Optical Power Reduction for Multiple-Subcarrier Modulated Indoor Wireless Optical Channels

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***Abstract*— In this paper, the unregulated bandwidth available in wireless optical channels is exploited to reduce the average optical power in multiple-subcarrier modulated (MSM) systems. Data transmission is confined to a bandwidth located near DC, while out-of-band subcarrier signals are designed outside the data bandwidth to reduce the average optical power. Although the out-of-band signals at higher frequency are subject to severe corruption induced by multipath distortion, they are removed at the receiver and not used for detection. Optimizing the out-of-band carrier amplitudes over the set of real numbers yields gains as high as 2.6 dB over conventional MSM systems at the same bandwidth efficiency. We additionally apply in-band trellis coding and out-of-band signals, whose amplitudes are optimized over a discrete constellation. When no out-of-band signals are used, the system achieves an average optical power reduction of 0.9 dB over uncoded systems with a simultaneous peak power reduction of 0.4 dB. An additional average optical power reduction of 0.9 dB is realized at the expense of 4 out-of-band carriers and a moderate increase in peak power.**

I. Introduction

Indoor wireless optical communication links are attractive complements to indoor radio frequency (RF) links due to their low cost, high security and freedom from spectral licensing issues [1, 2]. Optical transmission is easily blocked by opaque objects, eliminating co-channel interference which is common in RF systems. Unlike RF channels, the electromagnetic spectrum in the optical band is unlicensed worldwide, providing virtually unlimited bandwidth for system design. Indoor optical wireless channels have sharp bandwidth constraints due to multipath distortion as well as photodiode depletion capacitance [1]. Previous approaches neglect this upper bandwidth and confine signal design to lowpass signals. In this paper, we design signals which exploit these higher frequencies, not to send additional data, but rather to reduce the average optical power requirement in wireless optical multiple-subcarrier modulated (MSM) systems. Data transmission is confined within a *data bandwidth* located near DC where channel attenuation is low. The signals sent outside the data bandwidth, denoted as *out-of-band* signals, do not carry information and are designed to reduce the average optical power, i.e. amplitude, of the transmitted signal. These out-of-band signals are filtered out by the channel and do not cause co-channel interference due to the inherent isolation of optical wireless channels. We develop novel techniques to design out-of-band signals and show potential optical power gains are as high as 2.6 dB over conventional MSM systems at the same bandwidth efficiency.

In diffuse indoor wireless optical channels, information is transmitted by modulating the instantaneous intensity of an optical source which is emitted over a wide solid angle and allowed to reflect off surfaces in a room. At the receiver, a photodiode produces an electrical current proportional to the optical power impinging on its surface. These channels do not exhibit multipath fading, however, multiple reflections cause multipath distortion which leads to a low pass channel response [1]. Since the transmitted signals are intensities, all modulated amplitudes must be non-negative. Additionally, due to the limited dynamic range of the transmitter, the peak value must practically also be limited. Eye and skin safety regulations require that the average amplitude, i.e., optical power, is constrained. As a result of these amplitude constraints it is not possible to apply conventional electrical modulation directly to optical wireless channels.

Electrical MSM systems have been used extensively on frequency selective channels [3]. The addition of many independent frequency carriers in MSM systems leads to a high peak-to-average power ratio (PAPR). Since the transmitter has a limited dynamic range, MSM systems often experience nonlinear distortion due to clipping. In order to reduce the PAPR, both block coding [4–10] and cyclic coding [11] techniques have been considered. Additionally, optimization techniques have been used to assign amplitudes to a set of reserved subcarriers to minimize the PAPR [12]. Although these techniques reduce the PAPR effectively in electrical MSM systems, they cannot be applied directly to optical wireless channels due to the amplitude non-negativity and average constraints.

Optical MSM techniques have also been applied on the optical fibre channels to support multiple users on different frequencies [13]. The use of MSM in indoor wireless optical communications was first investigated by Carruthers *et al.* [14] as a means to provide bandwidth efficient communication as well as for multi-access. Independent data were transmitted on each carrier and a fixed bias was added to each symbol to ensure non-negative amplitude. A time-varying bias signal was introduced in [15] as a means of both reducing the average optical power and improving detection. In this paper, we adopt the conventional term "normal MSM system" to denote an optical MSM system with QPSK and symbol-by-symbol bias.

