Upper and Lower Bounds on the Capacity of Wireless Optical Intensity Channels

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***Abstract*— Improved upper and lower bounds on the capacity of wireless optical intensity channels under non-negativity and average optical power constraints are derived. We consider intensity modulated/direct detection (IM/DD) channels with pulse amplitude modulation (PAM). Utilizing the signal space geometry and a sphere packing argument, an upper bound is derived. Compared to previous work, the derived upper bound is tighter at low signal-to-noise ratios. In addition, a lower bound is derived based on source entropy maximization over discrete distributions. The proposed distribution provides a tighter lower bound compared to previous continuous distributions. The derived bounds asymptotically describe the capacity of PAM optical intensity channels at both low and high SNR.**

I. Introduction

In this paper, we study the capacity of wireless optical intensity modulated/direct detection (IM/DD) channels. In these channels, information is modulated as the instantaneous optical intensity and hence the information bearing signal is restricted to be non-negative. An average amplitude, i.e., average optical power, constraint is imposed to ensure eye-safety. The direct application of techniques from electrical channels to this channel is thus not straight forward due to the amplitude constraints. Here, we present improved upper and lower bounds for wireless optical channels which take the amplitude constraints into account explicitly.

Wireless optical channels can be well modelled as conditionally Gaussian channels with signal independent noise [1]. For conditionally Gaussian channels with bounded-input and power constraints the capacity-achieving distribution, under certain conditions, is shown to be discrete with a finite number of probability mass points [2], [3]. Similar results were obtained for optical photon counting channels, i.e. Poisson channels, with optical power constraints [4]. Since the channel capacity is the maximum mutual information between transmitter and receiver over all possible input distributions, any input distribution results in a lower bound for the channel capacity. Based on this reasoning, a lower bound for the capacity of wireless optical IM/DD channels was computed using the maxentropic continuous exponential distribution satisfying the amplitude constraints [5].

The channel capacity of wireless optical intensity channels can be upper bounded by applying a similar sphere-packing argument presented by Shannon [6] in a region which guarantees that the amplitude constraints are met. You and Kahn utilized sphere-packing to derive an upper bound for the optical IM/DD channel capacity with multiple-subcarrier modulation [7]. Results for band and power-limited optical intensity channels were presented in [5] where the total volume is approximated by a generalized n-cone. As a result, the derived bound is only tight at high signal-to-noise ratio (SNR) and loose at low SNR.

In this work, tight upper and low bounds on the capacity of pulse amplitude modulated (PAM) wireless optical IM/DD channels are derived. Using the intuition from previous studies, a tight lower bound is derived using a family of entropy maximizing *discrete* distributions. Although not necessarily capacity achieving, these distributions are shown to provide a tight lower bound for the capacity of wireless optical IM/DD channels at both low and high SNRs. Compared to previous bounds based on continuous distributions, the presented bound has approximately double the channel capacity at SNR=0 dB. In addition, an analytic upper bound to the channel capacity is derived using a sphere packing argument. Unlike previous work [5], the Minkowski sum of convex bodies is utilized to obtain the exact volume of the outer parallel body at fixed distance from a regular n-simplex. As a result, the derived upper bound is tighter than previous bounds [5] at low signal-to-noise ratios. Since most wireless optical links typically operate at low SNRs, the tightness of the derived bounds at low SNR provides a useful benchmark for communication system design.

II. System Model

Wireless optical communication links transmit data by modulating the transmitted optical power of a laser. In practical links, only the optical intensity is modulated and detected. In the following analysis, we consider pulse amplitude modulation (PAM). The transmitted optical signal is constrained to be non-negative due to physical constraints. Due to eye safety concerns, a constraint is also imposed on the average optical power transmitted P, i.e., the average amplitude. The output electrical signal is related to the incident power by the detector responsivity coefficient R. Without loss of generality, we consider $R = 1$. A good statistical channel model for this channel is [1],

$$y = x + z$$

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where $x \geq 0$ is the transmitted optical signal with average optical power $\mathbb{E}\{x\} \leq P$, y is the output electrical signal and z is thermal noise which is well modelled as zero-mean, signal independent, Gaussian distributed with variance σ^2. We define the optical signal-to-noise ratio as $\mathrm{SNR} = P/\sigma$ as in previous work [3], [5].

