MODULATION AND CONSTRAINED CODING TECHNIQUES FOR
WIRELESS INFRARED COMMUNICATION CHANNELS

by

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A thesis submitted in conformity with the requirements
for the degree of Master of Applied Science
Graduate Department of Electrical & Computer Engineering
University of Toronto

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Abstract

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Short-distance, point-to-point wireless infrared optical links provide a cost-effective means of high speed data transfer between portable devices. To investigate such links, a test-bench and circuits were constructed to determine the limitations of existing optoelectronics. The results of these measurements are used to formulate a signal-space channel model which is employed for the subsequent analysis of candidate bandwidth efficient modulation schemes.

The modulation scheme Adaptively Biased QAM (AB-QAM) is developed based on the channel model. AB-QAM provides an asymptotic 3 dB optical SNR improvement over PAM while maintaining the same bandwidth efficiency. The use of constellation shaping is shown to further improve the average optical power efficiency of AB-QAM.

This thesis proposes the use of constrained coding techniques to satisfy the non-negativity constraint of the optical channel. These coding techniques are illustrated through an example and contrasted to a baseline case. Constrained coding techniques allow greater flexibility in the choice of pulse shapes used in the channel, leading to possible optical power and bandwidth gains.
Acknowledgements

This thesis has required a great deal of sacrifice and effort to complete. However, as with any significant endeavour it was not accomplished without the help of a great number of individuals.

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This work also reflects numerous conversations and debates with my colleagues in room EA104. I thank them for the interest they took in my work as well as for their helpful criticisms. However, foremost I would like to thank them for the friendship and camaraderie they have displayed through my time at UofT.

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Chapter 1

Introduction and Motivation

1.1 Context

In recent years, there has been a migration of computing power from the desktop to portable, mobile formats. Devices such as digital still and video cameras, portable digital assistants and laptop computers offer users the ability to process and capture vast quantities of data. Although convenient, the interchange of data between such devices remains a challenge due to their small size, portability and low cost. A high performance link is necessary to allow data exchange from these portable devices to established computing infrastructure such as backbone networks, data storage devices and user interface peripherals. Also, the ability to form ad hoc networks between portable devices remains an attractive application. The links considered need to operate over relatively short distances, on the order of centimetres. These types of links would be useful for the interchange of data when the two communicating devices were in close proximity to one another. This type of link can be seen as a means of transferring data between portable devices or between a portable device and a fixed computer. Several solutions have been proposed to fulfil this need for a short distance high speed link.

The use of a direct electrical connection between portable devices and a host is a simple means of establishing a link. This electrical connection is made via a cable and connectors on both ends or by some other direct connection method. The connectors can be expensive due to the small size of the portable device. In addition, these connectors are prone to wear and break with repeated use. The physical pin-out of the link is fixed and incompatibility among various vendors solutions may exist. Also, the need to carry the
physical medium for communication makes this solution inconvenient for the user.

Wireless radio frequency (RF) solutions alleviate most of the disadvantages of a fixed electrical connection. RF wireless solutions allow a short distance link to be established without any physical connection. However, these solutions remain relatively expensive and have low data rates. A recently proposed “low cost” RF link over short distances provides data rates of 730 kbps in the 2.4 GHz band for a cost of US$10 per module [14]. Another factor which increases the cost of RF wireless links are the spectrum licensing fees paid to federal regulatory bodies. These frequency allocations are determined by local authorities and may vary from country to country, making a standard interface difficult. In addition, the broadcast nature of the RF channel creates problems with interference between devices communicating to a host in close proximity. Containment of electromagnetic energy at RF frequencies is difficult and if improperly done can impede system performance.

Wireless optical links provide a high data rate, low cost option to fulfil this short distance communication application. Present day wireless optical links can transmit at 4 Mbps over short distances using optoelectronic devices which cost approximately US$1 [15]. Since the electromagnetic spectrum is not licensed in the infrared range, spectrum licensing fees are avoided, further reducing system cost. Optical radiation in the infrared or visible range is easily contained by opaque boundaries. As a result, interference between adjacent devices can be minimised easily and economically. In contrast to direct electrical connections, a wireless infrared link can be programmed to support many different protocols which can be managed automatically by the device. Wireless optical links are also suited to portable devices since small surface mount light emitting and light detecting components are available in high volumes at relatively low cost.

