A Signal Space Model for Intensity Modulated Channels

Steve Hranilovic and Frank R. Kschischang
Department of Electrical and Computer Engineering
University of Toronto
Email: {steve,frank}@comm.utoronto.ca

Abstract— **We present a signal space model which incorporates explicitly the non-negativity constraint of intensity modulated channels. A basis function which is constant in the symbol interval is defined for every intensity signalling set with the result that coordinate values in this direction quantify the average optical power of the symbol. Using this structure, the model defines bounding regions, containing the set of transmittable signal points, and shaping regions which select a subset of transmittable points. Constellations can then be formed through the intersection of these regions with an appropriate lattice. A general expression for the optical power gain over a baseline is presented and shown to be separable into terms depending on lattice packing density and on the geometry of the region. The optimal shaping region is found to be a region defined by placing an upper bound on the coordinate values in the basis direction which sets the symbol average optical power. Using the geometric symmetries which arise from this model, an expression for the shaping gain using the optimal shaping region is developed. An asymptotic shaping gain of 1.33 dB is calculated in the case of an N-fold Cartesian product of the baseline constellation shaped using the optimal region. The bounding regions of 3-dimensional QAM, AB-QAM and PAM intensity signalling schemes are also presented.**

I. Introduction

Currently, two-level (on-off) modulation techniques prevail in optical channels. In order to achieve higher data rates, wireless optical links have begun to turn towards the use of bandwidth efficient multilevel modulation schemes. Indeed, the use of multilevel modulation techniques in fibre optical channels may be necessary to meet future high data rate requirements.

Due to the physical constraint that all intensity modulated signals be non-negative, a general signal space model cannot be applied directly to determine the performance of modulation schemes. This paper presents a signal space model particularly suited to the constraints imposed by the optical intensity physical layer. This model satisfies intuition about the the design of signal constellations for the channel and effectively separates optical and electrical power measures.

Section II provides a brief introduction to the optical intensity channel and the constraints it imposes on signals that may be transmitted. A signal space model for the intensity channel is presented in Section III. Section IV presents a formal definition for lattice codes in the signal space model. Using these codes, Section V presents a framework for the comparison of intensity modulation schemes. Expressions for the gain are given in the general case, and for the case of optimal shaping for average optical power. The asymptotic gain in a specific example is computed using the derived expressions. Section VI presents some examples of modulation schemes in the proposed model. The paper concludes with a brief review and suggestions for future work.

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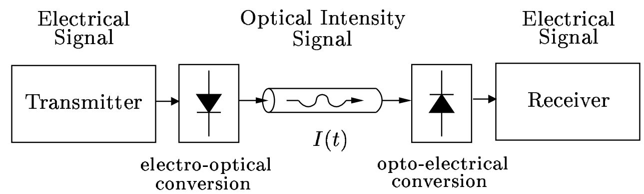

Fig. 1. Basic channel structure of an optical intensity channel.

II. The Optical Intensity Channel

Figure 1 depicts the basic structure of an optical intensity channel. The transmitter constructs signals in the electrical domain which represent the symbol to be transmitted. The signal is converted to optical form, and launched into a medium, here indicated as a fibre optical cable. After propagating through the channel, the signal is received and converted to electrical form. Detection occurs at the receiver in the electrical domain.

Technological limitations of the optoelectronics allow for only the *intensity* of the optical signal to be modulated and detected. The phase and amplitude of the underlying optical carrier are not modified directly, but rather the optical power transmitted is modulated. As a result, the optical intensity signal transmitted, $I(t)$, must satisfy the constraint

$$(\forall t \in \mathbb{R})\ I(t) \geq 0. \tag{1}$$

Since $I(t)$ is an optical power signal, the average optical power transmitted, P, can be computed as

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} I(t)\, dt. \tag{2}$$

This is in marked contrast to conventional channels in which the power transmitted depends on the square of transmitted signal.