III. Lower bound on channel capacity

The capacity of the wireless optical IM/DD channel is defined as the maximum mutual information between channel input and output over all possible input distributions satisfying the non-negativity and average optical power constraints. Consequently, the mutual information obtained by any input distribution satisfying the amplitude constraints is a lower bound for the channel capacity. Since the capacity achieving distribution of conditional Gaussian channels under amplitude and average power constraints was shown to be discrete, a discrete distribution is proposed and a lower bound on channel capacity is derived. The proposed distribution is obtained through input source entropy maximization.

Consider a discrete distribution for x over the alphabet $\frac{1}{\ell}\mathbb{Z}^+$, where $\mathbb{Z}^+$ is the set of non-negative integers and $1/\ell > 0$ is the spacing between mass points. A probability mass of $p_x(k;\ell)$, $k \in \mathbb{Z}^+$, is assigned to each point such that

$$\sum_{k=0}^{\infty} p_x(k;\ell) = 1 \quad \text{and} \quad \sum_{k=0}^{\infty} \frac{k}{\ell}\, p_x(k;\ell) = P. \tag{1}$$

Thus, $p_x(k;\ell)$ satisfies both the non-negativity and average amplitude constraints of wireless optical IM/DD channels. The entropy of the source is defined as,

$$H_\ell(x) = \sum_{k=0}^{\infty} -p_x(k;\ell)\ \log_2\ p_x(k;\ell).$$

Although any pmf $p_x(k;\ell)$ is sufficient to provide a lower bound, we propose selecting the *maxentropic* distribution subject to (1) under the intuition that it will be close to the capacity at high SNRs. In other words, for a given $\ell > 0$,

$$p_x^*(k;\ell) = \arg\max_{p_x(k;\ell)} \quad H_\ell(x)$$
$$\text{s.t} \quad \text{Eqn. 1 is satisfied.}$$

Applying the method of Lagrange multipliers, the solution is given by,

$$p_x^*(k;\ell) = \frac{1}{1+\ell P}\left(\frac{\ell P}{1+\ell P}\right)^k.$$

Notice that although $p_x^*(k;\ell)$ is a family of distributions parameterized in ℓ, P is independent of ℓ. Using this family of distributions, the maximum mutual information obtained over this set will be a function of both ℓ and σ. For a given σ there is an optimum value for ℓ that maximizes the mutual information. This results in a lower bound, C_L, for the channel capacity and can be formulated as,

$$C_L = \max_{\ell} \quad I_\ell(x;y) = h(y) - \frac{1}{2}\log_2\ 2\pi e\sigma^2,$$
$$\text{s.t} \quad f_y(y) = \sum_{k=0}^{\infty} p_x^*(k;\ell)\delta(y-k/\ell) \circledast f_z(y), \quad \ell > 0$$

where $\circledast$ is the convolution operator and $\delta(\cdot)$ is the Dirac delta functional. Substituting the discrete distribution $p_x^*(k;\ell)$ results in the mutual information,

$$I_\ell(x;y) = -\int_{-\infty}^{\infty}\left[\sum_{k=0}^{\infty} p_x^*(k;\ell)\ \frac{e^{-(y-k/\ell)^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}} \log_2\left(\sum_{k=0}^{\infty} p_x^*(k;\ell)\ \frac{e^{-(y-k/\ell)^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}\right)\right] dy - \frac{1}{2}\log_2(2\pi e\sigma^2). \tag{2}$$