1.2 Survey of Current Implementations

The Infrared Data Association (IrDA) is a group of industrial partners which specify hardware and software specifications for short distance infrared links [15]. Currently, the most popular link is a 4 Mbps serial point-to-point link which operates over a distance of a metre with optical emissions conforming to stringent class 1 eye safety levels [1]. Table 1.1 summarises some key points of the IrDA short distance wireless specification. The classification of optical radiating devices in terms of their eye safety limits is discussed in Section 2.1.2. Chapter 3 defines PPM and presents an analysis of error performance.
Section 1.2. Survey of Current Implementations

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<td>Bit Rate</td>
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<tr>
<td>Bit Error Rate</td>
<td>$10^{-8}$</td>
</tr>
<tr>
<td>Range</td>
<td>Min : 0 m ; Max : 1 m</td>
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Table 1.1: Some key points of the IrDA 4 Mbps wireless infrared link specification [1].

restrictions of this specification are commonly satisfied with inexpensive optoelectronics and without the need of extensive signal processing at the receiver. Over 60 million devices satisfying the IrDA specification have already been installed in laptop computers, digital cameras, printers, cellular phones and other portable computing equipment. It is projected that the short distance infrared market is growing at 40% per annum and will reach world wide sales of US$290 million dollars by 2002 [15, 16].

The IrDA has also recently extended its specification to 16 Mbps links over short distances. The standard was finalised in March 1999, and products are expected to ship by year end. Work has already started on extending this new standard to 32 Mbps in the near future [17].

A simple extension of point-to-point links are telepoint links in which a wider diverging beam is used to establish a one to many connection. A network base station may be mounted on the ceiling and provide access to a limited number of terminals. Each terminal, however, requires a line-of-sight link to the base station. The IrDA has started work on an advanced infrared (AIR) specification for just such a link. Prototype links are expected by year end which would provide 4 Mbps operation in a radius of 4 m, and 250 kbps in a radius of 8 m [18]. Others have extended this system to provide a shared 10 Mbps wireless infrared extension to an Ethernet network over a range of 10 m from the base station. Approximately six users can be accommodated per base station [19].

Another direction for wireless infrared links has been towards diffuse multipoint links. These types of links require base station satellites which are mounted on the ceiling or wall and communicate to many portable devices. These devices offer the freedom to move about a room while maintaining the data link. A line of site path is not required for the link to be established, since reflected paths are used as well. An example of a diffuse optical link operates at rates as high as 4 Mbps over a coverage area of approximately 90 m² [20]. The operation of this type of link is, however, highly dependent on the room layout and the composition of the walls [3].
Section 1.3. Research Direction

Longer distance free-space optical links have been constructed to provide a high data rate wireless link. They are used primarily for inter-building communications or to provide a portable redundant link for fibre optic and video systems. These systems operate exclusively outdoors due to the high optical power used. Long distance wireless infrared links have been shown to allow up to 622 Mbps data rates for distances as long as one kilometre [21]. Prototype links have also been constructed which provide a 1 Gbps data rate over 40 m distances [22]. All long distance wireless optical systems are degraded by atmospheric conditions, especially fog. The availability of these links has been shown to be approximately 99.99% which is insufficient for some backbone telecommunication systems.

1.3 Research Direction

Short distance wireless optical links are an economical and convenient means to transfer data to a portable device. The use of inexpensive optoelectronics in the 880 nm band will allow these inexpensive links to be used in a wide variety of applications.

The goal of this research is to initiate work on the construction of a short range, wireless optical link capable of data rates in the gigabit per second range. This link must be constructed using inexpensive optoelectronic components, so as to encourage its application over a wide variety of computing devices. The proposed range of communication is on the order of centimetres. It is also required that the output of the link remain eye safe so that the conditions of operation are not limited.

A potential application for this type of high speed, short distance link would be for data transfer between a small, portable personal computing device and other computing infrastructure. This type of link would allow a user to carry a note pad sized device which would house all data processing and storage elements. The portable device would have some rudimentary user interface, perhaps using handwriting recognition, to allow work to be done while away from network connections. The short infrared link could be used to connect this device to an interface unit when in a wired location. This interface unit would provide the portable device with a network connection, a connection to user interface peripherals and would display video images sent from the portable unit. In this manner, the interface unit would become an inexpensive shell for use with any unit, and could be placed in public locations. The infrared link would provide a robust means of linking the portable device to the interface unit. This wireless optical link could also be used to link two portable devices to transfer data when away from computing infrastructure. In this manner, the wireless
optical link provides a universal port for all data entering or leaving the portable personal computing device.

Another application for this type of link is in the digital video imaging arena. Vast quantities of data can be collected in short times using digital video cameras. Transferring these large amount of data directly into a network via a short distance infrared link would be convenient and efficient. In order to move these large amounts of data in a reasonable amount of time, a high data rate link is required. An inexpensive, short range, wireless optical link would be well suited to this task.

