathbf{h}_k$ as a row vector is a mild abuse of notation, it is common practice.

A. Channel Uncertainty Models

We consider additive uncertainty models for the CSI available at the transmitter:

$$\mathbf{h}_k = \hat{\mathbf{h}}_k + \mathbf{e}_k, \tag{6}$$

where $\hat{\mathbf{h}}_k$ is the transmitter's estimate of $\mathbf{h}_k$, and $\mathbf{e}_k$ is the corresponding error. This can be equivalently written as $\mathbf{H} = \hat{\mathbf{H}} + \mathbf{E}$, where $\mathbf{e}_k$ is the k^{th} row of $\mathbf{E}$. We will develop design formulations for robust transceivers under two broad models for the channel uncertainty.

The first model is suitable for communication schemes with reciprocity between the uplink and the downlink, which allows the transmitter to estimate the users' channels on the uplink. We will adopt a model in which the estimation errors are modelled by zero-mean random variables with covariances $\mathrm{E}\{\mathbf{e}_k^H \mathbf{e}_k\} = \sigma_{e_k}^2 \mathbf{I}$, where $\sigma_{e_k}^2$ depends on the uplink SNR of user k. This model is appropriate for scenarios in which the elements of $\mathbf{h}_k$ have zero mean and are uncorrelated with each other and those of other users, and linear minimum mean-square error estimation is used to estimate the channels on the uplink.[2] For this stochastic uncertainty model, robust transceivers based on the average MSE will be presented in Section III.

In the second model, the error $\mathbf{e}_k$ is assumed to be deterministically bounded, $\|\mathbf{e}_k\| \leq \delta_k$, for some given δ_k. This model is a convenient one for systems in which the channel state information is quantized at the receivers and fed back to the transmitter; e.g., [8], [20], [19], [21]. In particular, if the quantizer is (almost) uniform, then the quantization cells in the interior of the quantization region can be "covered" by balls of size δ_k. Using this model, the channel uncertainty set of each user can be described by the following (spherical) region:

$$\mathcal{U}_k(\delta_k) = \{\mathbf{h}_k \mid \mathbf{h}_k = \hat{\mathbf{h}}_k + \mathbf{e}_k,\ \|\mathbf{e}_k\| \leq \delta_k\}. \tag{7}$$

For this "per-user" bounded uncertainty model, minimax robust downlink transceivers based on the worst-case MSE will be presented in Section IV, for the uncertainty region in (7), as well as other regions. As an alternative to this "per-user" uncertainty model, the transmitter can consider a bounded model for the error matrix $\|\mathbf{E}\| \leq \Delta$, where an estimate of $\|\mathbf{E}\|$ is

$$\|\mathbf{E}\| \leq \sqrt{\textstyle\sum_{k=1}^{K} \|\mathbf{e}_k\|^2}. \tag{8}$$

For this "system-wide" uncertainty model, the channel uncertainty set can be described by

$$\mathcal{U}(\Delta) = \{\mathbf{H} \mid \mathbf{H} = \hat{\mathbf{H}} + \mathbf{E},\ \|\mathbf{E}\| \leq \Delta\}, \tag{9}$$

and a minimax robust downlink transceiver will be presented in Section V.

III. Downlink Statistically Robust Design Via BC-MAC Duality

For the stochastic uncertainty model, our objective is to jointly design the precoding matrix $\mathbf{P}$ and the receivers' equalizing gains g_k so as to minimize the average, over the channel estimation error, of the total MSE:

$$\overline{\mathrm{MSE}} = \sum_{k=1}^{K} \overline{\mathrm{MSE}}_k, \tag{10}$$

where each $\overline{\mathrm{MSE}}_k$ is given by:

$$\overline{\mathrm{MSE}}_k = \sum_{j=1}^{K} |g_k|^2 \mathbf{p}_j^H (\hat{\mathbf{h}}_k^H \hat{\mathbf{h}}_k + \sigma_{e_k}^2 \mathbf{I}) \mathbf{p}_j + \sigma_n^2 |g_k|^2 - g_k \hat{\mathbf{h}}_k \mathbf{p}_k - g_k \hat{\mathbf{h}}_k^H \mathbf{p}_k^H + 1. \tag{11}$$

It can be seen from (11), that each $\overline{\mathrm{MSE}}_k$ is not a jointly convex function of $\mathbf{P}$ and g_k.[3] To overcome this problem, previous approaches to the design of robust BC transceivers have considered simplifying the design by restricting all g_k to be equal [11], [12], or by using a simpler detection model [13]. In our approach, we will obtain a computationally efficient solution for the $\mathbf{P}$ and g_k that jointly minimize (10) by exploiting the duality between the broadcast channel (BC) and the multiple access channel (MAC). We will start by briefly reviewing (e.g., [15], [16], [23], [24], [25], [26], [27]) the dual MAC for the BC presented in Section II.

A. Dual Multiple Access Channel

By switching the roles of the transmitter and the receiver in the broadcast channel, we obtain the dual MAC that consists of K transmitters, each with a single antenna, and a receiver with N_t antennas. The channel matrix for the dual MAC is $\mathbf{H}^H$. Similar to the MSE expressions obtained for the BC in (11), we will be interested in obtaining corresponding expressions of individual MSEs in the dual MAC with linear precoding and linear multiuser reception. Because the transmitters in the dual MAC are decentralized and each have only one transmit antenna, linear precoding reduces to power loading:

$$x_k^{\text{MAC}} = p_k^{\text{MAC}} s_k^{\text{MAC}}, \tag{12}$$

where s_k^{MAC} and x_k^{MAC} are the data symbol and the transmitted signal of the k^{th} transmitter. Without loss of generality, we will assume that $\mathrm{E}\{\mathbf{s}^{\text{MAC}} \mathbf{s}^{\text{MAC}^H}\} = \mathbf{I}$. Hence, a total power constraint on all the transmitters can be written as $\sum_{k=1}^{K} |p_k^{\text{MAC}}|^2 \leq P_{\text{total}}$.

The vector of received signals $\mathbf{y}^{\text{MAC}}$ is given by

$$\mathbf{y}^{\text{MAC}} = \mathbf{H}^H \mathbf{x}^{\text{MAC}} + \mathbf{n}^{\text{MAC}}, \tag{13}$$

where $\mathbf{n}^{\text{MAC}}$ is the zero-mean receiver noise vector whose covariance matrix is $\mathrm{E}\{\mathbf{n}^{\text{MAC}} \mathbf{n}^{\text{MAC}^H}\} = \sigma_n^2 \mathbf{I}$. Using a linear multiuser receiver, $\mathbf{g}_k^{\text{MAC}} \in \mathbb{C}^{1 \times N_t}$, the base station obtains an estimate of the symbol transmitted by the k^{th} user, $\hat{s}_k^{\text{MAC}} = \mathbf{g}_k^{\text{MAC}} \mathbf{y}^{\text{MAC}}$.

Using the stochastic channel uncertainty model, the average over the channel estimation errors of the MSE associated with

[2] All our derivations extend directly to the case in which $\mathrm{E}\{\mathbf{e}_k^H \mathbf{e}_k\}$ is an arbitrary symmetric positive definite matrix, but for simplicity we will focus on the stated model.