Notice that a relation between noise standard deviation σ and the spacing between successive points $1/\ell$ must exist. Let

$$\ell\sigma = \beta, \qquad w = \ell\, y. \tag{3}$$

Rearranging (2) with respect to ℓ and substituting (3) yields,

$$I_\beta(x;y) = -\int_{-\infty}^{\infty}\left[\sum_{k=0}^{\infty} p_x^*(k;\beta)\ \frac{e^{-(w-k)^2/2\beta^2}}{\sqrt{2\pi\beta^2}} \log_2\left(\sum_{k=0}^{\infty} p_x^*(k;\beta)\ \frac{e^{-(w-k)^2/2\beta^2}}{\sqrt{2\pi\beta^2}}\right)\right] dw - \frac{1}{2}\log_2(2\pi e\beta^2), \tag{4}$$

where

$$p_X^*(k;\beta) = \left(\frac{1}{1+\beta\frac{P}{\sigma}}\right)\left(\frac{\beta\frac{P}{\sigma}}{1+\beta\frac{P}{\sigma}}\right)^k.$$

Thus, for a given P/σ, $I_\beta(x;y)$ is a function of β which quantifies the ratio between the noise variance σ and mass point separation $1/\ell$. A lower bound for the capacity of wireless optical IM/DD channels can be obtained as,

$$C_L = \max_{\beta} \quad I_\beta(x;y).$$

Note that, the optimum value for β is a function of P/σ. For a given P/σ, this maximization can be solved numerically using the bisection method over wide range of β to find C_L.

The lower bound, C_L, obtained from the proposed discrete distribution $p_x^*(k;\beta)$ is tighter at both low and high SNR than the previously reported bounds based on continuous distributions. Although no analytical form is provided, the bound can be computed efficiently through numerical integration. An advantage of this approach, however, is that it avoids a costly search procedure to find the capacity achieving distribution. In addition, it provides a closed form for the input distribution $p_x^*(k;\beta)$.

IV. Channel Capacity Upper Bound

Due to the non-negativity and average optical power constraints, any sequence of n transmitted PAM symbols can be represented geometrically as the set of points contained inside a regular n-simplex [8]. For conditionally Gaussian channels, the set of the received codewords approaches the parallel body to this regular n-simplex for large n. However, the maximum achievable rate can be upper bounded by the maximum asymptotic number of non-overlapping spheres packed in this volume, i.e. via a sphere packing argument. Unlike previous approaches, we use an exact expression for the volume of the set of received codewords to compute the bound, yielding greater accuracy at low SNRs.

A. Set of Received Codewords and Volumes

Consider transmitting a sequence of n independent PAM symbols to form the codeword $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$. The admissible set of transmitted codewords, $\Psi(P)$, is defined as,

$$\Psi(P) = \{\boldsymbol{x} \in \mathbb{R}^n : x_i \geq 0,\ \frac{1}{n}\sum_i x_i \leq P, i = 1, 2, \ldots, n\}.$$

The set Ψ is a regular n-simplex of equal side lengths nP located at the origin as shown in Fig. 1. According to the Gaussian noise model presented, the received vector $\boldsymbol{y}$ has a Gaussian distribution with mean $\boldsymbol{x}$ as follows,

$$\boldsymbol{y} = \boldsymbol{x} + \boldsymbol{z},$$

where $\boldsymbol{z}$ has i.i.d. Gaussian components. Define $\mathcal{B}_n$ as the n-dimensional ball. In the signal space representation, for large enough n, $\boldsymbol{y}$ will, with high probability, be on the surface of $\rho\mathcal{B}_n$, centered on $\boldsymbol{x}$ where

$$\rho = \sqrt{n\sigma^2}.$$

Define the set $\Phi(P, \rho)$ as the outer parallel body to $\Psi(P)$ at distance ρ which results as the Minkowski sum of $\Psi(P)$ and $\rho\mathcal{B}_n$. Formally,

$$\Phi(P, \rho) = \{\boldsymbol{y} \in \mathbb{R}^n : \boldsymbol{y} = \boldsymbol{x} + \boldsymbol{b},\ \boldsymbol{x} \in \Psi(P),\ \boldsymbol{b} \in \rho\mathcal{B}_n\}.$$