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As in electrical systems, large positive and negative amplitude peaks may occur in optical MSM systems. The negative peaks require a large bias to ensure non-negativity which in turn implies a large average optical power requirement. In [16], optimized block codes were developed to minimize the average optical power in normal MSM systems realizing gains of 0.1 dB over a normal system at the same bandwidth efficiency. In [17], a single reserved subcarrier is added to the in-band signal and the transmitted amplitude optimized to provide the minimum average optical power when BPSK modulation is employed. In the case of more reserved subcarriers a suboptimal search procedure was employed. The PCMS technique [10] was applied to wireless optical MSM systems by simply adding a symbol-by-symbol bias [18]. In [19], a group of codewords were chosen from a subset of MSM signals with low average optical power such that the minimum distance among each pair of codewords is maximized. Notice that all previous work for average power reduction of optical wireless MSM systems considered block coding and did not code over a *sequence* of outputted signals. Additionally, the reduction of both peak and average optical powers was not treated in any previous work.

In this work, we present novel methods which exploit the unregulated bandwidth and inherent isolation of wireless optical channels to reduce the average optical power requirement of wireless optical MSM systems. By optimizing the amplitude of a set of out-of-band carriers over the set of real numbers, the results in [16] are extended to provide an upper bound for average optical power reduction using out-of-band reserved subcarriers. Gains as high as 2.6 dB optical over normal system are realized at the same bandwidth efficiency using 4 additional carriers. To alleviate transmitter complexity, out-of-band subcarrier amplitudes are also optimized over a discrete constellation using an exhaustive search. Moreover, trellis-coded wireless optical MSM systems are developed by expanding the signal set and designing trellis codes over the increased degrees of freedom. Convolutional coding is applied to sequences of MSM symbols to reduce average optical power, which is not treated in previous work. In this manner, an average optical power reduction of 0.9 dB is achieved with a simultaneous peak power reduction up to 0.4 dB over conventional MSM system at the same bandwidth efficiency without out-of-band signals. Using 4 out-of-band carriers with discrete constellation, an additional average optical power reduction as high as 0.9 dB is achieved.

Section II describes the indoor wireless optical channel model and the optical MSM system. Section III presents the design of trellis-codes for the in-band carriers of the wireless optical MSM systems. The design of out-of-band carriers are discussed in Sec. IV. Simulation results of the proposed techniques, along with comparison to existing techniques are shown in Sec. V. The paper concludes in Sec. VI with some directions for future work.

II. IM/DD Channel Model and Wireless Optical MSM System

Indoor wireless optical channels can be modeled as the baseband linear system [1]

$$y(t) = rx(t) \otimes h(t) + n(t) \tag{1}$$

where $\otimes$ represents convolution, r[A/W] is the photodetector responsivity, $x(t)$[W] is the input intensity-modulated signal, $h(t)$ is the channel impulse response and $y(t)$[A] is the received photocurrent. The channel is often modeled as low-pass and we assume it is flat in the bandwidth used for data transmission. In this paper we denote this flat lowpass region of spectrum as *in-band* while all higher frequencies are termed *out-of-band*. The dominant noise source in indoor wireless optical channels is shot noise resulting from background light and receiver preamplifier noise. Due to the high-intensity of ambient illumination, the noise is well modeled as being white, signal independent and Gaussian distributed [1].

As an instantaneous optical power signal $x(t)$ must satisfy,

$$x(t) \geq 0.$$

Due to eye safety requirements, the average optical power P must be limited. From the channel model (1), a constraint is placed on the average amplitude

$$P = E[x(t)]$$

rather than $E[x^2(t)]$ as in electrical channels. Thus, a reduction of electrical power in dB is equivalent to half of that value in optical power. For example, 3 dB electrical power gain corresponds to 1.5 dB optical gain [2]. Moreover, the maximum amplitude,

$$I = \max_t x(t)$$

should also be limited due to the limited dynamic range of the transmitter.