[3] It can also be seen from (11) that the phase component of g_k can be absorbed into $\mathbf{p}_k$ without changing $\overline{\mathrm{MSE}}_k$, and hence that the base station need only send $|g_k|$ to receiver k.

the estimation of $\hat{s}_k^{\text{MAC}}$ can be written as

$$\begin{aligned}\overline{\text{MSE}}_k^{\text{MAC}} &= \sum_{j=1}^{K} |p_j^{\text{MAC}}|^2 \mathbf{g}_k^{\text{MAC}} (\hat{\mathbf{h}}_j^H \hat{\mathbf{h}}_j + \sigma_{e_j}^2 \mathbf{I}) \mathbf{g}_k^{\text{MAC}^H} \\ &+ \sigma_n^2 \mathbf{g}_k^{\text{MAC}} \mathbf{g}_k^{\text{MAC}^H} - p_k^{\text{MAC}^H} \hat{\mathbf{h}}_k \mathbf{g}_k^{\text{MAC}^H} \\ &- p_k^{\text{MAC}} \mathbf{g}_k^{\text{MAC}} \hat{\mathbf{h}}_k^H + 1. \end{aligned} \quad (14)$$

B. BC-MAC Duality with Stochastic Channel Uncertainty

In this section, we will present the MSE duality result for the BC and MAC channels under the stochastic channel uncertainty model described in Section II-A. This duality result generalizes the MSE duality between the BC and MAC channels for the perfect channel knowledge case [15], [16], [23], [24], [25], [26] to scenarios with uncertain CSI.[4] The duality relation will be useful in obtaining a robust BC transceiver that minimizes the average MSE in terms of the corresponding transceiver of the dual MAC that minimizes the same objective.

Theorem 1: Under the same total transmitted power constraint, the sets of individual average MSEs for the BC, $\{\overline{\text{MSE}}_k\}$, and for the dual MAC, $\{\overline{\text{MSE}}_k^{\text{MAC}}\}$, are equal when one uses the following transceiver designs:

$$\mathbf{p}_k = \omega_k \mathbf{g}_k^{\text{MAC}^H}, \qquad g_k = \omega_k^{-1} p_k^{\text{MAC}^H}, \quad (15)$$

where the vector of positive constants $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_K)$ is given by:

$$\boldsymbol{\omega}^2 = \mathbf{M}^{-1} \left[\ |p_1^{\text{MAC}}|^2, \ldots, |p_K^{\text{MAC}}|^2\ \right]^T, \quad (16)$$

and the matrix $\mathbf{M}$ is given by:

$$[\mathbf{M}]_{k,j} = \begin{cases} \sum_{i \neq k}^{K} \frac{|p_i^{\text{MAC}}|^2}{\sigma_n^2} \mathbf{g}_k^{\text{MAC}} (\hat{\mathbf{h}}_i^H \hat{\mathbf{h}}_i + \sigma_{e_i}^2 \mathbf{I}) \mathbf{g}_k^{\text{MAC}^H} \\ \quad + \mathbf{g}_k^{\text{MAC}} \mathbf{g}_k^{\text{MAC}^H} & k = j, \\ -\frac{|p_k^{\text{MAC}}|^2}{\sigma_n^2} \mathbf{g}_j^{\text{MAC}} (\hat{\mathbf{h}}_k^H \hat{\mathbf{h}}_k + \sigma_{e_k}^2 \mathbf{I}) \mathbf{g}_j^{\text{MAC}^H} & k \neq j. \end{cases} \quad (17)$$

A sketch of the proof of this result is provided in the appendix. It is a generalization of the proof in [16] to scenarios with channel uncertainty.[5] Using Theorem 1, the broadcast precoder $\mathbf{P}$ and receiver gains g_k that jointly minimize a general function of the users' average MSEs under a total power constraint can be obtained by first obtaining the MAC transceiver that jointly minimizes the same objective and then applying the transformation in (16) to obtain the optimal BC transceiver. In the following section we will consider the sum of the average MSEs as an example, and we will obtain an efficiently solvable formulation for the jointly optimal transceivers for the dual MAC that minimize that objective.

[4]Note that SINR duality does not extend to the statistical model of uncertain CSI [28].

[5]In fact, the MSE duality result for the stochastic uncertainty model extends directly to the case of multiple antennas at the receivers and multiple data streams per user, analogous to the case of perfect channel knowledge in [16].

C. Statistically Robust Transceiver Design for the Dual MAC

Our objective here is to find the dual MAC transmitters p_k^{MAC} and receivers $\mathbf{g}_k^{\text{MAC}}$ that jointly minimize the average MSE, $\overline{\text{MSE}}^{\text{MAC}} = \sum_{k=1}^{K} \overline{\text{MSE}}_k^{\text{MAC}}$. First, we will obtain an analytic expression for the optimal receiver $\mathbf{g}_k^{\text{MAC}}$ for a given set of transmitters p_i^{MAC}. Using these expressions we will then obtain a convex formulation for the optimal p_k^{MAC} under a total power constraint.

To design the $\mathbf{g}_k^{\text{MAC}}$, we observe from (14) that each $\overline{\text{MSE}}_k^{\text{MAC}}$ is a convex function of $\mathbf{g}_k^{\text{MAC}}$ and is independent of the other $\mathbf{g}_j^{\text{MAC}}$, $j \neq k$, and hence that it can be minimized independently. Setting the derivative of $\overline{\text{MSE}}_k^{\text{MAC}}$ with respect to $\mathbf{g}_k^{\text{MAC}}$ to zero, we obtain the following expression for the optimal $\mathbf{g}_k^{\text{MAC}}$:

$$\mathbf{g}_k^{\text{MAC}} = p_k^{\text{MAC}^H} \hat{\mathbf{h}}_k \Big(\sum_{i=1}^{K} |p_i^{\text{MAC}}|^2 (\hat{\mathbf{h}}_i^H \hat{\mathbf{h}}_i + \sigma_{e_i}^2 \mathbf{I}) + \sigma_n^2 \mathbf{I} \Big)^{-1}. \quad (18)$$

Using this optimal value, the average total MSE reduces to

$$\overline{\text{MSE}}^{\text{MAC}} = K - N_t + \sigma_n^2 \text{tr}(\mathbf{B}^{-1}), \quad (19)$$

where $\mathbf{B} = \sum_{i=1}^{K} |p_i^{\text{MAC}}|^2 (\hat{\mathbf{h}}_i^H \hat{\mathbf{h}}_i + \sigma_{e_i}^2 \mathbf{I}) + \sigma_n^2 \mathbf{I}$.