In summary, the proposed high data rate infrared link over short distances would provide a robust and inexpensive data port for a variety of next generation portable computing devices.

1.4 Thesis Structure

The goal of this thesis is to investigate system issues involved in transmitting data over wireless infrared links at high data rates. In order to begin to consider the system issues of such a link, preliminary channel characteristics are required. Chapter 2 describes the basic channel structure of wireless infrared channels. A more detailed description of channel characteristics follows through the use of device physics and measurements from an experimental channel.

Chapter 3 uses the characteristics and constraints of the wireless optical channel to evaluate a variety of modulation schemes for the channel. Conventional binary level modulation schemes are shown to provide insufficient bandwidth efficiency, while multi-level schemes do not provide sufficient power efficiency. The modulation scheme adaptively biased QAM is proposed and shown as a bandwidth efficient alternative with favourable power efficiency. The use of constellation shaping is also investigated to improve the power efficiency of the scheme.

Chapter 4 describes an alternate method of satisfying the non-negativity constraint of the wireless optical channel. The use of constrained coding to satisfy channel constraints allows the use of symbol pulses which individually do not satisfy the channel constraints. This gives the system designer the freedom to choose pulses with favourable spectral or average optical power characteristics.

The thesis concludes in chapter 5 with a summary of the results as well as some directions for future work.
Chapter 2

Channel Modelling and Characterisation

In order to proceed with the design of a high-speed wireless optical link, a basic knowledge of the channel characteristics is required. This chapter presents a high-level overview of the characteristics and constraints of wireless optical links. The basic channel characteristics are further illuminated by an overview of the device physics governing optoelectronic devices. On the basis of device behaviour, a comparison between popular devices is used to justify the design choices. The chapter continues with a description of the experimental apparatus constructed to determine channel bandwidth and linearity. Experimental results are presented on the performance of test circuitry as well as on the free-space optical link. Various noise sources present in the free space optical link are also discussed to determine which are dominant.

The chapter concludes with a summary of the main topics covered. The choice of optoelectronic components is justified. Conclusions on the channel structure are drawn based on measurements and noise considerations.

2.1 The Wireless Optical Channel

Wireless optical channels differ in several key ways from conventional communications channels treated extensively in literature. This section presents introductory remarks on the channel characteristics and structure.
2.1.1 Basic Channel Structure

In a wireless optical channel, information is transmitted by sending a time varying optical signal between the transmitter and receiver. The information sent on this channel is not contained in the amplitude, phase or frequency of the transmitted optical waveform, but rather in the intensity of the transmitted signal. Present day optoelectronics cannot operate directly on the frequency or phase of the $10^{14}$ Hz range optical signal. Instead, only the intensity, defined as the power per area in W/m$^2$, can be modulated or detected by the optoelectronics.

On a conceptual level, the operation of optoelectronic devices can be seen as performing a conversion between the optical domain, where the signal is a time varying intensity, and the electrical domain, where the information is sent as a current signal. In ideal devices, the conversion between the two signal domains is governed by a linear proportionality constant. The operation of some popular optoelectronic components is described in more detail in Sections 2.2.1 and 2.2.2.

The operation of a wireless optical channel is outlined in Figure 2.1. The transmit electronics convert an input data stream into a time varying current, $i_{tx}(t)$. This current is used to drive a light emitting device to produce the output optical radiation. The electrical characteristics of the light emitter can be modelled as a diode, as shown in the figure. The electrical current signal is converted to an optical intensity signal, $x(t)$, by the light emitter. The intensity signal $x(t)$ ideally is proportional to the magnitude of the electrical signal current, $i_{tx}(t)$.

This optical radiation propagates through free-space until it reaches the light detector at the receive side of the link. The light detector is often termed a square law device.
since its operation is modelled as squaring the amplitude of the incoming electromagnetic signal and integrating over time to find the intensity. The output of the light detector is a current signal, $i_{rx}(t)$, proportional to the intensity of the received optical signal. The electrical operation of this device is most often modelled as a reverse biased diode, as shown in Figure 2.1. This received electrical current signal is amplified and the digital information detected.

The fact that the optical channel is an intensity modulated one, adds constraints on the class of signals which may be transmitted. The information bearing intensity signal must remain positive for all time since the transmitted power can physically never be negative. Thus, the physics of the link imposes the fundamental constraint that the transmitted signals remain non-negative for all time.

### 2.1.2 Eye Safety

Safety considerations must be taken into account when designing a wireless optical link. Since the energy is propagated in a free-space channel, the impact of this radiation on eye safety must be considered.