To facilitate comparison of average and peak optical power requirement of various techniques, define a reference on-off keying (OOK) wireless optical system with bit error rate (BER) 10^{-6}, rectangular pulse shape, average optical power P_{OOK} and maximum amplitude I_{OOK} as is conventionally done [1, 16]. All schemes are designed to achieve the same bit rate and BER as the reference system. The average optical power, P, and maximum amplitude, I, of various systems are compared versus the reference using

$$\rho = 10\log_{10}\frac{P}{P_{\text{OOK}}} \quad [\text{dB}] \tag{2}$$

$$\psi = 10\log_{10}\frac{I}{I_{\text{OOK}}} \quad [\text{dB}] \tag{3}$$

where ρ is the normalized average optical power requirement and ψ is the normalized peak amplitude.

Fig. 1 presents a diagram of a wireless optical MSM transmitter with W in-band carriers and L out-of-band carriers giving a total number of $N = W + L$ carriers. It is assumed that the symbol period is limited to T seconds and that a

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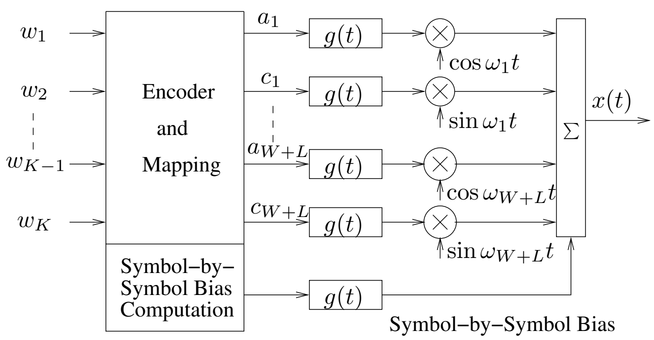

Fig. 1. Wireless optical MSM transmitter

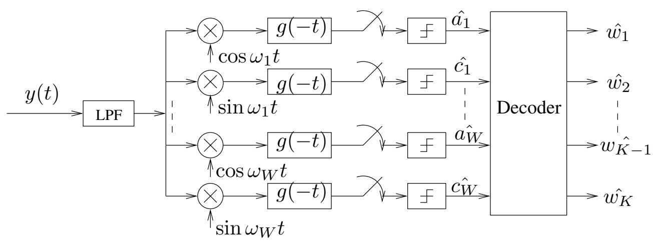

Fig. 2. Wireless optical MSM receiver

rectangular pulse shape $g(t)$ is employed. On each of the W in-band carriers a data bearing signal is transmitted while the L out-of-band carriers are set to reduce average optical power, as will be demonstrated shortly. We consider a symbol-by-symbol bias which is equal to the minimum of the N-MSM waveform at each symbol period since it offers significant optical power reduction [15, 16]. Thus, the goal of an optical power reduction technique is to reduce the magnitude of the negative peak of the N-MSM waveform for each symbol period. The amplitudes of the L-carrier out-of-band signals are designed to minimize the required bias and hence average optical power of the wireless optical MSM system.

The receiver in Fig. 2 produces an estimate of the transmitted data symbol over the W in-band carriers. The out-of-band signals are removed at the receiver by low-pass filtering. Therefore, distortion of the out-of-band signals at higher frequencies does not affect detection of data symbols. Assuming K bits are sent per symbol period, the null-to-null bandwidth efficiency of the system is defined as

$$\eta = \frac{K}{W+1} \qquad [\text{bits/sec/Hz}]. \tag{4}$$

Note that η is independent of the L out-of-band carriers. This is because these carriers do not carry any data and there is no constraint imposed on the spectrum of optical emissions. Thus, adding out-of-band subcarriers does not affect the bandwidth efficiency of the system.