The next step is to design the p_k^{MAC} that minimize (19) subject to a total transmitted power constraint $\sum_{k=1}^{K} |p_k^{\text{MAC}}|^2 \leq P_{\text{total}}$. By defining $q_k = |p_k^{\text{MAC}}|^2$, that problem can be formulated as:

$$\min_{q_i} \quad \text{tr}\Big(\sum_{i=1}^{K} q_i (\hat{\mathbf{h}}_i^H \hat{\mathbf{h}}_i + \sigma_{e_i}^2 \mathbf{I}) + \sigma_n^2 \mathbf{I} \Big)^{-1} \quad (20a)$$

$$\text{s. t.} \quad q_i \geq 0,\ i = 1, \ldots, K, \qquad \textstyle\sum_{i=1}^{K} q_i \leq P_{\text{total}}. \quad (20b)$$

Using techniques similar to those in [29], this problem can be transformed into the following (convex) Semidefinite Program (SDP):

$$\min_{q_i, \mathbf{S}} \text{tr}(\mathbf{S}) \quad (21a)$$

$$\text{s. t.} \begin{bmatrix} \mathbf{S} & \mathbf{I} \\ \mathbf{I} & \big(\sum_{i=1}^{K} q_i (\hat{\mathbf{h}}_i^H \hat{\mathbf{h}}_i + \sigma_{e_i}^2 \mathbf{I}) + \sigma_n^2 \mathbf{I} \big) \end{bmatrix} \geq \mathbf{0}, \quad (21b)$$

$$q_i \geq 0,\ i = 1, \ldots, K, \qquad \textstyle\sum_{i=1}^{K} q_i \leq P_{\text{total}}. \quad (21c)$$

This SDP can be efficiently solved using self-dual interior point methods; e.g., [30].[6]

IV. Downlink Minimax Robust Design with Individual Channel Uncertainties

In this section we present a robust transceiver design that does not rely on a statistical model of channel uncertainty, but merely assumes that the each user's channel lies within a given uncertainty set $\mathcal{U}_k(\delta_k)$; c.f. (7). As mentioned in Section II-A, this uncertainty model is a convenient one for systems in which a channel estimate is quantized at the receiver and fed back to the transmitter. For this type of channel uncertainty, our goal is to jointly design the precoder $\mathbf{P}$ and equalization gains g_k so as to minimize the worst-case MSE over all

[6]Without loss of generality, we can choose each p_k^{MAC} to be the positive square root of q_k. Since ω_k in (15) is then real, each optimal g_k will be real.

admissible channels $\mathbf{h}_k \in \mathcal{U}_k(\delta_k)$, subject to a total power constraint. That is,

$$\min_{\mathbf{P},\,\mathbf{g}} \max_{\mathbf{h}_k \in \mathcal{U}_k(\delta_k)} \sum_{k=1}^{K} \|g_k \mathbf{h}_k \mathbf{P} - \mathbf{m}_k\|^2 + \sigma_n^2 \|\mathbf{g}\|^2 \tag{22a}$$

$$\text{s. t.} \quad \|\text{vec}(\mathbf{P})\|^2 \leq P_{\text{total}}. \tag{22b}$$

By introducing the auxiliary variables $t_k,\ 0 \leq k \leq K$, this minimax problem can be written as the following minimization problem:

$$\min_{\mathbf{P},\,\mathbf{g},\mathbf{t}} \sum_{k=0}^{K} t_k^2 \tag{23a}$$

$$\text{s. t.} \quad \|g_k \mathbf{h}_k \mathbf{P} - \mathbf{m}_k\| \leq t_k \qquad \forall\, 1 \leq k \leq K,\ \mathbf{h}_k \in \mathcal{U}_k(\delta_k), \tag{23b}$$

$$\sigma_n \|\mathbf{g}\| \leq t_0, \tag{23c}$$

along with (22b).[7] The constraint in (23b) represents K infinite sets of second order cone (SOC) constraints (e.g., [18], [17]), with one constraint for each $\mathbf{h}_k \in \mathcal{U}_k(\delta_k)$. However, these infinite sets of constraints can be precisely characterized by the following set of $2K$ inequalities [31]:

$$\begin{bmatrix} \lambda_k - \mu_k & \mathbf{0} & (g_k \hat{\mathbf{h}}_k \mathbf{P} - \mathbf{m}_k) \\ \mathbf{0} & \mu_k \mathbf{I} & \delta_k (g_k \mathbf{P}) \\ (g_k \hat{\mathbf{h}}_k \mathbf{P} - \mathbf{m}_k)^H & \delta_k (g_k \mathbf{P})^H & \lambda_k \mathbf{I} \end{bmatrix} \succeq \mathbf{0},$$

$$t_k - \lambda_k \geq 0, \quad 1 \leq k \leq K. \tag{24}$$

Using the characterization in (24), the robust transceiver design can be formulated as:

$$\min_{\substack{\mathbf{P},\,\mathbf{g},\\ \boldsymbol{\lambda},\,\boldsymbol{\mu},\,\alpha}} \alpha \tag{25a}$$

$$\text{s. t.} \quad \left\| \begin{bmatrix} \boldsymbol{\lambda} \\ \sigma_n \mathbf{g} \end{bmatrix} \right\|^2 \leq \alpha, \tag{25b}$$

$$\begin{bmatrix} \lambda_k - \mu_k & \mathbf{0} & (g_k \hat{\mathbf{h}}_k \mathbf{P} - \mathbf{m}_k) \\ \mathbf{0} & \mu_k \mathbf{I} & \delta_k (g_k \mathbf{P}) \\ (g_k \hat{\mathbf{h}}_k \mathbf{P} - \mathbf{m}_k)^H & \delta_k (g_k \mathbf{P})^H & \lambda_k \mathbf{I} \end{bmatrix} \succeq \mathbf{0} \quad 1 \leq k \leq K, \tag{25c}$$

$$\|\text{vec}(\mathbf{P})\|^2 \leq P_{\text{total}}, \tag{25d}$$

where we have used the fact that the optimal value for t_0 is $\sigma_n \|\mathbf{g}\|$, and that for t_k is λ_k. The constraint in (25c) represents a set of K bilinear matrix inequalities and hence the optimization problem in (25) is NP hard [32]. However, given initial values for $\mathbf{P}$ and $\mathbf{g}$, one can find a locally optimal solution by iteratively optimizing over $\mathbf{P}$ for fixed $\mathbf{g}$, and over $\mathbf{g}$ for fixed $\mathbf{P}$. Each of those problems is implicit in (25) and is a convex conic program that can be efficiently solved; e.g., [30]. One natural choice of the starting point for this iterative design would be the transceiver designed for the case in which the estimates $\hat{\mathbf{h}}_k$ are assumed to be the actual channels; e.g., [15], [4].

[7] As was the case in the previous section, the formulation in (23) shows that phase component of g_k can be absorbed into $\mathbf{p}_k$. Indeed, if $\{|g_k|\ e^{j\theta_k}\}$ and $\mathbf{P}$ form an optimal solution of (23), then $\{|g_k|\}$ and $\mathbf{P}\,\text{Diag}(e^{j\theta_1}, \ldots, e^{j\theta_K})$ are also optimal.