The International Electrotechnical Commission (IEC) is the standards body which classifies exposure limits of optical sources. Table 2.1 includes a list of the primary classes under which an optical radiator can fall. Class 1 operation is most desirable for a wireless optical system since emissions from products are safe under all circumstances. Under these conditions, no warning labels need to be applied and the device can be used without special safety precautions. This is important since these optical links are destined to be inexpensive, portable and convenient for the user. Longer distance free-space links often operate in class 3B mode, and are used for high data rate transmission over moderate distances (40 m in [22]). The safety of these systems is maintained by locating optical beams on rooftops or on towers to prevent inadvertent interruption [3].

The critical parameter which determines whether a source falls into a given class depends on the application. The allowable exposure limit (AEL) depends on the wavelength of the optical source, the geometry of the emitter and the intensity of the source. For high frequency modulated sources, the average transmitted power of modulation scheme used sets the AEL for a given geometry and wavelength. At modulation frequencies greater than about 24 kHz, the AEL can be calculated based on average output power of the source [4].
Section 2.1. The Wireless Optical Channel

<table>
<thead>
<tr>
<th>Class</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1</td>
<td>Safe under reasonably foreseeable conditions of operation.</td>
</tr>
<tr>
<td>Class 2</td>
<td>Eye protection afforded by aversion responses including blink reflex (for visible sources only $\lambda=400–700$ nm).</td>
</tr>
<tr>
<td>Class 3A</td>
<td>Safe for viewing with unaided eye. Direct intra-beam viewing with optical aids may be hazardous.</td>
</tr>
<tr>
<td>Class 3B</td>
<td>Direct intra-beam viewing is always hazardous. Viewing diffuse reflections is normally safe.</td>
</tr>
</tbody>
</table>

Table 2.1: Interpretation of IEC safety classification for optical sources [2].

The choice of which optical wavelength to use for the wireless optical link also impacts the AEL. Table 2.2 presents the limits for the average transmitted optical power for the IEC classes listed in Table 2.1 at four different wavelengths. The allowable average optical power is calculated assuming that the source is a point emitter, in which the radiation is emitted from a small aperture and diverges slowly as is the case in laser diodes. Wavelengths in the 650 nm range are visible red light emitters. These are seldom used due to the high background ambient light noise present in the channel. Infrared wavelengths are typically used in optical networks. The wavelengths $\lambda=880$ nm, 1310 nm and 1550 nm correspond to the loss minima in typical silica fibre systems, at which wavelengths optoelectronics are commercially available [5]. The trend apparent in Table 2.2 is that for class 1 operation the allowable average optical power increases as does the optical wavelength. This would suggest that the “far” infrared wavelengths above 1 $\mu$m are best suited to wireless optical links due to their higher optical power budget for class 1 operation. The difficulty in using this band is the prohibitive cost associated with these far infrared devices. Photodiodes for far infrared bands are made from exotic III-V semiconductor compounds while photodiodes for the 880 nm band are manufactured in low cost silicon technologies. As a result, the 880 nm “near” infrared optical band is typically used for wireless optical links, and is assumed as the optical wavelength in the balance of this work.

The power levels listed in Table 2.2 are pessimistic when applied to light sources which emit less concentrated beams of light, such as light emitting diodes. For a diode with $\lambda=880$ nm, a diameter of 1 mm and emitting light through cone of angle 30°, the allowable average power for class 1 operation is 28 mW [4]. However, the allowed average optical power for class 1 operation still increases with wavelength. Section 2.2.1 discusses the tradeoff between the use of lasers or light emitting diodes as light emitters.
Section 2.1. The Wireless Optical Channel

### Table 2.2: Point source safety classification based on allowable average optical power output for a variety of optical wavelengths [3, 2].

<table>
<thead>
<tr>
<th>Class</th>
<th>650 nm visible</th>
<th>880 nm infrared</th>
<th>1310 nm infrared</th>
<th>1550 nm infrared</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1</td>
<td>&lt; 0.2 mW</td>
<td>&lt; 0.5 mW</td>
<td>&lt; 8.8 mW</td>
<td>&lt; 10 mW</td>
</tr>
<tr>
<td>Class 2</td>
<td>0.2–1 mW</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>Class 3A</td>
<td>1–5 mW</td>
<td>0.5–2.5 mW</td>
<td>8.8–45 mW</td>
<td>10–50 mW</td>
</tr>
<tr>
<td>Class 3B</td>
<td>5–500 mW</td>
<td>2.5–500 mW</td>
<td>45–500 mW</td>
<td>50–500 mW</td>
</tr>
</tbody>
</table>

Eye safety considerations limit the average optical power which can be transmitted. This is another fundamental limit on the performance of free-space optical links. Therefore, the constraint on any modulation scheme constructed for wireless optical links is that the average optical power is limited.