The regions defined by $\Psi(P)$ and $\Phi(P, \rho)$ are illustrated in Fig. 1 for the two-dimensional case. An upper bound for the wireless optical IM/DD channel capacity can be obtained by applying a sphere-packing argument and finding the maximum number of non-overlapping spheres that can be packed in $\Phi(P, \rho)$ as $n \to \infty$. Let $V(\cdot)$ denote the volume of a closed set. The volume of $\rho\mathcal{B}_n$ is given by,

$$V(\rho\mathcal{B}_n) = \kappa_n \rho^n = \frac{\pi^{n/2}}{(n/2)!}\rho^n \tag{5}$$

where κ_n denotes the volume of the n-dimensional unit ball. The maximum rate can be expressed in terms of the asymptotic number of transmissible signals as,

$$C \leq \lim_{n\to\infty} \frac{1}{n} \log_2 \frac{V(\Phi(P, \rho))}{V(\rho\mathcal{B}_n)}. \tag{6}$$

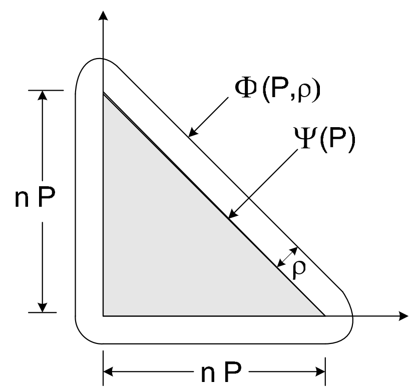

Fig. 1. Two-dimensional representation of an n-simplex and its parallel body at distance ρ.

B. Volume Approximation

Since $\Phi(P, \rho)$ results as the Minkowski sum of two sets, analytic expressions exist to compute its volume. The volume $V(\Phi(P, \rho))$ can be expressed in terms of the intrinsic volumes $V_m(P)$ of an n-simplex as [9],

$$V(\Phi(P, \rho)) = \sum_{m=0}^{n} V_m(P)\kappa_{n-m}\rho^{n-m}. \tag{7}$$

The $V_m(P)$ are given as,

$$V_m(P) = \gamma_m \frac{(nP)^m}{m!}, \tag{8}$$

where $\gamma_n = 1$ and when $0 \leq m \leq n-1$ [9],

$$\gamma_m = \binom{n}{m}\frac{1}{2^{n-m}} + \binom{n}{m+1}\frac{m+1}{\sqrt{\pi}} \times \int_0^\infty e^{-(1+m)v^2}\left[1 - \frac{1}{2}\mathrm{erfc}(v)\right]^{n-m-1} dv. \tag{9}$$

The ratio of the outer volume $V(\Phi(P, \rho))$ to the volume of the n-dimensional ball $V(\rho\mathcal{B}_n)$ is thus,

$$\frac{V(\Phi(P, \rho))}{V(\rho\mathcal{B}_n)} = \sum_{m=0}^{n} \frac{\kappa_{n-m}\,\gamma_m}{\kappa_n\, m!}\left(\frac{nP}{\rho}\right)^m \tag{10}$$

In order to find a simple analytic upper bound for the channel capacity, γ_m needs to be simplified to a more compact form. Since $(1 - \frac{1}{2}\mathrm{erfc}(v)) \geq \frac{1}{2}$ for $v \geq 0$ it follows that,

$$\frac{m+1}{\sqrt{\pi}} \int_0^\infty e^{-(1+m)v^2}\left[1 - \frac{1}{2}\mathrm{erfc}(v)\right]^{n-m-1} dv \geq \frac{1}{2^{n-m}},$$

and (9) can be bounded as,

$$\gamma_m < \binom{n+1}{m+1}\frac{m+1}{\sqrt{\pi}} \times \int_0^\infty e^{-(1+m)v^2}\left[1 - \frac{1}{2}\mathrm{erfc}(v)\right]^{n-m-1} dv.$$