III. Design of In-Band Trellis Codes

Inspired by Ungerboeck's work on trellis codes [20] and Frenger's work on PCMS [10], in this section we expand the signal set for the W in-band carriers by adding a zero symbol to each subcarrier constellation. Trellis coded modulation is

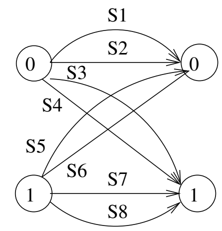

Fig. 3. Trellis structure for $K = 2$ and $M = 2$.

applied over these in-band symbols using this expanded signal set resulting in a reduction in both peak and average optical powers.

A. Constellation Expansion and Set Partitioning

Given a wireless optical MSM system with W in-band subcarriers, an *MSM constellation point* is composed of W *sub-constellations* on each frequency carrier. In our scheme, each sub-constellation is expanded by adding a zero amplitude to the QPSK constellation or to the 8-PSK constellation. The resulting constellations are called 5-APSK and 9-APSK constellations respectively. The trellis structure is fixed to have M states with M branches leaving each state. If K bits are to be sent per symbol interval, each branch in the trellis must therefore send $K - \log_2 M$ bits using parallel transitions. An example of $K = 2$,$M = 2$ is shown in Fig. 3. One bit selects the branch leaving the current state, the other bit selects which of the MSM constellation points S_i to transmit. To develop a good code, a search algorithm is designed in Sec. III-B to find sequences of MSM constellation points with good distance properties.

The expanded constellations could be constrained to have zero amplitude on *at least* U subcarriers for *all* data symbols. The search results obtained by using the constrained constellation should be a subset of results obtained by using the more general expanded search. However, as will be shown later, using our sub-optimal search algorithms, searching over this constrained set often yields codes with better performance. Therefore, expanded and constrained constellations are treated separately in this paper. Denote systems with $U = 0$ as coded MSM systems, and systems with $U \geq 1$ as constrained MSM systems.

B. Trellis-Coded MSM and Constrained MSM System

The block diagram of a coded-MSM transmitter is shown in Fig. 1 using a convolutional coder in the encoder block. Convolutional codes are designed to impose a structure on the transmitted sequences of symbols. Code development involves the design of a trellis structure and the mapping between MSM constellation points and branches.

Given the trellis structure, the mapping between MSM constellation points and branches in the trellis determines the free Euclidean distance, d_{free}, of the code. The d_{free} asymptotically determines the code's error performance and is the minimum distance between any two paths in the trellis. In this scheme, this distance can be limited by two MSM

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constellation points on the same branch or two different paths that start and end at the same state. Denote these distances as d_p and d_s respectively. Motivated by Ungerboeck's trellis code design heuristics [20], the following rules are applied when searching for a mapping:

1) Any MSM constellation point can occur only once at each stage of the trellis.
2) Pick parallel edges with maximum distance.
3) Pick points on edges leaving or entering the same state with maximum distance.

Rule 1 avoids confusion in decoding at each stage. Rules 2 and 3 are designed to maximize d_p and d_s respectively. For an M-state code using W data carriers with m-APSK sub-constellation and constraint U, the total number of MSM constellation points is

$$n_s = \sum_{i=U}^{W} \binom{W}{i} \times m^{W-i}.$$

To transmit K bits per symbol, the number of MSM constellation points at each stage of the trellis is $M \times 2^K$. Thus, the number of all possible mappings is

$$n_s^{M\times 2^K}.$$

As a result, any exhaustive search is prohibitively expensive, even for small K.

However, the search is greatly constrained if an MSM constellation point is selected at each step. Suppose at the fth step, $f-1$ MSM constellation points have been assigned. Let $d_i, i \in [1, f-1]$ denote the Euclidean distances between the $f\mathrm{th}$ point and the ith assigned point. Define P_f as,

$$P_f = \sum_{d=d_1}^{d_{f-1}} Q\left(\frac{d}{2\sigma}\right). \tag{5}$$

The function P_f is an upper bound on the probability that a wrong decision has been made regarding the $f\mathrm{th}$ MSM constellation point over each $f-1$ assigned points. At each step, a point is chosen to minimize this upper bound. Thus, we propose the following sub-optimal search algorithm for both the coded MSM system and constrained MSM system,

Algorithm Search for an MSM constellation point mapping for a given constraint $U \geq 0$.