#### 2.1.3 Channel Propagation Properties

As is the case in radio frequency transmission systems, multipath propagation effects are important for wireless optical networks. The power launched from the transmitter may take many reflected and refracted paths before arriving at the receiver. In radio systems, the sum of the transmitted signal and its images at the receive antenna cause spectral nulls in the transmission characteristic. These nulls are located at frequencies where the phase shift between the paths causes destructive interference at the receiver. This effect is known as multipath fading [13].

Unlike radio systems, multipath fading is not a major impairment in wireless optical transmission. The “antenna” in a wireless optical system is the light detector which typically has an active radiation collection area of approximately 1 cm². The relative size of this antenna with respect to the wavelength of the infrared light is immense, on the order of $10^4 \lambda$. The multipath propagation of light produces fades in the amplitude of the received electromagnetic signal at spacings on the order of half a wavelength apart. As mentioned earlier, the light detector is a square law device which integrates the square of the amplitude of the electromagnetic radiation impinging on it. The large size of the detector with respect to the wavelength of the light provides a degree of inherent spatial diversity in the receiver which mitigates the impact of multipath fading [23].

Although multipath fading is not a major impediment to wireless optical links, temporal dispersion of the received signal due to multipath propagation remains a problem. This dispersion is often modelled as a linear time invariant system since the channel...
properties change slowly over many symbol periods. The impact of multipath dispersion is most noticeable in diffuse infrared communication systems. In short distance line-of-sight (LOS) links, multipath dispersion is seldom an issue. Indeed, channel models proposed for LOS links assume the LOS path dominates and model the channel as a linear attenuation and delay. Thus, for the short range optical links considered in this work, multipath effects are not significant [24, 25, 26].

Section 2.2. Optoelectronic Components

The basic channel characteristics can be investigated more fully by considering the operation of the optoelectronic devices alone. Device physics provides significant insight into the operation of these optoelectronic devices. This section presents an overview of the basic device physics governing the operation of certain optoelectronic devices, emphasising their benefits and disadvantages for wireless optical applications.

2.2.1 Light Emitting Devices

Solid state light emitting devices are essentially diodes operating in forward bias which output an optical intensity approximately linearly related to the drive current. This output optical intensity is due to the fact that a large proportion of the injected minority carriers recombine giving up their energy as emitted photons.

To ensure a high probability of recombination events causing photon emission, light emitting devices are constructed of materials known as direct band gap semiconductors. In this type of crystal, the extrema of the conduction and valence bands coincide at the same value of wave vector. As a result, recombination events can take place across the band gap while conserving momentum, represented by the wave vector (as seen in Figure 2.2)[9]. A majority of photons emitted by this process have energy $E_{\text{photon}} = E_g = h\nu$, where $E_g$ is the band gap energy, $h$ is Planck's constant and $\nu$ is the photon frequency in hertz. This equation can be re-written in terms of the wavelength of the emitted photon as

$$
\lambda = \frac{1240}{E_g}
$$

where $\lambda$ is the wavelength of the photon in nm and $E_g$ is the band gap of the material in electron-Volts. Commercial direct band gap materials are typically compound semiconductors of group III and group V elements. Examples of these types of crystals include: GaAs, InP, InGaAsP and AlGaAs (for Al content less than $\approx 0.45$) [8].
Figure 2.2: An example of a one dimensional variation of band edges with wave number \((k)\) for (a) direct band gap material, (b) indirect band gap material [7].

Elemental semiconducting crystals silicon and germanium are *indirect band gap* materials. In these types of materials, the extrema of conduction and valence bands do not coincide at the same value of wave vector \(k\), as shown in Figure 2.2. Recombination events cannot occur without a variation in the momentum of the interacting particles. The required change in momentum is supplied by collisions with the lattice. The lattice interaction is modelled as the transfer of phonon particles which represent the quantization of the crystalline lattice vibrations. Recombination is also possible due to lattice defects or due to impurities in the lattice which produce energy states within the band gap [8, 11]. Due to the need for a change in momentum for carriers to cross the band gap, recombination events in indirect band gap materials are less likely to occur. Furthermore, when recombination does take place, most of the energy of recombination process is lost to the lattice as heat and little is left for photon generation. As a result, indirect band gap materials produce highly inefficient light emitting devices [7].

The structure of light emitting devices fabricated in direct band gap III-V compounds greatly varies the properties of the emitted optical intensity signal. The two most popular solid-state light emitting devices are light emitting diodes (LEDs) and laser diodes (LDs).
Light Emitting Diodes

As was mentioned in Section 2.1.2, the use of the 780 – 950 nm optical band is preferable due to the availability of low cost optoelectronic components. The direct band gap, compound semiconductor GaAs has a band gap of approximately 1.43 eV which corresponds to a wavelength of approximately 880 nm following (2.1).