Consider the substitution $\mathrm{erfc}(v) = 2u$ and applying the bound

$$e^{-(\mathrm{erfc}^{-1}(2\mathrm{u}))^2} \leq \sqrt{eu}\,, \qquad 0 \leq u \leq \frac{1}{2},$$

then γ_m can be upper bounded as follows,

$$\begin{aligned}
\gamma_m &< \binom{n+1}{m+1}(m+1)(\sqrt{e})^m \int_0^{1/2} u^{\frac{m}{2}}(1-u)^{n-m-1}du,\\
&< \binom{n+1}{m+1}(m+1)(\sqrt{e})^m \underbrace{\int_0^{1} u^{\frac{m}{2}}(1-u)^{n-m-1}du}_{\mathrm{B}(\frac{m}{2}+1,n-m)}\\
&= \binom{n+1}{m+1}(m+1)(\sqrt{e})^m \frac{(\frac{m}{2})!(n-m-1)!}{(n-\frac{m}{2})!}\\
&= \lambda_m
\end{aligned}$$

where $\mathrm{B}(\frac{m}{2}+1, n-m)$ is the beta function. Substituting into (10) results in

$$\frac{V(\Phi(P,\rho))}{V(\rho\mathcal{B}_n)} < \sum_{m=0}^{n} \underbrace{\frac{\kappa_{n-m}\,\lambda_m}{\kappa_n\; m!}\left(\frac{nP}{\rho}\right)^m}_{\psi_m} = \sum_{m=0}^{n}\psi_m.$$

The capacity can be upper bounded as follows,

$$\begin{aligned}
C &\leq \lim_{n\to\infty}\frac{1}{n}\log_2\frac{V(\Phi(P,\rho))}{V(\rho\mathcal{B}_n)}\\
&< \lim_{n\to\infty}\frac{1}{n}\log_2[n\max_m \psi_m]\\
&= \lim_{n\to\infty}\frac{1}{n}\log_2[\max_m \psi_m]\\
&= \max_m \lim_{n\to\infty}\frac{1}{n}\log_2[\psi_m] \qquad (11)
\end{aligned}$$

where the last inequality is due to the monotonic increase of the $\log_2(\cdot)$ function. Let

$$m = \alpha n \qquad 0 \leq \alpha \leq 1,$$

then the capacity can be expressed as,

$$C < \max_\alpha \lim_{n\to\infty}\frac{1}{n}\log_2\left[\frac{\kappa_{(1-\alpha)n}\,\lambda_{\alpha n}}{\kappa_n\;(\alpha n)!}\left(\frac{nP}{\rho}\right)^{\alpha n}\right]$$

Bounding the factorial using Stirling's approximation

$$\sqrt{2\pi n}\left(\frac{n}{e}\right)^n e^{\left(\frac{1}{12n+1}\right)} < n! < \sqrt{2\pi n}\left(\frac{n}{e}\right)^n e^{\left(\frac{1}{12n}\right)}$$

results in the following upper bound

$$C < \max_\alpha \log_2\left[\left(\sqrt{\frac{e^2}{4\pi}}\right)^\alpha \left(\frac{P}{\sigma}\right)^\alpha \frac{1}{\Theta(\alpha)}\right] \qquad (12)$$

where,

$$\Theta(\alpha) = \alpha^{\frac{3\alpha}{2}}\;(1-\alpha)^{\frac{(1-\alpha)}{2}}\;\left(1-\frac{\alpha}{2}\right)^{(1-\frac{\alpha}{2})}$$

In the following we will show that there is a unique root $0 \leq \alpha^* < 1$ that maximizes the capacity upper bound given in (12) and explicitly derive a closed form expression for α^*.