III. A Signal Space Model

Let $\{x_1(t), x_2(t), \ldots, x_M(t)\}$, $t \in [0, T_s]$, be an intensity signalling set. Define $\phi_i(t)$, for $i = 1, 2, \ldots, N$, $N \leq M$ and $t \in [0, T_s]$, as a set of real-valued, orthonormal basis functions such that

$$x_i(t) = \sum_{k=1}^{N} x_{i,k}\phi_k(t), \tag{3}$$

where $\vec{x}_i = (x_{i,1}, x_{i,2}, \ldots, x_{i,N})$ is the vector of coordinates of $x_i(t)$ with respect to the given basis. The *constellation* of the intensity signalling scheme can then be written as the collection of such vectors or $\Omega = \{\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_M\}$.

The non-negativity constraint in (1) implies that $x_i(t) \geq 0$ for every i. Thus, the average amplitude value of the signals transmitted is always non-negative. Without loss of generality, it is possible to set the function

$$\phi_1(t) = \sqrt{\frac{1}{T_s}}\,\mathrm{rect}(t) \tag{4}$$

where,

$$\mathrm{rect}(t) = \begin{cases} 1 & : \quad 0 \leq t \leq T_s \\ 0 & : \quad \text{otherwise} \end{cases},$$

as a basis function for every intensity modulation scheme. This basis function contains the average amplitude of each symbol, and as a result represents the average optical power of each symbol. Due to the orthogonality of the other basis functions,

$$\int_0^{T_s} \phi_i(t)dt = \begin{cases} \sqrt{T_s} & : \quad i = 1 \\ 0 & : \quad i > 1 \end{cases}.$$

In this manner, the average optical power requirement is represented in a single dimension. The average optical power of an intensity signalling set can then be computed as

$$P = \frac{1}{\sqrt{T_s}} \sum_{\vec{x}_i \in \Omega} p_{\vec{x}_i} x_{i,1}, \tag{5}$$

where $p_{\vec{x}_i}$ is the probability of transmitting $\vec{x}_i$. Note that average optical power of the signalling scheme is not completely described by the geometry of the constellation but also depends on the symbol period. This is due to the fact that the in (4), $\phi_1(t)$ is set to have unit *electrical* energy. As a result, the average amplitude and hence average optical power depends on the signalling interval. This scaling is appropriate since detection of the signal is performed in electrical domain where the orthonormal basis defined is appropriate. Section V discusses the implications of this dependence.

IV. Definition of Lattice Codes

We define the *bounding region* of the modulation scheme as the set of all points in the signal space which describe pulses satisfying (1), or formally

$$\Upsilon = \left\{ (v_1, v_2, \ldots, v_N) \in \mathbb{R}^N : (\forall t \in \mathbb{R}), \sum_{i=1}^{N} v_i\phi_i(t) \geq 0 \right\}.$$

It is clear that $\vec{0} \in \Upsilon$ and that the set is determined solely by the choice of basis functions for the signal space. This set is convex since for $\vec{b}_1, \vec{b}_2 \in \Upsilon$, $\alpha\vec{b}_1 + (1-\alpha)\vec{b}_2 \in \Upsilon$, for $0 \leq \alpha \leq 1$, describes a non-negative signal.

Additionally, the bounding region exhibits a great deal of structure when observed from points of equal average optical power. Define the set,

$$\Upsilon_r = \{(v_1, v_2, \ldots, v_N) \in \Upsilon : v_1 = r, r \in \mathbb{R}^+\}$$

as the set of all signal points which require a fixed average optical power of $r/\sqrt{T_s}$. Take two such sets, Υ_u and Υ_v for $u, v > 0$ and $K = v/u$. The set $K\Upsilon_u$ is a set of pulses with average optical power $v/\sqrt{T_s}$, so $K\Upsilon_u \subseteq \Upsilon_v$. Similarly, $K^{-1}\Upsilon_v \subseteq \Upsilon_u$ which implies that $\Upsilon_v \subseteq K\Upsilon_u$. Therefore, $\Upsilon_v = K\Upsilon_u$. This implies that the Υ_r are directly similar regions with dimensions which scale linearly as the coordinate value in the ϕ_1 dimension. Therefore it is only necessary to look at a single Υ_r to characterise the entire set.