Most modern LEDs in the band of interest are constructed from GaAs and AlGaAs as double heterostructure devices. This type of structure is formed by depositing two wide band gap materials on either side of a lower band gap material, and doping the materials appropriately to give diode action. An example of an AlGaAs/GaAs/AlGaAs double heterostructure LED is illustrated in Figure 2.3. Under forward bias conditions, the band diagram forms a potential well in the low band gap material (GaAs) into which carriers are injected. This region is known as the active region where recombination of the injected carriers takes place. The recombination process in the active region occurs randomly and as a result the photons are generated incoherently (i.e., the phase relationship between emitted photons is random in time). This type of radiation is termed spontaneous emission [8].

The advantages of using a double heterostructure stem from the fact that the injected carriers are confined to a well defined region. This confinement results in large concentration of injected carriers in the active region. This in turn reduces the radiative recombination time constant, improving the frequency response of the device. Another advantage of this carrier confinement is that the generated photons are also confined to a well defined area. Since the adjoining regions have a larger band gap than the active region, the losses of due to absorption in these regions is minimised [9].

Using the structure for the LED in Figure 2.3, it is possible to derive an expression for the output optical power of the device as a function of the drive current in the following form:

\[ P_{vol} = h\nu \frac{J}{qd} B \tau_n \left( p_o + n_o + \frac{\tau_n J}{qd} \right), \]  

(2.2)

where \( P_{vol} \) is the output power per unit device volume, \( J \) is the current density applied, \( h\nu \) is the photonic energy, \( d \) is the thickness of the active region, \( B \) is the radiative recombination coefficient, \( \tau_n \) is electron lifetime in the active region and \( p_o, n_o \) are carrier concentrations at thermal equilibrium in the active region [8].

Equation 2.2 shows that for low levels of injected current, \( p_o > \tau_n J/qd \) and \( P_i \) is approximately proportional to the current density. As the applied current density increases
Section 2.2. Optoelectronic Components

Figure 2.3: An example of an AlGaAs/GaAs/AlGaAs double heterostructure LED (a) construction and (b) band diagram under forward bias [8, 9].
by increasing drive current) the optical output of the device exhibits more non-linear components. The choice of active region thickness, $d$, is a critical design parameter for source linearity. By increasing the thickness of the active region, the device has a wider range of input currents over which the behaviour is linear. However, an increase in the active region thickness reduces the confinement of carriers. This, in turn, limits the frequency response of the device as mentioned above. Thus, there is a trade-off between the linearity and frequency response of LEDs.

Another important characteristic of the LED is the performance of the device due to self-heating. As the drive current flows through the device, heat is generated due to the Ohmic resistance of the regions as well as the inefficiency of the device. This increase in temperature degrades the internal quantum efficiency of the device by reducing the confinement of carriers in the active region since a large majority have enough energy to surmount the barrier. This non-linear drop in the output intensity as a function of input current can be seen in Figure 2.4. The impact of self-heating on linearity can be improved by operating the device in pulsed operation and by the use of compensation circuitry [27, 28, 29]. Prolonged operation under high temperature environments reduces output optical intensity at a given current and can lead to device failure [10, 5].

The central wavelength of the output photons is approximately equal to the result given in (2.1). The typical width of the output spectrum is approximately 40 nm around the centre wavelength of 880 nm. This variation is due to the temperature effects as well as the energy distributions of holes and electrons in the active region [8].

**Laser Diodes**

Laser diodes (LDs) are a more recent technology which has grown from underlying LED fabrication techniques. LDs still depend on the transition of carriers over the band gap to produce radiant photons, however, modifications to the device structure allow such devices to efficiently produce coherent light over a narrow optical bandwidth.

As mentioned above, LEDs undergo spontaneous emission of photons when carriers traverse the band gap in a random manner. LDs exhibit a second form of photon generation process: *stimulated emission*. In this process, photons of energy $E_g$ are incident on the active region of the device. In the active region, an excess of electrons is maintained such that in this region the probability of an electron being in the conduction band is greater
Section 2.2. Optoelectronic Components

Optical Power Emitted (mW)

Figure 2.4: An example of an optical intensity versus drive current for LED and LD [10]

than it being in the valence band. This state is called population inversion and is created by the confinement of carriers in the active region and the carrier pumping of the forward biased junction. The incident photon induces recombination processes to take place. The emitted photons in this process have the same energy, frequency, and phase as the incident photon. The output light from this reaction is said to be coherent [8, 7, 30].