A. Other power constraints

The formulation in (22) employs a simple constraint on the transmitted power. However, other types of power constraints can be incorporated into the design without compromising the convex conic nature of the steps in the proposed iterative algorithm. In particular, one can incorporate:

- Per-antenna power constraints: In practice, each antenna at the base station has its own power amplifier and hence a constraint on the average power transmitted by each individual antenna, $\text{E}\{|x_n|^2\} \leq P_n,\ 1 \leq n \leq N_t$, is of practical importance [33]. These constraints can be formulated as the second order cone (SOC) constraints $\|\mathbf{m}_n \mathbf{P}\|^2 \leq P_n,\ 1 \leq n \leq N_t$.
- Per-cell power constraints: These constraints arise naturally in the design of multi-cell downlinks, in which neighboring base stations cooperate in the downlink transmission by acting as a sparse antenna array; e.g., [34]. Similar to per-antenna constraints, per-cell power constraints can be formulated as SOC constraints.
- Spatial masking constraints: These constraints arise from the imposition of a spatially-shaped bound (e.g., [35]) on the transmitted power, $\text{E}\{\mathbf{x}^H \mathbf{Q}(\theta) \mathbf{x}\} \leq P_{\text{shape}}(\theta)$ for all $\theta \in \Theta$, where $\mathbf{Q}(\theta) = \mathbf{v}(\theta)\mathbf{v}^H(\theta)$, with $\mathbf{v}(\theta)$ being the "steering vector" (e.g., [36]) of the transmitter's antenna array in the direction θ, $P_{\text{shape}}(\theta)$ is the maximum allowable power in the direction of θ, and Θ is the set of angles of interest. Such constraints are of interest in cellular systems in which interference to neighboring cells needs to be controlled; e.g., [34]. A convenient way in which this constraint can be incorporated into (25) is through the following set of (weighted) second order cone constraints:

$$\|\text{vec}\big(\mathbf{Q}(\theta)^{1/2} \mathbf{P}\big)\| \leq \sqrt{P_{\text{shape}}(\theta)} \qquad \forall\, \theta \in \Theta. \tag{26}$$

B. Other uncertainty regions

The formulation in (22) is based on the simple spherical uncertainty region in (7). In some applications, other uncertainty regions might be more appropriate, but many such regions lead to design problems that are at least as hard as that in (25). As an example, in some applications the quantization regions may be better covered by ellipses of the form

$$\tilde{\mathcal{U}}_k(\delta_k) = \Big\{ \mathbf{h}_k \mid \mathbf{h}_k = \hat{\mathbf{h}}_k + \sum_{j=1}^{J} \mathbf{a}_j u_j,\ \|\mathbf{u}\| \leq \delta_k \Big\} \tag{27}$$

rather than the spheres in (7). For uncertainties in this form, the robust transceiver design problem takes a form similar to (25), but with each constraint in (25c) replaced by

$$\begin{bmatrix} \lambda_k - \mu_k & \mathbf{0} & (g_k \hat{\mathbf{h}}_k \mathbf{P} - \mathbf{m}_k) \\ \mathbf{0} & \mu_k \mathbf{I} & \delta_k g_k \mathbf{A}\mathbf{P} \\ (g_k \hat{\mathbf{h}}_k \mathbf{P} - \mathbf{m}_k)^H & \delta_k (g_k \mathbf{A}\mathbf{P})^H & \lambda_k \mathbf{I} \end{bmatrix} \succeq \mathbf{0}, \tag{28}$$

where $\mathbf{A}$ is the matrix whose rows are $\mathbf{a}_j$. Hence, this problem is also NP-hard.

In other applications, uncertainty regions with box-like constraints on the elements of $\mathbf{h}_k$ may be more appropriate.

However, that model, which is the intersection of a set of ellipsoidal regions of the form in (27), is more difficult to deal with because there is no known tractable characterization of the corresponding infinite set of constraints of the form in (23b), [31].

V. Downlink Minimax Robust Design with Overall Channel Uncertainty

The robust minimax design in (25) for the "per-user" channel uncertainty model contains K bilinear matrix inequalities, one for each user. In this section, we consider the alternative "system-wide" channel uncertainty model in (9), namely $\|\mathbf{E}\| \leq \Delta$, and we will show that the resulting robust minimax design involves only one nonlinear matrix inequality. Therefore, the computational cost of the conic programs used in the iterative algorithm is reduced. This approach may be suitable for downlink systems involving cells with large number of users or for multi-cell designs.

As in the previous section, our goal is to jointly design the precoder $\mathbf{P}$ and the equalization gains g_k so as to minimize the worst-case MSE over all admissible channels, subject to a total power constraint. The design problem can be formally stated as:

$$\min_{\substack{\mathbf{P},\\ \mathbf{G}=\mathrm{Diag}(\mathbf{g})}} \quad \max_{\|\mathbf{E}\|\leq\Delta} \ \mathrm{tr}\{(\mathbf{I}-\mathbf{GHP})^H(\mathbf{I}-\mathbf{GHP})\} + \sigma_n^2\|\mathbf{g}\|^2 \tag{29a}$$

$$\text{s. t.} \quad \|\mathrm{vec}(\mathbf{P})\|^2 \leq P_{\text{total}}, \tag{29b}$$

and using the auxiliary variables w_0 and w_1, that minimax problem can be precisely transformed into the following minimization problem:

$$\min_{\substack{\mathbf{P},\, w_0, w_1\\ \mathbf{G}=\mathrm{Diag}(\mathbf{g})}} \quad w_0 + w_1 \tag{30a}$$

$$\text{s. t.} \quad \mathrm{tr}(\mathbf{I}-\mathbf{G}(\hat{\mathbf{H}}+\mathbf{E})\mathbf{P})^H(\mathbf{I}-\mathbf{G}(\hat{\mathbf{H}}+\mathbf{E})\mathbf{P}) \leq w_1 \quad \forall\,\|\mathbf{E}\| \leq \Delta, \tag{30b}$$

$$\sigma_n^2\|\mathbf{g}\|^2 \leq w_0, \tag{30c}$$

$$\|\mathrm{vec}(\mathbf{P})\|^2 \leq P_{\text{total}}. \tag{30d}$$

Like (23), this problem has an infinite set of constraints, namely (30b). (Furthermore, we can also choose $\mathbf{g}$ to be a real vector, without loss of generality.) The first step in the transformation of (30b) into a single constraint is the application of the following lemma.

Lemma 1 ([37]): Let $\mathbf{M} \in \mathbb{C}^{K\times K}$ be a Hermitian matrix. Then there exists a scalar s and a matrix $\mathbf{Z} \geq \mathbf{0}$ such that the constraint $\mathrm{tr}(\mathbf{M}) \leq t$ is equivalent to the following representation:

$$t - Ks - \mathrm{tr}(\mathbf{Z}) \geq 0, \tag{31}$$

$$\mathbf{M} \leq \mathbf{Z} + s\mathbf{I}. \tag{32}$$

While Lemma 1 considers a single matrix $\mathbf{M}$, it can be directly extended to a set of matrices by applying the lemma to an element of that set of matrices with the largest trace. Applying that extension to (30b) yields a single constraint of the form in (31) and the set of constraints $(\mathbf{I}-\mathbf{G}(\hat{\mathbf{H}}+\mathbf{E})\mathbf{P})^H(\mathbf{I}-\mathbf{G}(\hat{\mathbf{H}}+\mathbf{E})\mathbf{P}) \leq \mathbf{Z}+s\mathbf{I}$, $\forall\|\mathbf{E}\| \leq \Delta$. Using the Schur Complement Theorem [22], that set of quadratic matrix inequalities can be transformed into the following set of bilinear matrix inequalities:

$$\begin{bmatrix} \mathbf{Z}+s\mathbf{I} & (\mathbf{I}-\mathbf{G}(\hat{\mathbf{H}}+\mathbf{E})\mathbf{P})^H \\ (\mathbf{I}-\mathbf{G}(\hat{\mathbf{H}}+\mathbf{E})\mathbf{P}) & \mathbf{I} \end{bmatrix} \geq \mathbf{0} \quad \forall\,\|\mathbf{E}\| \leq \Delta. \tag{33}$$