1) Fix the first MSM constellation point.
2) For the fth MSM constellation point ($f > 1$), select the point with at least U zero points which minimizes P_f.
3) If number of occurrences of current point $n > 1$, return to step 2 and pick the point with the next smallest P_f.
4) Fix current point, add 1 to the number of occurrences of this point. If assignment not finished, go to step 2.

End.

Note that the same search algorithm is implemented for both constrained MSM system ($U > 0$) and coded MSM system ($U = 0$), despite the fact that constrained MSM system constellation is a subset of coded MSM system constellation. In fact, as shown in Sec. V, the search for constrained MSM system often returned codes with smaller average optical power requirement than search results for the more general coded MSM search. This is due to the sub-optimal search algorithm which improves the distance properties of the code stepwise rather than globally. Additionally, for each system the search is run using the following three sub-constellations for each non-zero in-band carrier: QPSK, 5-APSK and 9-APSK.

For each system with a specific sub-constellation type at a given N, U, K, and M, the search algorithm runs n_s times to search all possible starting points in step 1. In the case of large n_s, 100 random starting points are selected. After searching using all three sub-constellation types, the search terminates with a set of codes having some maximum d_free and smallest number of d_free paths. In the case where there is more than one such code, the average optical power of each of the codes are then compared and the code with minimum average optical power is chosen. The performance of both coded MSM systems and constrained MSM systems are shown in Sec. V.

IV. Design of Out-of-Band Reserved Subcarrier Signals

In this section, two methods for designing the out-of-band signals to reduce the average optical power are discussed. The first method optimizes the amplitudes of the out-of-band subcarriers over the set of real numbers by using a standard optimization algorithm. Although the results serve as an upper bound for the average optical power reduction using out-of-band carriers, this technique greatly complicates the transmitter. The second method optimizes out-of-band carrier amplitudes over a 9-APSK constellation using an exhaustive search. This technique is shown to provide significant average optical power reduction while maintaining a simple transmitter architecture.

A. Optimization of Amplitudes Over Real Numbers

In this section, we extend the results in [16] by finding the optimum amplitudes on L out-of-band subcarriers over the set of real numbers. For each possible selection of data on the W in-band carriers, the L out-of-band carriers are selected which maximize the minimum amplitude of the unbiased N-carrier signal. The minimum amplitude is approximated by discretizing the signal into samples. To minimize approximation error, the signal is oversampled and the approximation error can be shown to be negligible. The maximization problem can be formulated as a linear program which can be solved effectively using standard optimization algorithms [21]. The receiver removes the L out-of-band subcarriers by low-pass filtering and only detects the W in-band data subcarriers.

Note that this technique requires the transmitter to send real-amplitude carriers and to solve the optimization problem for each data symbol. Given the complexity of the transmitter design, implementation of this technique for current wireless optical MSM systems is complex. However, this technique demonstrates a upper bound for average optical reduction using out-of-band subcarrier signals. Sec. V presents performance of this technique applied to normal MSM system for

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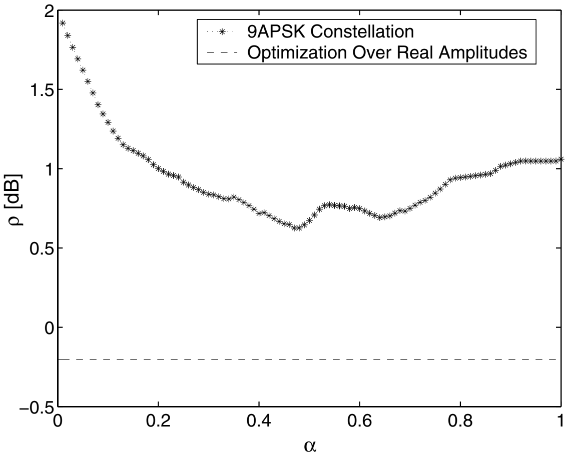

Fig. 4. Normalized Average Optical Power Requirement versus scaling factor for coded MSM system with $N = 3$, $U = 0$, $K = 4$, $M = 2$ and $L = 4$

the cases of $L = 1, 2, 4$ with $W = 1, \ldots, 7$, along with comparison to previous work.