In order for this process to be sustainable, the double heterostructure is modified to provide optical feedback. This optical feedback occurs essentially by placing a reflective surface to send generated photons back through the active region to re-initiate the recombination process. There are many techniques to provide this optical feedback, each with their merits and disadvantages. A Fabry-Perot laser achieves photon confinement by having internal reflection inside the active region. This is accomplished by adjusting the refractive-index of surrounding materials. The ends of the device have mirrored facets which are cleaved from the bulk material. One facet provides nearly total reflection while the other allows some transmission to free-space [8].

The operation of this optical feedback structure is analogous to microwave resonators which confine electromagnetic energy by high conductivity metal. These structures
resonate at fixed set of modes depending on the physical construction of the cavity. As a result, due to the structure of the resonant cavity LDs emit their energy over a very narrow spectral width. Also, the resonant nature of the device allows for the emission of relatively high power levels.

Unlike LEDs which emit a light intensity approximately proportional to the drive current, lasers are threshold devices. As shown in Figure 2.4, at low drive currents spontaneous emission dominates and the device behaves essentially as a low intensity LED. After the current surpasses the threshold level, $I_{\text{threshold}}$, stimulated emission dominates and the device exhibits a high optical efficiency as indicated by the large slope in the figure. In the stimulated emission region, the device exhibits an approximately linear variation of optical intensity versus drive current.

**Comparison**

The chief advantage of LDs over LEDs is in the speed of operation. Under conditions of stimulated emission, the recombination time constant is approximately one to two orders of magnitude shorter than during spontaneous recombination [9]. This allows LDs to operate at pulse rates in the gigahertz range, while LEDs are limited to megahertz range operation.

The variation of optical characteristics over temperature and age are more pronounced in LDs than in LEDs. As is the case with LEDs, the general trend is to have lower radiated power as temperature increases. However, a marked difference in LDs is that the threshold current as well as the slope of the characteristic can change drastically as a function of temperature or age of the device. For commercial applications of these devices, such as laser printers, copiers or optical drives, additional circuitry is required to stabilize operating characteristics over the life of the device [31, 32].

For LDs the linearity of the optical output power as a function of drive current above $I_{\text{threshold}}$ also degrades with device aging. Abrupt slope changes, known as *kinks*, are evident in the characteristic due to defects in the junction region as well as due to device degradation in time [10]. LEDs do not suffer from kinks over their lifetimes. Few manufactures quote linearity performance of their devices over their operating lifetimes.

LDs are more difficult to construct and as a result can be more expensive than LEDs. As stated in Chapter 1, the use of inexpensive optical components is a key factor
Section 2.2. Optoelectronic Components

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>LED</th>
<th>LD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optical Spectral Width</td>
<td>25–100 nm</td>
<td>0.1 to 5 nm</td>
</tr>
<tr>
<td>Modulation Bandwidth</td>
<td>Tens of kHz to</td>
<td>Tens of kHz to</td>
</tr>
<tr>
<td></td>
<td>Hundreds of MHz</td>
<td>Tens of GHz</td>
</tr>
<tr>
<td>Special Circuitry Required</td>
<td>None</td>
<td>Threshold and</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Temperature</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Compensation Circuitry</td>
</tr>
<tr>
<td>Eye Safety</td>
<td>Considered Eye Safe</td>
<td>Must be rendered eye safe</td>
</tr>
<tr>
<td>Reliability</td>
<td>High</td>
<td>Moderate</td>
</tr>
<tr>
<td>Cost</td>
<td>Low</td>
<td>Moderate to High</td>
</tr>
</tbody>
</table>

Table 2.3: Comparison of LEDs versus LDs for wireless optical links (based on [4, 5])

determining the implementation of a wireless IR link.

An important limitation for the use of LDs for wireless optical applications is the fact that it is difficult to render laser output eye safe. Due to the coherency and high intensity of the emitted radiation, the output light must be diffused. This requires the use of filters which reduce the efficiency of the device and increase system cost. LEDs are not optical point sources, as are LDs, and can launch greater radiated power while maintaining eye safety limits [4, 3].

As a result of these issues, LEDs were chosen as the light emitting devices for the target application. The strengths and weakness of LDs and LEDs for wireless applications are summarised in Table 2.3.

2.2.2 Photodetectors

Photodetectors are solid-state devices which perform the inverse operation of light emitting devices: they convert the incident radiant light into an electrical current. Photodetectors are essentially reverse biased diodes on which the radiant optical energy is incident, and are also referred to as photodiodes. The incident photons, if they have sufficient energy, generate free electron-hole pairs. The drift or diffusion of these carriers to the contacts of the device constitutes the detected photocurrent.

