Outage Probability for Free-Space Optical Systems Over Slow Fading Channels With Pointing Errors

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***Abstract*— We investigate the outage capacity of free-space optical communication links in the presence of atmospheric turbulence and pointing errors. An expression for the outage probability is derived and we show that optimizing the transmitted beam width with respect to detector size and pointing error variance maximizes the achievable rates.**

I. Introduction

Free-space optical (FSO) systems establish point-to-point communication links by transmitting laser beams through the atmosphere. They provide high security and unregulated bandwidth supporting high data rate applications over distances in the range of 1-2 km. However, optical signals are affected by atmospheric turbulence and pointing errors which fade the signal at the receiver and deteriorate the link performance. Optical channels can be well modelled as slow fading or block fading channels. In this situation, the ergodic channel capacity can not be utilized to characterize the channel. Therefore, we study the outage probability for these optical links. In order to study the channel from an information theoretic perspective, it is necessary to accurately model the statistical distribution of the channel. Two parameters affect the link performance, namely atmospheric turbulence and pointing errors. Atmospheric turbulence causes fluctuations in the intensity of the received signal due to variations in the refractive index. Misalignment between transmitter and receiver due to building sway causes pointing error. Approaches for studying FSO system performance under the influence of atmospheric turbulence and pointing errors are limited to the assumption of negligible detector aperture size with respect to the beam width [1]. In this paper, we derive a statistical model for FSO channels due to atmospheric turbulence and pointing errors considering the beam width, pointing error variance and the detector size. We assume a Gaussian beam profile and derive a closed form expression for the outage probability of the FSO links. We show that the maximum rate for a given outage probability can be increased by optimizing the beam width with respect to detector size at a given pointing error variance.

II. System Model

The FSO links studied here employ intensity modulation and direct detection (IM/DD). The statistical channel model can be represented as,

$$y = h\,x + n,$$

where $x \in \{0, 2P_t\}$ is the binary transmitted signal, P_t is the average optical power, h is the channel state, y is the received electrical signal and n is noise which is modelled as zero-mean, Gaussian distributed with variance σ_n^2. Without loss of generality, we assume a unity responsivity coefficient. The transmitted signal is subjected to attenuation due to two factors: geometric spread with pointing errors and atmospheric turbulence. This dependence is formulated as,

$$h = h_p(r)\;h_a,$$

where h_p is the attenuation due to geometric spread with pointing error when the radial displacement between the beam footprint and the detector origins is r and h_a is the attenuation due to atmospheric turbulence as a function of time. For slow fading channels and equiprobable on-off keying (OOK) signaling, the instantaneous received electrical signal-to-noise ratio (SNR) is

$$\mathrm{SNR_{rx}} = h^2\,\frac{2P_t^2}{\sigma_n^2}.$$

The performance of the system under this slow fading state is measured by the probability of outage.

III. FSO Channel Capacity and Outage Probability

Channel capacity is the maximum data rate that can be transmitted over the channel with arbitrarily small probability of error. For slow fading channels there is probability bounded above zero that a realized channel state, h, fails to support a given rate. Therefore, the capacity is defined in terms of the probability of outage and the corresponding outage capacity [2]. The outage event occurs when the transmitted rate, $\mathcal{R}$, exceeds the channel capacity, $\mathcal{C}$. In this case, transmitted codewords can not be decoded correctly at the receiver with arbitrarily small probability of error. The outage probability is defined as

$$P_{\mathrm{out}}(\mathcal{R}) = \mathrm{Prob}(\mathcal{C} < \mathcal{R}).$$

Let SNR_0 denote the electrical signal-to-noise ratio that is required to support a rate $\mathcal{R}_0$ over an additive white Gaussian noise channel ($h = 1$). As the channel capacity is monotonically increasing with transmitted power for a given channel state, the outage event can be expressed in terms of the signal-to-noise ratio as,

$$P_{\mathrm{out}}(R_0) = \mathrm{Prob}(\mathrm{SNR_{rx}} < \mathrm{SNR_0}).$$

It is clear that, the outage probability depends on the statistical distribution of $\mathrm{SNR_{rx}}$.

IV. Channel Statistical Model

A. Atmospheric Turbulence Statistical Model

For weak turbulence, the distribution of the intensity fluctuations is generally modelled as a log-normal distribution which has been validated through measurements [3]. The log-amplitude variance, σ_X^2, is related to Rytov variance as $\sigma_R^2 = 4\sigma_X^2$. In the case that $\sigma_R^2 < 0.3$, the turbulence is termed *weak* and is well modelled by the unity mean distribution [4],

$$f_{h_a}(h_a) = \frac{1}{2h_a\sqrt{2\pi\sigma_X^2}} \exp\left(\frac{(\ln h_a + 2\sigma_X^2)^2}{8\sigma_X^2}\right).$$