The bounding region is unbounded in the ϕ_1 direction, since by adding arbitrary average optical power any coordinate value in the other $N-1$ dimensions is allowed. In other words, Υ is a generalised cone. By definition, a finite region Θ is formed through the intersection of Υ with a *shaping region* denoted Ψ. The set Ψ is independent of the basis functions or actual shape of Υ, so long as the resulting Θ is a finite region.

An N-dimensional constellation, or *lattice code*, can be constructed through the intersection of an N-dimensional lattice and the finite region Θ. Thus, the constellation under consideration can be formed as

$$\begin{aligned} \Omega(\Lambda, \Upsilon, \Psi) &= \Lambda \cap \Upsilon \cap \Psi \\ &= \Lambda \cap \Theta. \end{aligned}$$

V. Shaping and Coding Gain

A. Constellation Figure of Merit, Gain

In conventional channels, the *constellation figure of merit* (CFM) is a popular measure of the energy efficiency of a signalling scheme [1]. An analogous measure for optical intensity channels which quantifies the optical power efficiency of the scheme is [2, 3]

$$\mathrm{CFM}(\Omega) = \frac{d_{\min}(\Omega)}{P(\Omega)}, \tag{6}$$

where $d_{\min}(\Omega)$ is the minimum Euclidean distance between constellation points and $P(\Omega)$ is the average optical power. The CFM in (6) is invariant to scaling of the constellation, however, it is not unitless since the average symbol amplitude (i.e., P) depends on T_s while $d_{\min}$ is independent of the symbol interval.

The CFM can be used to determine the optical power gain of one scheme versus another. Consider two optical intensity constellations, Ω_1 and Ω_2, which transmit on an additive white Gaussian noise channel [1]. The probability

[1] The noise distribution approaches a Gaussian distribution in the case of high intensity optical transmission. This case is almost always true for wireless optical networks, for example.

of a symbol error can be approximated by the relation

$$P_{e,i} \approx \bar{N}_i Q(\mathrm{CFM}_i \cdot P_i/2\sigma)$$

where σ^2 is the variance of the noise source, P_i is the average optical power of Ω_i, $\bar{N}_i$ is the error coefficient, CFM_i is the constellation figure of merit for Ω_i and $Q(x) \triangleq (1/\sqrt{2\pi}) \int_x^\infty \exp(-u^2/2)du$. Further, assume that the schemes are operating at the same symbol error rate. If $P_{e,i}^{-1}$ is the P/σ required to achieve a symbol error rate of p, then the optical power gain (G_p) of Ω_1 with respect to Ω_2 is given by the ratio of P_2/P_1 or

$$\begin{aligned} G_p &= P_{e,2}^{-1}/P_{e,1}^{-1} \\ &= (\mathrm{CFM}_1)/(\mathrm{CFM}_2) \times [Q^{-1}(p/\bar{N}_2)/Q^{-1}(p/\bar{N}_1)]. \end{aligned}$$

The asymptotic optical power gain of Ω_1 over Ω_2, as $p \to 0$, can be shown to be [4],

$$G = \lim_{p \to 0} G_p = \mathrm{CFM}_1/\mathrm{CFM}_2$$

which is independent of the error coefficients.

In order to have a fair comparison, the spectral properties of the two schemes in question must also be taken into account. Define the *bandwidth efficiency*, η, of a modulation scheme as

$$\eta = \frac{R}{W}$$

where R is the bit rate in bits/second and W is the fractional power bandwidth of the power spectrum in Hertz. The fraction of in-band to out-of-band power can be thought of as determining the amount of distortion introduced by a bandlimited channel. If we let $\kappa = T_s W$, where T_s is the symbol period and κ is a constant for the given pulse shapes used, the bandwidth efficiency takes the form

$$\eta = \frac{\log_2 |\Omega|}{\kappa} \tag{7}$$

which is independent of T_s. Since the average optical power in (5) depends directly on the symbol interval, fixing bandwidth efficiencies of the two schemes is not sufficient to ensure a fair comparison. Rather, W and R are set to be identical in both schemes under comparison.