By moving terms containing $\mathbf{E}$ to the right-hand side of the inequality, we can re-write (33) as:

$$\begin{bmatrix} \mathbf{Z}+s\mathbf{I} & (\mathbf{I}-\mathbf{G}\hat{\mathbf{H}}\mathbf{P})^H \\ (\mathbf{I}-\mathbf{G}\hat{\mathbf{H}}\mathbf{P}) & \mathbf{I} \end{bmatrix} \geq \begin{bmatrix} \mathbf{0} \\ \mathbf{G} \end{bmatrix} \mathbf{E} \begin{bmatrix} \mathbf{P} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{P}^H \\ \mathbf{0} \end{bmatrix} \mathbf{E}^H \begin{bmatrix} \mathbf{0} & \mathbf{G}^H \end{bmatrix} \quad \forall\|\mathbf{E}\| \leq \Delta. \tag{34}$$

To cast (34) as a single matrix inequality we use the following lemma:

Lemma 2 ([38]): Let $\mathbf{A}$ be a Hermitian matrix. Then $\mathbf{A} \geq \mathbf{C}^H\mathbf{X}^H\mathbf{B} + \mathbf{B}^H\mathbf{X}\mathbf{C}$ for all $\|\mathbf{X}\| \leq \Delta$ if and only if there exists a $\lambda \geq 0$ such that

$$\begin{bmatrix} \mathbf{A}-\lambda\mathbf{C}^H\mathbf{C} & -\Delta\mathbf{B}^H \\ -\Delta\mathbf{B} & \lambda\mathbf{I} \end{bmatrix} \geq \mathbf{0}.$$

Applying Lemma 2 with $\mathbf{B} = [\mathbf{P} \quad \mathbf{0}]$, and $\mathbf{C} = [\mathbf{0} \quad \mathbf{G}^H]$, the robust minimax design in (29) can be formulated as

$$\min_{\substack{\mathbf{P},\mathbf{G}=\mathrm{Diag}(\mathbf{g}),\\ s,\lambda,w_0,w_1,\mathbf{Z}}} \quad w_0 + w_1 \tag{35a}$$

$$\text{s. t.} \quad \begin{bmatrix} \mathbf{Z}+s\mathbf{I} & (\mathbf{I}-\mathbf{G}\hat{\mathbf{H}}\mathbf{P})^H & -\Delta\mathbf{P}^H \\ (\mathbf{I}-\mathbf{G}\hat{\mathbf{H}}\mathbf{P}) & \mathbf{I}-\lambda\mathbf{G}\mathbf{G}^H & \mathbf{0} \\ -\Delta\mathbf{P} & \mathbf{0} & \lambda\mathbf{I} \end{bmatrix} \geq \mathbf{0}, \tag{35b}$$

$$w_1 - Ks - tr(\mathbf{Z}) \geq 0, \tag{35c}$$

$$s \geq 0, \tag{35d}$$

$$\sigma_n^2\|\mathbf{g}\|^2 \leq w_0, \tag{35e}$$

$$\|\mathrm{vec}(\mathbf{P})\|^2 \leq P_{\text{total}}. \tag{35f}$$

Although this problem has a finite number of inequalities, like (25), the presence of the non-linear matrix inequality in (35b) renders (35) NP-hard. However, one can use an iterative algorithm to obtain a locally optimal solution. For the iterations with fixed $\mathbf{g}$, the problem in (35) represents a convex conic optimization problem that can be solved more efficiently than the corresponding problem in the case of "per-user" channel uncertainty model, c.f., (25). For the iterations with fixed $\mathbf{P}$, one can interchange the choices of $\mathbf{B}$ and $\mathbf{C}$ in the application of Lemma 2 to obtain an equivalent inequality to (35b) that is linear in $\mathbf{g}$. The resulting problem is also an efficiently-solvable convex conic optimization problem.

As was the case with the results in Section III-B, the results in this section extend directly to the case of multiple antennas at the receivers and multiple data streams per user. For such scenarios, $\mathbf{G}$ is a block diagonal matrix (with rectangular blocks).

VI. Uplink Minimax Robust Designs

The proposed design framework for minimax robust transceivers for the downlink is quite general and can be applied to uplink systems as well. In this section we will provide explicit formulations of the minimax robust designs for the dual MAC. As mentioned in Section III, the channel matrix for the dual MAC is $\mathbf{H}^H$, and we will define $\mathbf{p}^{\mathrm{MAC}} = (p_1^{\mathrm{MAC}}, \ldots, p_K^{\mathrm{MAC}})$ and $\mathbf{G}^{\mathrm{MAC}}$ to be the matrix whose rows are $\mathbf{g}_k^{\mathrm{MAC}}$.

To derive the robust "per-user" minimax design, we first observe that MSE expression for the k^{th} user in the uplink is function is function of all channels, not just its own. While these multiple sources of uncertainty can complicate the design, one can write the total MSE as

$$\mathrm{MSE}^{\mathrm{MAC}} = \sum_{k=1}^{K} \|\mathbf{G}^{\mathrm{MAC}}\mathbf{h}_k^H p_k^{\mathrm{MAC}} - \mathbf{m}_k^T\|^2 + \sigma_n^2 \mathrm{tr}((\mathbf{G}^{\mathrm{MAC}})^H \mathbf{G}^{\mathrm{MAC}}), \quad (36)$$

where each term of the summation is subject to uncertainty from one source only. Using (36) and the analysis in Section IV, the uplink robust minimax design with the "per-user" uncertainty model can be formulated as

$$\min_{\mathbf{G}^{\mathrm{MAC}}, \mathbf{p}^{\mathrm{MAC}}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \beta} \quad \beta \quad (37a)$$

s. t.

$$\left\| \begin{bmatrix} \boldsymbol{\lambda} \\ \sigma_n \mathrm{vec}(\mathbf{G}^{\mathrm{MAC}}) \end{bmatrix} \right\|^2 \leq \beta, \quad (37b)$$

$$\begin{bmatrix} \lambda_k - \mu_k & \mathbf{0} & \phi_k^H \\ \mathbf{0} & \mu_k \mathbf{I} & \delta_k (\mathbf{G}^{\mathrm{MAC}} p_k^{\mathrm{MAC}})^H \\ \phi_k & \delta_k (\mathbf{G}^{\mathrm{MAC}} p_k^{\mathrm{MAC}}) & \lambda_k \mathbf{I} \end{bmatrix} \geq \mathbf{0}$$

$$1 \leq k \leq K, \quad (37c)$$

$$\|\mathbf{p}^{\mathrm{MAC}}\|^2 \leq P_{\mathrm{total}}, \quad (37d)$$

where ϕ_k is used as placeholder for $\mathbf{G}^{\mathrm{MAC}}\hat{\mathbf{h}}_k^H p_k^{\mathrm{MAC}} - \mathbf{m}_k^T$. Similarly, the uplink robust minimax design with the "system-wide" model of uncertainty can be formulated as

$$\min_{\substack{\mathbf{G}^{\mathrm{MAC}}, \mathbf{Z}, s, \lambda, w_0, w_1, \\ \mathbf{P}^{\mathrm{MAC}} = \mathrm{Diag}(\mathbf{p}^{\mathrm{MAC}})}} \quad w_0 + w_1 \quad (38a)$$

s. t.