B. Pointing Error Statistical Model

Consider a Gaussian spatial intensity profile of beam waist w_z (radius at e^{-2} of peak) on the receiver plane at distance z from the transmitter. The fraction of the collected power due to geometric spread with radial displacement r from the origin of the detector is obtained by integrating over a circular aperture of radius a and can be approximated as,

$$h_p(r) \approx A_0 \exp\left(-\frac{2r^2}{w_{z_{eq}}^2}\right),$$

where $w_{zeq}^2 = w_z^2 A_0/A_2$, $v = (\frac{\sqrt{\pi}a}{\sqrt{2}w_z})$, $A_0 = [\mathrm{erf}(v)]^2$ and $A_2 = \frac{2}{\sqrt{\pi}}\ \mathrm{erf}(v)\ \left[v\exp(-v^2)\right]$. Assuming independent identical Gaussian distributions for the elevation and the horizontal displacement (sway) both of variance σ_s^2, the radial displacement r has a Rayleigh distribution [1]. Let $\gamma = w_{z_{eq}}/2\sigma_s$, the probability distribution of h_p is given by

$$f_{h_p}(h_p) = \frac{\gamma^2}{A_0^{\gamma^2}}\ h_p^{\gamma^2-1}, \qquad 0 \le h_p \le A_0.$$

C. Channel Model

For weak turbulence the statistical model for the fading of the FSO channel is obtained as,

$$f_h(h) = \frac{\gamma^2}{2A_0^{\gamma^2}}\ h^{\gamma^2-1}\ \mathrm{erfc}\left(\frac{\ln(\frac{h}{A_0}) + \mu}{\sqrt{8}\sigma_X}\right) e^{2\sigma_X^2\gamma^2(1+\gamma^2)},$$

where $\mu = 2\sigma_X^2(1+2\gamma^2)$. The probability of outage at a given rate R_0 can be written in terms of $f_h(h)$ as,

$$P_{\mathrm{out}}(R_0) = \mathrm{Prob}(h < h_0).$$

where $h_0 = \sqrt{\frac{\mathrm{SNR}_0}{2P_t^2/\sigma_n^2}}$. Therefore, the probability of outage is the cumulative density function evaluated at h_0, namely

$$P_{\mathrm{out}}(R_0) = \frac{1}{2}\left[e^{\gamma^2\eta - 2\sigma_X^2\gamma^4}\mathrm{erfc}\left(\frac{\eta}{\sqrt{8}\sigma_X}\right) + \mathrm{erfc}\left(\frac{4\sigma_X^2\gamma^2 - \eta}{\sqrt{8}\sigma_X}\right)\right]$$

where $\eta = \ln(h_0/A_0) + \mu$.

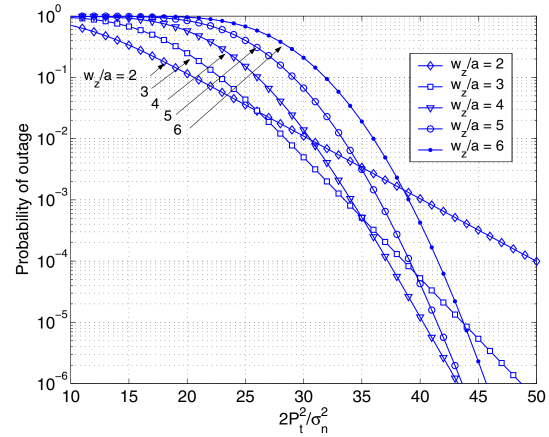

Fig. 1. Probability of outage versus $\frac{2P_t^2}{\sigma_n^2}$ at $\frac{\sigma_s}{a} = 0.8$

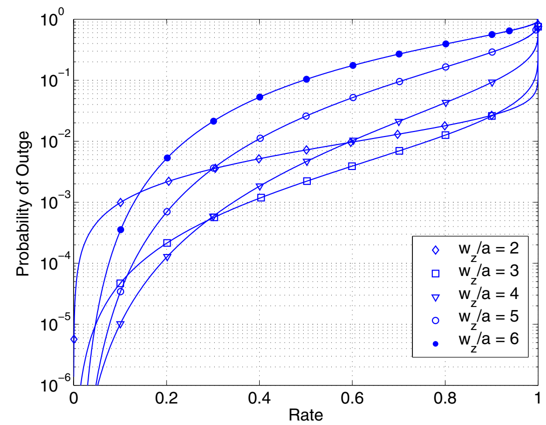

Fig. 2. Probability of outage versus rate for $\frac{\sigma_s}{a} = 0.8$ and $\frac{2P_t^2}{\sigma_n^2} = 35$ dB

V. Conclusion

For small jitter variance (i.e. $\sigma_s/a \approx 0.1$), the smaller the beam width w_z/a the lower the probability of outage obtained for a given transmitted power. However, the scenario is different when σ_s/a is increased as shown in Fig. 1. For a given transmitted power, there is an optimum beam width that achieves the minimum probability of outage. As shown in Fig. 2, when the link is designed to achieve a certain P_{out}, the optimum beam width can provide a higher data rate compared to other beam widths for a fixed transmitted power.

References

[1] S. Arnon, “Effects of atmospheric turbulence and building sway on optical wireless communication systems,” *Optical Letters*, vol. 28, pp. 129–131, Jan. 2003.

[2] A. Goldsmith, *Wireless Communications*. Cambridge, U.K: Cambridge Univ. Press, 2004.

[3] M. A. Al-Habash, L. C. Andrews, and R. L. Philips, “Mathematical model for the irradiance probability density function of a laser propagating through turbulent media,” *Optical Engineering*, vol. 40, pp. 1554–1562, Aug. 2001.

[4] X. Zhu and J. Kahn, “Free space optical communication through atmospheric turbulence channels,” *IEEE Transactions on Communications*, vol. 50, pp. 1293–1300, Aug. 2002.