B. Baseline Constellation

Define $\Omega_\oplus = \{0, d_{\min}, 2d_{\min}, \ldots, (M-1)d_{\min}\}$ as the baseline constellation with basis function $\phi_1(t)$ as defined in (4). This is commonly referred to as a PAM (pulse-amplitude modulated) constellation. The baseline CFM is thus

$$\mathrm{CFM}_\oplus = \frac{2}{|\Omega_\oplus| - 1}\sqrt{T_{s\oplus}},$$

where $T_{s\oplus}$ is the symbol time defined for the baseline constellation.

In N dimensions, the constellation $\Omega_\oplus^N$ is formed through the N-fold Cartesian product of $\Omega_\oplus$ with itself. The increase in the number of dimensions does not vary the minimum distance properties of the constellation, so, $d_{\min}(\Omega_\oplus^N) = d_{\min}(\Omega_\oplus)$. Since the distribution of amplitude values in each component of the N-dimensional constellation is identical to the 1-D case and the bandwidth of the two cases is identical, $P(\Omega_\oplus^N) = P(\Omega_\oplus)$. Hence, $\mathrm{CFM}(\Omega_\oplus^N) = \mathrm{CFM}(\Omega_\oplus)$, and there is no asymptotic optical power gain.

The asymptotic optical power gain of other schemes over this baseline can then be computed as

$$\begin{aligned} G(\Omega) &= \frac{\mathrm{CFM}(\Omega)}{\mathrm{CFM}_\oplus} \\ &\approx \frac{|\Omega_\oplus|}{2} \frac{1}{\sqrt{T_{s\oplus}}} \frac{d_{\min}(\Omega)}{P(\Omega)} \end{aligned} \tag{8}$$

assuming $|\Omega_\oplus|$ is large so that $(|\Omega_\oplus| - 1) \approx |\Omega_\oplus|$. For correct comparison as is Section V-A, we define $\kappa_\oplus = T_{s\oplus} W_\oplus$.

C. Optical Power Gain

Insight into the factors which govern the gain in (8) can be achieved through the application of the *continuous approximation.* This approximation allows for the replacement of discrete sums of a a function evaluated at every $\vec{x} \in \Omega$ by a normalised integral of the function over the region Θ [4]. This permits (8) to be re-cast in terms of the lattice properties and the region geometry.

Using this approximation, the average optical power in (5) for the case of equiprobable signalling is a function of the region Θ and takes the form

$$P(\Theta) \approx \int_{\vec{x} \in \Theta} x_1 \frac{1}{\sqrt{T_s}} \frac{1}{V(\Theta)} dV(\vec{x}), \tag{9}$$

where $V(\cdot)$ evaluates to the volume of the region. Similarly, $|\Omega|$ can be approximated by summing the constant 1 at each constellation point which through the continuous approximation gives

$$|\Omega| \approx \frac{V(\Theta)}{V(\Lambda)}.$$

Substituting these approximations, along with the fact that $d_{\min}(\Omega) = d_{\min}(\Lambda)$, into (8) yields

$$G(\Omega(\Lambda, \Theta)) \approx \frac{|\Omega_\oplus|}{2} \sqrt{\frac{T_s}{T_{s\oplus}}} \frac{d_{\min}(\Lambda)}{P'(\Theta)} \tag{10}$$