B. Optimization of Amplitudes Over 9*-APSK Constellation*

In this section, the L out-of-band subcarrier amplitudes are designed over 9-APSK constellation rather than real numbers to alleviate computational burden and transmitter complexity. For each data symbol, an exhaustive search is performed *once* to find the optimal amplitudes on the L out-of-band subcarriers over a 9-APSK constellation which minimize the average optical power. The number of possible amplitudes on L out-of-band carriers is 9^L for each data symbol, which makes exhaustive search practical to implement for moderate L. The search results for each data symbol is stored at the transmitter to generate out-of-band signals according to the input data symbol.

To improve the average optical power reduction capability, a scaling factor α, in the range $(0, 1]$, is introduced to scale the amplitude transmitted on *all* out-of-band subcarriers with respect to the amplitudes of the in-band subcarriers. Unlike the most general case of individual real amplitudes on each out-of-band carrier considered in Sec. V, α is fixed for all symbols. To find the best value of α, the range of $(0, 1]$ is discretized into 20 points. For each α, an exhaustive search over all L, 9-APSK constellation points was performed to minimize the average optical power. The performance of this technique, applied to both coded MSM systems and constrained MSM systems, is presented in Sec. V.

The gain achieved by changing α for out-of-band carriers is quantified in Fig.4 for a coded MSM code designed at $N = 3$, $U = 0$, $K = 4$, $M = 2$ and $L = 4$. The normalized average optical power requirement to achieve a BER of 10^{-6} is plotted versus α. The performance realized when optimizing out-of-band subcarrier amplitudes over real number as in Sec. IV-A is also presented. An additional 0.4 dB saving of ρ is realized from setting $\alpha = 1$.

V. Simulation Results

Figures 5(a) and 5(b) plot the average optical power and peak amplitude of the techniques presented here versus bandwidth efficiency. For comparison, a normal MSM system with no out-of-band carriers and the minimum-power block coding technique in [16] are simulated for $N = 1, \ldots, 8$. Additionally, a recent SSPS code given in [19, Tbl. II] is also simulated for comparison. The normal MSM system with out-of-band carriers designed according to Sec. IV-A was simulated for $L = 1, 2, 4$ and the Sedumi 1.05 optimization toolbox [21] was used for each symbol. Applying the algorithm of Sec. III-B, trellis-coded MSM codes were designed with $N = 3$ and $N = 4$, while constrained MSM codes were designed with $N = 3, U = 1$ and $N = 4, U = 1, 2$ for different K values. The performance of coded systems with no out-of-band subcarriers and with $L = 4$ out-of-band subcarriers designed according to Sec. IV-B are both shown.

In Fig. 5(a) it is evident that bandwidth efficiency is traded for average optical power efficiency for all power reduction schemes, as is conventional in optical signalling design [22]. All proposed systems outperform existing techniques at the same bandwidth efficiency. When no out-of-band signals are used, trellis-coded MSM offers a significant reduction of 0.5 dB in average optical power while increasing the maximum amplitude by at most 0.6 dB, as shown in Fig. 5(b). In contrast, constrained MSM without out-of-band signals achieved a greater reduction up to 0.9 dB in average optical power while simultaneously *reducing* the maximum amplitude by 0.4 dB, due to the presence of U zero-amplitude carriers.

When out-of-band signals are used, real-valued amplitudes for out-of-band signals achieved the best reduction in average optical power at the expense of greatly increased peak amplitude. When adding 4 out-of-band scaled carriers to trellis-coded systems, a significant additional average optical power reduction of 0.9 dB to 1.4 dB is observed over the $L = 0$ case. However, the maximum amplitude of the system is increased on the order of 2 dB. Since both trellis coding and 9-APSK out-of-band signals are practical for transmitter design, constrained MSM system with out-of-band signals is an attractive solution for average optical power reduction of wireless optical MSM systems.