$$w_1 - Ks - \mathrm{tr}(\mathbf{Z}) \geq 0, \quad (38b)$$

$$\begin{bmatrix} \mathbf{Z} + s\mathbf{I} & \Phi^H & -\Delta(\mathbf{P}^{\mathrm{MAC}})^H \\ \Phi & \mathbf{I} - \lambda \mathbf{G}^{\mathrm{MAC}} (\mathbf{G}^{\mathrm{MAC}})^H & \mathbf{0} \\ -\Delta \mathbf{P}^{\mathrm{MAC}} & \mathbf{0} & \lambda \mathbf{I} \end{bmatrix} \geq \mathbf{0}$$

$$1 \leq k \leq K, \quad (38c)$$

$$s \geq 0, \quad (38d)$$

$$\sigma_n^2 \|\mathrm{vec}(\mathbf{G}^{\mathrm{MAC}})\|^2 \leq w_0, \quad (38e)$$

$$\|\mathbf{p}^{\mathrm{MAC}}\|^2 \leq P_{\mathrm{total}}, \quad (38f)$$

where Φ is used as placeholder for $\mathbf{I} - \mathbf{G}^{\mathrm{MAC}}\hat{\mathbf{H}}^H \mathbf{P}^{\mathrm{MAC}}$. As with the case with the downlink, both problems are NP hard, but one can employ a local iterative algorithm in which a convex conic program is solved at each iteration.

VII. Simulation Studies

In order to compare the performance of the proposed robust designs with existing approaches, we have simulated these methods for the cases of uncoded QPSK and 16-QAM transmission over independent block fading Rayleigh channels (without shadowing). We considered downlink scenarios with $N_t = 4$ and 5 antennas, and $K = 4$ users, at different distances from the base station. The first two users are assumed to be far from the base station and their channels coefficients are modeled as being independent circularly symmetric complex Gaussian random variables with zero mean and unit variance. The other two users are assumed to be closer to the base station and their channel coefficients are generated using the above model but with variances equal to 10.[8] We will plot the average bit error rate (BER) over all users against the signal-to-noise-ratio, which is defined as $\mathrm{SNR} = P_{\mathrm{total}}/(K\sigma_n^2)$. We will also plot the average BER over each pair of near and far users. The BERs are averaged over 500 channel realizations, $\mathbf{H}$. For each realization, we construct 100 channel estimates, $\hat{\mathbf{H}}$, using (6). For each channel estimate, we compute the robust precoder $\mathbf{P}$ and the equalizing gains $\mathbf{g}$, inform each receiver of the equalizing gain g_k that it is to use, and transmit a block of 200 uncoded vector symbols $\mathbf{s}$. (The channel $\mathbf{H}$ is held constant over that block, as are the transmitter's estimate $\hat{\mathbf{H}}$ and the transceiver design $(\mathbf{P}, \mathbf{g})$.) Using (1)–(3), the inputs to the decision device of receiver k can be written as

$$\hat{s}_k = g_k \mathbf{h}_k \mathbf{p}_k s_k + \sum_{j \neq k} g_k \mathbf{h}_k \mathbf{p}_j s_j + g_k n_k. \quad (39)$$

Since $\mathbf{P}$ and $\mathbf{g}$ are designed using the transmitter's estimate of the channel, $\hat{\mathbf{H}}$, it can be seen from (39) that the uncertainty in $\hat{\mathbf{H}}$ affects the scaling of the desired symbol, the interference from signals transmitted to other users, and the scaling of the additive noise.

A. Statistically robust transceiver design

The channel estimation error $\mathbf{e}_k = \mathbf{h}_k - \hat{\mathbf{h}}_k$ was modelled by generating $\mathbf{e}_k$ from a zero-mean Gaussian distribution with $\mathrm{E}\{\mathbf{e}_k^H \mathbf{e}_k\} = \sigma_{e_k}^2 \mathbf{I}$, where we will use the same $\sigma_{e_k}^2$ for all users. This model is appropriate for a scenario in which the uplink power is controlled so that the received SNRs on the uplink are equal and independent from the downlink SNR. For convenience, we define $\epsilon^2 = \mathrm{E}\{\mathbf{e}_k \mathbf{e}_k^H\} = N_t \sigma_{e_k}^2$.

In Fig. 1 we compare the performance of the statistically robust transceiver proposed in Section III with that of the regularized channel inversion approach in [4], [39], and that of the channel inversion approach in [2], [3], for a system with 4 transmit antennas, 4 users, QPSK signalling, and $\epsilon^2 = 0.01$. It can be seen that the performance of a linear transceiver in the broadcast channel is rather sensitive to the mismatch between the actual CSI and the transmitter's estimate of CSI; see also [8]. It can be also seen that while the effect of noise is dominant at low SNR, the channel uncertainty dominates at high SNR, where the proposed robust transceiver design performs significantly better than the other two approaches.

[8] In practice, a scheduler may select the users to which data is transmitted, but in order to focus on the impact of the proposed designs, no scheduling will be considered in the simulations.

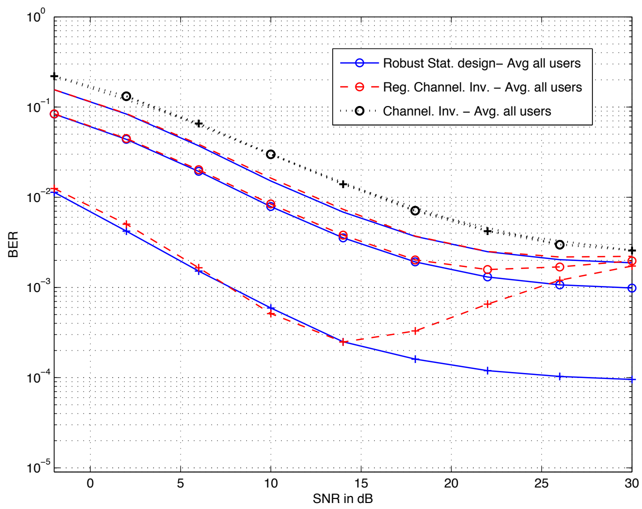

Fig. 1. Comparison between the performance of the proposed statistically robust transceiver, the channel inversion approach in [2], [3], and regularized channel inversion [4], [39] for values of channel uncertainty $\epsilon^2 = 0.01$ for a system with $N_t = 4$ and $K = 4$ using QPSK signalling. The curves with (+) markers and no markers represent the average BER of the two near and the two far users, respectively.