where $P'(\Theta) = \sqrt{T_s} P(\Theta)$. Say $\kappa = T_s W$ for the modulation scheme under consideration. Since the bandwidth efficiency is the same between the schemes, from (7), $|\Omega_\oplus| = |\Omega|^{\frac{1}{\nu}}$ where $\nu = \kappa/\kappa_\oplus$. Similarly, since the bandwidth is also set to be the same between schemes, $T_s/T_{s\oplus} = \nu$. Applying these facts and re-arranging (10) gives a general expression for power gain

$$G(\Omega(\Lambda, \Upsilon, \Psi)) \approx \underbrace{\frac{d_{\min}(\Lambda)}{V(\Lambda)^{\frac{1}{\nu}}}}_{\gamma_c(\Lambda)} \cdot \underbrace{\frac{\sqrt{\nu}}{2} \frac{V(\Upsilon, \Psi)^{\frac{1}{\nu}}}{P'(\Upsilon, \Psi)}}_{\gamma_s(\Upsilon, \Psi)}. \tag{11}$$

The *coding gain*, $\gamma_c(\Lambda)$, is a function of the lattice used. This term is a measure of the packing density of the lattice.

The *shaping gain*, $\gamma_s(\Upsilon, \Psi)$ is a measure of the gain in optical power of implementing a constellation of shape Θ with respect to the baseline constellation geometry.

D. Specific Bound on Shaping Gain

The coding gain is determined by the choice of lattice. High density lattices are known which provide good coding gain as dimension increases, at the penalty of higher implementation cost.

Shaping gain is determined by the choice of bounding region (i.e., basis functions) and the shaping region. For a given Υ, the optimum shaping region which maximises shaping gain is

$$\Psi_{\text{opt}} = \{(\psi_1, \psi_2, \ldots, \psi_N) \in \mathbb{R}^N : 0 \le \psi_1 \le \psi_{1\max}\}. \quad (12)$$

This assertion can be justified by noting that all points with equal components in the ϕ_1 dimension, have the same average optical power. For a given volume and Υ, the optimal shaping region can be formed by successively connecting points of the smallest possible average optical energy until the volume is achieved. Clearly, the region in (12) will result.

Suppose that we form the region $\Theta = \Upsilon \cap \Psi_{\text{opt}}$. It is possible to exploit the symmetries of Υ to determine expressions for $V(\Upsilon, \Psi_{\text{opt}})$ and $P(\Upsilon, \Psi_{\text{opt}})$ in (11). In Section IV, it was shown that the Υ_r are directly similar and scale linearly in r. As a result, the volume of each of the Υ_r must scale as r^{N-1} for an N dimensional signal space. Formally,

$$V(\Upsilon_r) = V(\Upsilon_1) r^{N-1} \quad (13)$$

where $V(\Upsilon_1)$ is the volume of the set of points Υ_1. The volume of $\Upsilon \cap \Psi_{\text{opt}}$ can then be computed simply as $\int_0^{\psi_{1\max}} V(\Upsilon_r) dr$ which evaluates to

$$V(\Upsilon, \Psi_{\text{opt}}) = \frac{1}{N} V(\Upsilon_1) \psi_{1\max}^N. \quad (14)$$

Exploiting the symmetry of the region in the ϕ_1 dimension, the average optical power expression in (9) can be computed as an integral over the ϕ_1 direction only. Noting that $dV(\Upsilon, \Psi_{\text{opt}}) = dV(\Upsilon_r)dr$ and substituting (13) and (14) into (9) gives

$$P(\Upsilon, \Psi_{\text{opt}}) = \int_0^{\psi_{1\max}} r \frac{1}{\sqrt{T_s}} \frac{N}{V(\Upsilon_1)\psi_{1\max}^N} (V(\Upsilon_1) r^{N-1}) dr$$

which simplifies to

$$P(\Upsilon, \Psi_{\text{opt}}) = \frac{1}{\sqrt{T_s}} \frac{N}{N+1} \psi_{1\max}. \quad (15)$$