Note that conventionally, ρ is plotted against N or K [16–19]. Under such plot constrained MSM with $L = 0$ achieved an average optical power reduction of 2.3 dB over normal MSM systems with a simultaneous peak power reduction of 3.2 dB. Constrained MSM with $L = 4$ realized an average optical power reduction of 3.2 dB. However, the plot of ρ and ψ versus η more clearly justifies the performance of various schemes versus their cost and is therefore a better comparison.

VI. Conclusions

In this paper, the huge bandwidth available for wireless optical MSM channels is exploited to reduce the average optical power requirement. Out-of-band carriers are added at higher frequencies and their amplitudes optimized to reduce the average optical power requirement. By optimizing the real

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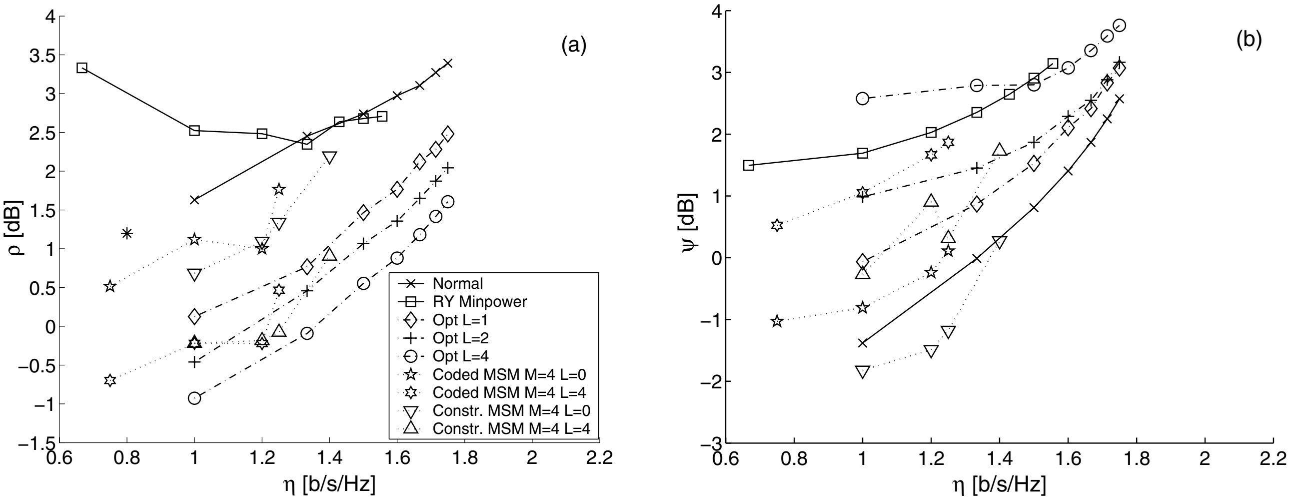

Fig. 5. Normalized average optical power (ρ) (a) and normalized maximum amplitude (ψ) (b) versus bandwidth efficiency (η).

amplitudes of out-of-band carriers using a standard optimization algorithm, gains as high as 2.6 dB over conventional MSM systems are realized at the same bandwidth efficiency. Moreover, trellis-coded MSM and constrained MSM systems expand the signal set and realize a peak and average optical power gain by coding over the increased degrees of freedom. An average optical power reduction up to 0.9 dB over uncoded systems is achieved with a simultaneous peak power reduction of 0.4 dB without out-of-band signals. An additional average optical power reduction of 0.9 dB is realized using 4 out-of-band 9-APSK amplitude carriers, making constrained MSM systems with out-of-band subcarriers an attractive solution for wireless optical MSM systems.

Although coded MSM and constrained MSM systems achieve excellent performance, decoding complexity increases as number of data carriers increases. Sub-optimal decoders and modified coding schemes are currently being developed to address these needs. Additionally, improved search techniques over the expanded MSM constellations may yield further improvements. The work presented here justifies further study into peak and average amplitude reduction techniques for indoor wireless optical MSM systems.

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