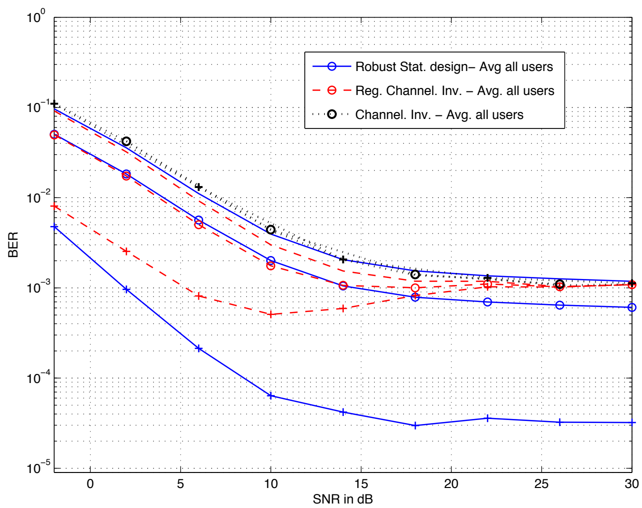

Fig. 2. Comparison between the performance of the transceivers considered in Figure 1 for a system with $N_t = 5$ and $\epsilon^2 = 0.1$. The legend is the same as that in Figure 1.

Fig. 1 also shows that in the presence of channel uncertainty, both the regularized channel inversion and channel inversion designs have the same performance limit at high SNR. This is due to the fact that the regularized method involves the addition of a regularization term whose value is inversely proportional to $P_{\text{total}}/(K\sigma_n^2)$; see [4]. In Fig. 2 we compare the performance of the statistically robust transceiver with that of channel inversion approach in [2], [3], and regularized channel inversion approach in [4], [39], for a system with 5 transmit antennas, 4 users, QPSK signalling, and uncertainty value $\epsilon^2 = 0.1$. The impact of the robust design is apparent in the average performance of the two near users for the whole SNR range, and in the average performance of all users at high SNRs.

For Fig. 3 we consider a system with 16-QAM signalling, 5 transmit antennas and 4 users, and we compare the performance of the proposed statistically robust transceiver with that of the robust regularized channel inversion approach in [12], which restricts all the receiver gains g_k to be equal. It can be seen from Fig. 3 that significant improvement in the performance of the near users can be achieved by the proposed robust design, as it offers more degrees of freedom in the choice of the gains g_k.

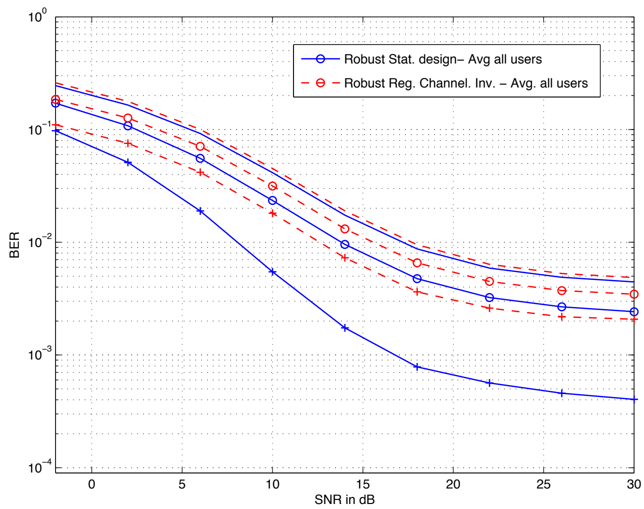

Fig. 3. Comparison between the performance of the proposed statistically robust transceiver and the robust regularized channel inversion approach in [12] for values of channel uncertainty $\epsilon^2 = 0.03$ for a system with $N_t = 5$ and $K = 4$ using 16-QAM signalling. The curves with (+) markers and no markers represent the average BER of the two near and the two far users, respectively.

B. Robust minimax transceiver designs

In systems that use feedback to provide the transmitter with quantized version of the CSI, the information available to the transmitter will include the designed quantization codebooks and the statistics of the error resulting from the use of these codebooks; e.g., $\mathrm{E}\{(\mathbf{h}_k - \hat{\mathbf{h}}_k)(\mathbf{h}_k - \hat{\mathbf{h}}_k)^H\} = \epsilon^2$. Since we assume each user's channel is independent from the others, the transmitter can model the error matrix $\mathbf{E}$ as being zero mean with independent rows and second order statistics given by $\mathrm{E}\{\mathbf{E}\mathbf{E}^H\} = \epsilon^2\mathbf{I}$. Thus, we have $\|\mathrm{E}\{\mathbf{E}\mathbf{E}^H\}\| = \epsilon^2$. To simulate quantization errors, we will generate matrices $\mathbf{E}$ such that the real and imaginary parts of each element E_{ij} are drawn independently from uniform distribution $U\left(-\sqrt{\frac{3}{2N_t}}\epsilon, \sqrt{\frac{3}{2N_t}\epsilon}\right)$, and hence $\mathrm{E}\{\mathbf{E}\mathbf{E}^H\} = \epsilon^2\mathbf{I}$. Given that the transmitter will have access to ϵ, and since $\Delta^2 = \|\mathbf{E}^H\mathbf{E}\|$, an appropriate choice for Δ, for the "system-wide" uncertainty model, is ϵ. For the "per-user" uncertainty model, when all users are using the same codebooks, all δ_k are equal and one can use equation (8) to set $\delta_k = \epsilon/\sqrt{K}$.

In Fig. 4 the performance of the proposed robust minimax approaches with "per-user" and "system-wide" uncertainty models is compared to that of the regularized channel inversion approach in [4], [39] in the presence of uniformly distributed quantization errors with $\epsilon^2 = 0.03$ for a system with $N_t = 5$, $K = 4$ and 16-QAM signalling. It can be seen that performance of the minimax approach with the "system-wide" uncertainty model is reasonably close to the minimax approach with "per-user" uncertainty, especially in terms of

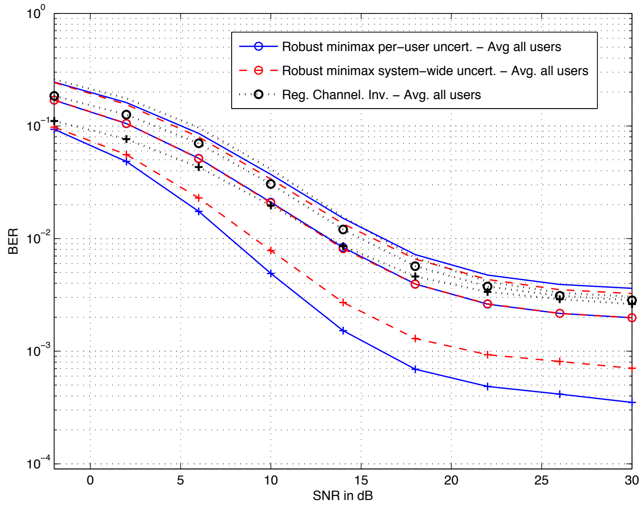

Fig. 4. Comparison between the performance of the proposed robust minimax approaches with "per-user" and "system-wide" uncertainty models, and the regularized channel inversion approach in [4], [39] in the presence of uniformly distributed quantization errors with $\epsilon^2 = 0.03$ for a system with $N_t = 5$ and $K = 4$ using 16-QAM signalling. The curves with (+) markers and no markers represent the average BER of the two near and the two far users, respectively.