The expression for the shaping gain in the case of general bounding region Υ and optimal shaping region Ψ_{opt} (for power efficiency) is computed by substituting (14) and (15) into (11) to yield

$$\gamma_s(\Upsilon, \Psi_{\text{opt}}) = \frac{\sqrt{\nu}}{2} \left(\frac{(N+1)^\nu}{N^{\nu+1}} V(\Upsilon_1) \psi_{1\max}^{N-\nu} \right)^{\frac{1}{\nu}} \quad (16)$$

The asymptotic gain as $N \to \infty$ can be determined in the specific case where the basis functions are chosen to correspond to an N-fold Cartesian product of the baseline case in Section V-B. Denote the resulting N-dimensional bounding region as $\Upsilon_\oplus$ and the set $\Upsilon_{1\oplus}$ as the $(N-1)$-dimensional set of all signals with average amplitude value $1/\sqrt{NT_{s\oplus}}$. It is helpful to consider the coordinate values of each of the N constituent constellations to determine the form of $\Upsilon_{1\oplus}$. Due to the average amplitude requirement, the largest amplitude value for any of the N constituent symbols is $\sqrt{N/T_{s\oplus}}$. Let the vectors $\vec{v}_i$, $i = 1, 2, \ldots, N$, be the N points in $\Upsilon_{1\oplus}$ at which this maximum occurs corresponding to the case when a single sub-constellation has a non-zero amplitude value. It is clear that $\|\vec{v}_i - \vec{v}_j\|^2 = 2N$ for every $i \ne j$. Define the set Δ as a *regular* $(N-1)$-*simplex* with N vertices $\vec{v}_i$, or formally

$$\Delta = \left\{ \vec{\delta} \in \mathbb{R}^N \ : \ \vec{\delta} = \sum_{i=1}^N \alpha_i \vec{v}_i, \sum_{i=1}^N \alpha_i = 1, \alpha_i \in [0,1] \right\}.$$

From the definitions of $\vec{v}_i$ and Δ, the average amplitude of signal represented by a $\vec{\delta} \in \Delta$ is $1/\sqrt{NT_{s\oplus}}$. Therefore, $\Delta \subseteq \Upsilon_{1\oplus}$. The set of $\vec{v}_i$ vectors is orthogonal since each attains the maximum amplitude in a disjoint time interval. So, it is possible to write $v \in \Upsilon_{1\oplus}$ as $\sum_{i=1}^N \beta_i \vec{v}_i$ for some $\beta_i \in [0,1]$. Since the average amplitude value of signal represented by v is $1/\sqrt{NT_{s\oplus}}$, the relation simplifies to $\sum_{i=1}^N \beta_i = 1$ which implies $\Upsilon_{1\oplus} \subseteq \Delta$. Therefore, $\Upsilon_{1\oplus}$ is a regular $(N-1)$-simplex, with sides of length $\sqrt{2N}$. The volume of such a region can be computed using symmetry arguments similar to those in used in (14) to yield the expression,

$$V(\Upsilon_{1\oplus}) = \frac{N^{\frac{N}{2}}}{(N-1)!}.$$

Noting that in the case of constellations formed from constituent 1-D baseline constellations $\nu = N$ and substituting into (16) gives the result,

$$\gamma_s(\Upsilon_\oplus, \Psi_{\text{opt}}) = \frac{1}{2} \left(\frac{(N+1)^N}{N!} \right)^{\frac{1}{N}}.$$

Using Stirling's approximation, $N! \approx (N/e)^N$ for large N, the asymptotic gain is found as

$$\lim_{N \to \infty} \gamma_s(\Upsilon_\oplus, \Psi_{\text{opt}}) = \frac{e}{2}.$$

This corresponds to the result reported in [3] which was calculated for this one specific case from purely geometrical considerations.

VI. Example Bounding Regions

Figure 2 illustrates examples of two dimensional Υ_1 regions for three intensity modulated signalling schemes.