C. Uniqueness of α^*

Consider the right hand side of (12). To simplify the analysis, the natural logarithmic function is considered instead of $\log_2$ where the optimum value for α will not be affected. Denote this term by J where,

$$J = \alpha\ln K - \ln[\Theta(\alpha)]\,, \qquad K = \sqrt{\frac{e^2}{4\pi}}\,\frac{P}{\sigma}.$$

In order to maximize J with respect to α, let $\partial J/\partial\alpha = 0$, resulting in the cubic equation,

$$\Lambda(\alpha) = \alpha^3 - a\alpha^2 + 3a\alpha - 2a = 0, \qquad (13)$$

where

$$a = \frac{1}{2}\exp(2\ln K - 1) \geq 0.$$

Proposition 1: For all $a > 0$, there exists a unique root for $\Lambda(\alpha)$, denoted α^*, lying in the interval $0 \leq \alpha^* < 1$.

Proof: Note that, $\Lambda(0) = -2a \leq 0$ and $\Lambda(1) = 1$. As a result, there exists at least one root of $\Lambda(\alpha)$ in $[0, 1)$. The extrema, α_+ and α_-, of $\Lambda(\alpha)$ are,

$$\alpha_\pm = \frac{1}{3}\left(a \pm \sqrt{a^2 - 9a}\right)$$

The existence of these extrema over $\mathbb{R}$ depends on a as follows,

$$\alpha_\pm \to \begin{cases} \text{No extrema,} & a < 9;\\ \text{One extrimum at } \alpha = 3, & a = 9;\\ \text{Two extrema,} & a > 9. \end{cases}$$

To prove the existence of a unique root for $\Lambda(\alpha)$ in $[0, 1)$, it is sufficient to prove that $\alpha_\pm > 1$ for all values of $a > 0$. For $a < 9$, there are no extrema, and the single root in $[0, 1)$ is unique. For $a > 9$, consider the following lemma.

Lemma 1: If $a > 9$ then $\alpha_\pm > 1$.

Proof: It is clear that $\alpha_+ > 3$ when $a > 9$. In addition, α_- is decreasing in a and its asymptotic value as $a \to \infty$ is greater than one as follows,

$$\begin{aligned}
&\frac{d\alpha_-}{da} = \frac{1}{3}\left(1 - \frac{1}{\sqrt{1-9/a}}\right) < 0, \text{ when } a > 9,\\
&\lim_{a\to\infty}\alpha_- = \frac{3}{2} > 1.
\end{aligned}$$

As a result $\alpha_\pm > 1$ whenever $a > 9$. ■

Therefore, for all values of $a > 0$ there is a unique real root for $\Lambda(\alpha)$ in $[0, 1)$ and is denoted as α^*. ■

D. Expression for α^*

To find the unique root α^*, define

$$\Lambda_o = \Lambda(a/3), \quad \delta^2 = (a^2 - 9a)/9, \quad h = 2\delta^3.$$

The following three cases are considered [10] to solve (13).

(i) $\Lambda_o^2 > h^2$: One root exists for $\Lambda(\alpha)$ and is given by,

$$\alpha^* = \frac{a}{3} + \Omega_+ + \Omega_-, \qquad (14)$$

$$\text{where} \quad \Omega_\pm = \left[\frac{1}{2}\left(-\Lambda_o \pm \sqrt{\Lambda_o^2 - h^2}\right)\right]^{1/3}.$$

(ii) $\Lambda_o^2 = h^2$: Two roots coincide and α^* depends on the sign of Λ_o,

$$\alpha^* = \min\left\{\frac{a}{3} + \left(\frac{\Lambda_o}{2}\right)^{1/3}, \frac{a}{3} - 2\left(\frac{\Lambda_o}{2}\right)^{1/3}\right\}.$$

(iii) $\Lambda_o^2 < h^2$: Three distinct roots, and α^* is obtained as,

$$\alpha^* = \min_{i=0,1,2}\left\{\frac{a}{3} + 2\delta\cos\left(\frac{2\pi i}{3} + \theta\right)\right\},$$

$$\text{where} \quad \cos(3\theta) = \frac{-\Lambda_o}{h}.$$

The upper bound for the optical channel capacity can be explicitly written as follows,

$$C_U = \log_2\left[\left(\sqrt{\frac{e^2}{4\pi}}\right)^{\alpha^*}\left(\frac{P}{\sigma}\right)^{\alpha^*}\frac{1}{\Theta(\alpha^*)}\right]. \tag{15}$$