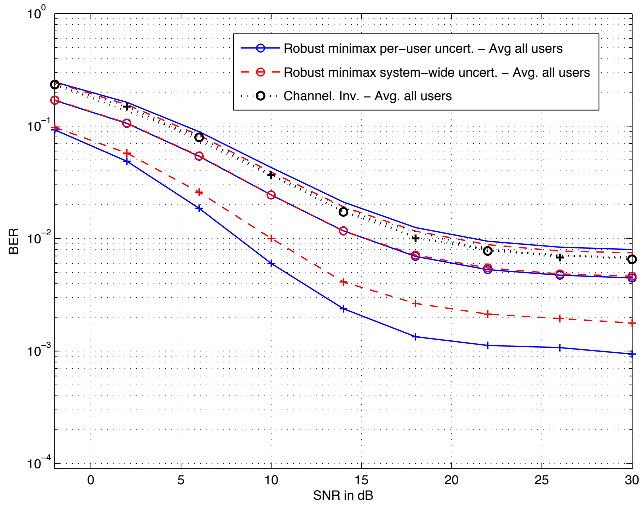

Fig. 5. Comparison between the performance of the proposed robust minimax approaches with "per-user" and "system-wide" uncertainty models, and the channel inversion approach in [2], [3] in the presence of uniformly distributed quantization errors with $\epsilon^2 = 0.05$ for a system with $N_t = 5$ and $K = 4$ using 16-QAM signalling. The curves with (+) markers and no markers represent the average BER of the two near and the two far users, respectively.

the average performance of all users. Both approaches provide improved performance over the non-robust approach in terms of the average BER and significantly improved performance in terms of the BER of the near users. In Fig. 5, a comparison is made with the non-robust of channel inversion approach in [2], [3], for a similar system with $\epsilon^2 = 0.05$, and similar performance advantages are observed.

VIII. Conclusion

We have considered statistical and minimax robust joint designs for linear transceivers for multiuser communication systems. For the statistical approach, we have presented a robust design for the broadcast channel transceivers that jointly minimize the average, over the channel estimation errors, of the sum of the MSEs of each user. By generalizing the MSE duality between the broadcast channel (BC) and multiple access channel (MAC) to schemes with channel estimation errors, we have shown that the robust design for the broadcast channel can be obtained from an efficiently-solvable conic programming formulation for the robust transceivers for the dual MAC. For the minimax approach, we have provided a formulation for the robust downlink transceivers that maximize the worst-case performance for "per-user" and "system-wide" channel uncertainty models. We also proposed computationally-tractable iterative algorithms for obtaining locally optimal solutions to these two design problems. The problem formulation and proposed algorithms can be applied to systems with per-antenna, per-cell, and spatial-shaping power constraints, as well as a constraint on the total transmitted power. We showed that proposed minimax downlink transceiver designs can be applied to the design of uplink transceivers as well. Our simulation results demonstrated that the proposed approaches to the robust linear transceiver design can significantly reduce the sensitivity of the downlink to uncertain CSI, and can provide improved performance over that of existing robust designs.

Appendix
Proof of Theorem 1

We start by considering linearly related transceivers for BC and dual MAC:

$$\mathbf{p}_k = \omega_k \mathbf{g}_k^{\mathrm{MAC}^H}, \qquad g_k = \chi_k p_k^{\mathrm{MAC}^H}, \tag{40}$$

and we find the necessary conditions for ω_k and χ_k such that set of MSEs in BC and dual MSE are equal. By setting $\overline{\mathrm{MSE}}_k = \overline{\mathrm{MSE}}_k^{\mathrm{MAC}}$ and substituting the values $\mathbf{p}_k$ and g_k from (40), we obtain a set of K equations. From the equality of coefficients the term in $p_k^{\mathrm{MAC}} \mathbf{g}_k^{\mathrm{MAC}} \hat{\mathbf{h}}_k^H$ (or $p_k^{\mathrm{MAC}^H} \hat{\mathbf{h}}_k \mathbf{g}_k^{\mathrm{MAC}^H}$) on both sides we have $\chi_k = 1/\omega_k$. Using this relation, the set of K equations reduces to the following linear system in $\boldsymbol{\omega}^2$:

$$\mathbf{M}\,\boldsymbol{\omega}^2 = \left[\ |p_1^{\mathrm{MAC}}|^2, \ldots, |p_K^{\mathrm{MAC}}|^2\ \right]^T, \tag{41}$$

where $\mathbf{M}$ was defined in (17). We observe that $\mathbf{M}$ has strictly dominant diagonal elements and negative off-diagonal elements, hence it is non-singular and the elements of $\mathbf{M}^{-1}$ are non-negative. Adding all equations in the linear system in (41) results in $\sum_{k=1}^{K} \omega_k^2 \mathbf{g}_k^{\mathrm{MAC}} \mathbf{g}_k^{\mathrm{MAC}^H} = \sum_{k=1}^{K} |p_k^{\mathrm{MAC}}|^2$, i.e., total transmitted power in BC and dual MAC are the same.

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Michael Botros Shenouda received the B.Sc. (Hons. 1) degree in 2001 and the M.Sc. degree in 2003, both in electrical engineering and both from Cairo University, Egypt. He is currently working toward the Ph.D. degree at the Department of Electrical and Computer Engineering, McMaster University, Canada. His main areas of interest include wireless and MIMO communication, convex and robust optimization, and signal processing algorithms. He is also interested in majorization theory, and its use in the development of design frameworks for non-linear MIMO transceivers. Mr. Botros Shenouda was awarded an IEEE Student Paper Award at ICASSP 2006, and was a finalist in the IEEE Student Paper Award competition at ICASSP 2007.

Tim Davidson (M'96) received the B.Eng. (Hons. I) degree in Electronic Engineering from the University of Western Australia (UWA), Perth, in 1991 and the D.Phil. degree in Engineering Science from the University of Oxford, U.K., in 1995.

He is currently an Associate Professor in the Department of Electrical and Computer Engineering at McMaster University, Hamilton, Ontario, Canada, where he holds the (Tier II) Canada Research Chair in Communication Systems, and is currently serving as Acting Director of the School of Computational Engineering and Science. His research interests lie in the general areas of communications, signal processing and control. He has held research positions at the Communications Research Laboratory at McMaster University, the Adaptive Signal Processing Laboratory at UWA, and the Australian Telecommunications Research Institute at Curtin University of Technology, Perth, Western Australia.

Dr. Davidson was awarded the 1991 J. A. Wood Memorial Prize (for "the most outstanding [UWA] graduand" in the pure and applied sciences) and the 1991 Rhodes Scholarship for Western Australia. He is currently serving as an Associate Editor of the IEEE Transactions on Signal Processing and as an Editor of the IEEE Transactions on Wireless Communications. He has also served as an Associate Editor of the IEEE Transactions on Circuits and Systems II, and as a Guest Co-editor of issues of the IEEE Journal on Selected Areas in Communications and the IEEE Journal on Selected Topics in Signal Processing.