The use of QAM signalling in optical systems is quite rare, but it is included here for comparison purposes. The

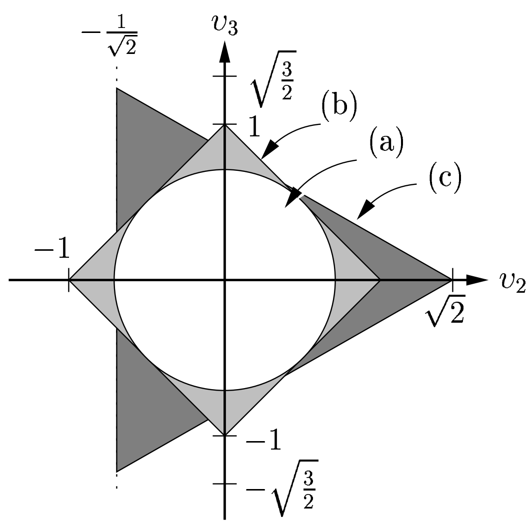

Fig. 2. Example cross-sections of bounding regions, Υ_1, for (a) QAM, (b) AB-QAM (c) $\Omega^3_\oplus$.

basis functions used in the example are

$$\begin{aligned}\phi_2(t) &= \sqrt{\frac{2}{T_s}}\cos(2\pi t/T_s)\\ \phi_3(t) &= \sqrt{\frac{2}{T_s}}\sin(2\pi t/T_s).\end{aligned}$$

The boundary of Υ_1 is circular as expected, since all QAM signals with the same peak values have the same electrical energy.

Adaptively-Biased QAM (AB-QAM) is a signalling scheme originally envisioned for wireless optical communications [5, 2]. The basis functions for this scheme are chosen to correspond to binary level waveforms where sign changes occur according to a subset of the rows of a Hadamard matrix. It is possible to show that this pulse set maximises the set of allowed coordinate values in the ϕ_i direction ($i > 1$) for functions that have the same values for positive and negative extrema. The basis functions used in the example in Figure 2 are

$$\begin{aligned}\phi_2(t) &= \frac{1}{\sqrt{T_s}}\mathrm{rect}(t/T_s) - \frac{2}{\sqrt{T_s}}\,\mathrm{rect}(2t/T_s - 1)\\ \phi_3(t) &= \frac{1}{\sqrt{T_s}}\mathrm{rect}(t/T_s) - \frac{2}{\sqrt{T_s}}\,\mathrm{rect}(2t/T_s - 1/2).\end{aligned}$$

Figure 2 also shows the region $\Upsilon_{1\oplus}$ for the case of $N = 3$. The basis functions used for the example are

$$\begin{aligned}\phi_2(t) &= \sqrt{\frac{2}{T_s}}\mathrm{rect}(t/T_s) - \frac{3}{\sqrt{2T_s}}\,\mathrm{rect}\left(\frac{3t/T_s - 1}{2}\right)\\ \phi_3(t) &= \sqrt{\frac{3}{2T_s}}\,\mathrm{rect}(3t/Ts - 1) - \sqrt{\frac{3}{2T_s}}\,\mathrm{rect}(3t/T_s - 2).\end{aligned}$$

The region corresponds to a 2-D simplex (i.e., an equilateral triangle) with sides of length $\sqrt{6}$, as discussed in Section V-D.

VII. Conclusions

This paper has illustrated a modification to the traditional signal space model to incorporate the non-negativity constraint of intensity modulated channels. The average optical power per symbol is represented in one dimension which allows for a geometric representation of the channel constraints in the signal space model. The definition of a bounding region, which arises due to the structure of the basis functions, and a shaping region allows for the definition of a subset of points in the signal space in which transmittable signals are possible. Constellations can then be constructed by the appropriate distribution of points within the defined regions.

The geometry of bounding regions was discussed and the optimum shaping region was also presented. General expressions for the gain over a baseline constellation were given, and shown to have separable coding and shaping gains. An expression for the shaping gain arising from a general bounding region and Ψ_{opt} was presented, and the asymptotic gain was calculated for the case of an optimally shaped, N-fold Cartesian product of the baseline model.

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