V. Results

Fig. 2 presents the lower bound, C_L, derived using the discrete source distribution $p_x^*(k;\ell)$ and the upper bound, C_U in (15). The proposed lower bound is tight at both low and high SNRs and asymptotically describes the channel capacity. In addition, the mutual information is shown for one-sided continuous exponential and Gaussian input distributions which satisfy non-negativity and average optical power constraints.

The lower bound proposed here, indicates that a significant increase in rate is possible at low SNRs, where most wireless optical links operate. An SNR margin of 3.7 and 2 dB can be noticed between C_L and the bound obtained from the one-sided exponential distribution, proposed in [5], at a channel capacity of 0.5 and 1 bits/channel use respectively. In addition, the presented bound has approximately double the channel capacity (0.85 and 0.45 respectively) at SNR=0 dB. For the sake of comparison, the mutual information with uniform M-ary source distributions are also presented. Notice that a significant gap of 3.5 dB exists between C_L and the lower bound from a uniform (2-PAM) discrete source distribution at $C = 0.5$ bits/channel use.

The derived upper bound C_U is tight at low SNRs and asymptotically incurs an increase of $\log_2(\sqrt{e/2})$ in channel capacity at high SNRs. Compared to the previous upper bound, C_U^o [5, Eq. 20], C_U is a significantly better representation for the channel capacity at low SNRs. As a result, C_U is a better metric for comparison at low SNRs (SNR <-3.5 dB) over C_U^o, since a majority of wireless optical IM/DD channels operate in this low SNR regime. Note that, the unique root α^* depends on SNR through a. Numerical simulations indicate that (14) is utilized to find α^* when SNR<9.9 dB. As a result, the upper bound can be defined by (14) and (15) at low and moderate SNRs.

VI. Conclusion

Lower and upper bounds for the capacity of PAM wireless optical IM/DD channels are derived. The proposed lower bound is tight at both low and high SNRs. Although no

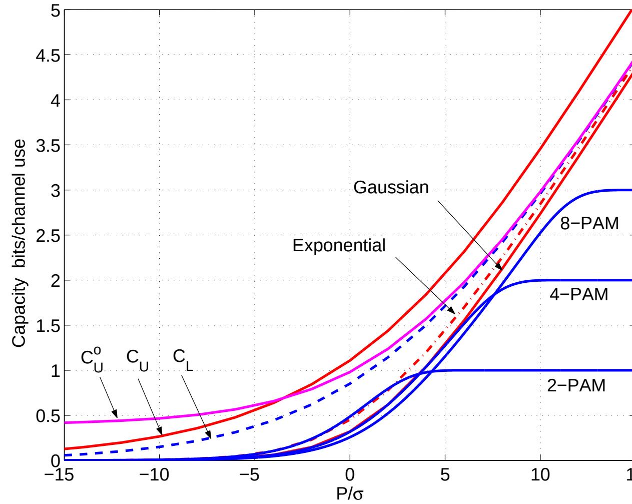

Fig. 2. Capacity bounds and mutual information for continuous one-sided exponential, Gaussian and discrete uniform PAM.

analytical form is provided, the bound can be efficiently computed numerically avoiding a search procedure to find the capacity achieving distribution. The proposed discrete distribution achieves higher mutual information than the continuous one-sided exponential and Gaussian distributions or discrete uniform M-ary distributions. In addition, an analytical expression for a tight upper bound at low SNRs is derived based on a sphere packing argument. The asymptotic behavior of the upper bound at high SNRs incurs a constant increase over the actual channel capacity. Since most wireless optical links operate at relatively low SNRs, the tightness of the derived lower and upper bounds at low SNRs provides a useful benchmark for modulation and coding design.

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