Inexpensive photodetectors can be constructed of silicon (Si) for the 780–950 nm optical band. The photonic energy at the 880 nm emission peak of GaAs is approximately \( E_g = 1.43 \text{ eV} \), by rearranging (2.1). Since the band gap of silicon is approximately 1.15 eV, these photons have enough energy to promote electrons to the conduction band, and hence are able to create free electron-hole pairs. Figure 2.5 shows that the sensitivity of a silicon
Section 2.2. Optoelectronic Components

Figure 2.5: Relative sensitivity curve for a silicon photodiode [11]. Note that the position of the GaAs emission line is located near the peak in sensitivity of the photodiode.

The basic steady-state operation of a solid-state photodiode can be modelled by the expression,

\[ I_p = q \eta_i \frac{P_p}{h \nu}, \]  

(2.3)

where \( I_p \) is the average photocurrent generated, \( \eta_i \) is the internal quantum efficiency of the device, \( P_p \) is the incident optical power and \( h \nu \) is the photonic energy. The internal quantum efficiency of the device, \( \eta_i \), is the probability of an incident photon generating an electron-hole pair. Typical values of \( \eta_i \) range from 0.7 to 0.9. This value is less than 1 due to current leakage in the device, absorption of light in adjacent regions and due to device defects [5].

Equation (2.3) can be re-arranged to yield the responsivity of the photodiode in the following manner,

\[ R = \frac{I_p}{P_p} = \frac{q \eta_i}{h \nu} \]  

(2.4)

The units of responsivity \( (R) \) are in amperes per watt, and it represents the optoelectronic
conversion factor from optical to electrical domain. Responsivity is a key parameter in photodiode models, and is taken at the central optical frequency of operation.

Two popular examples of photodiodes currently in use include p-i-n photodiodes and avalanche photodiodes.

**p-i-n Photodiodes**

As the name implies, p-i-n photodiodes are constructed by placing a relatively large region of intrinsic semiconducting material between p+ and n+ doped regions (Figure 2.6). Once placed in reverse bias, an electric field extends through most of the intrinsic region. Incident photons first arrive upon an anti-reflective coating which improves the coupling of energy from the environment into the device. The photons then proceed into the p+ layer of the diode. The thickness of the p+ layer is made much thinner than the absorption depth of the material so that a majority of the incident photons arrive in the intrinsic region. The incident light is absorbed in the intrinsic region, producing free carriers. Due to the high electric field in this region (ε), these carriers are swept up, and collected across the junction at a saturation velocity on the order of $10^7$ cm/s. This generation and transport of carriers through the device is the origin of the photocurrent.

Although carrier transit time is an important factor limiting the frequency response of photodiodes for fibre applications, the main limiting factor for wireless applications is the junction capacitance of the device. In wireless applications, devices must be made with
relatively large areas so as to be able to collect as much radiant optical power as possible. As a result, the capacitance of the device can be relatively large. Additionally, the junction capacitance is increased due to the fact that low reverse bias voltages must be used. This is due to the fact that these devices are destined for applications in portable devices where power consumption and hence voltage rails are minimised. Typical values for this junction depletion capacitance at a reverse bias of 3.3 V range from 2 pF for expensive devices used in some fibre applications to 20 pF for very low speed, and cost devices. Careful design of receiver structures is necessary so as not to unduly reduce system bandwidth or increase noise [33].

The relationship between generated photocurrent and incident optical power for p-i-n photodiodes in (2.3) has been shown to be linear over six to eight decades of input level [6, 11]. Second order effects appear when the device is operated at high frequencies as a result of variations in transport of carriers through the high-field region. These effects become prevalent at frequencies above approximately 5 GHz and do not limit the linearity of links at lower frequencies of operation [34]. Since the frequency of operation is limited due to junction capacitance, the non-linearities due to charge transport in the device are negligible. The p-i-n photodiode behaves in a linear fashion over a wide range for the proposed application.

Avalanche Photodiodes

The basic construction of avalanche photodiodes (APDs) is very similar to that of a p-i-n photodiode. The difference is that for every photon which is absorbed by the intrinsic layer, more than on electron-hole pair may be generated. As a result, APDs have a photocurrent gain of greater than unity, while p-i-n photodiodes are fixed at unit gain.

The process by which this gain arrives is known as avalanche multiplication of the generated carriers. A high intensity electric field is established in the depletion region. This field accelerates the generated carriers so that collisions with the lattice generate more carriers. The newly generated carriers are also accelerated by the field, repeating the impact generation of carriers. The photocurrent gain possible with this type of arrangement is of the order $10^2$ to $10^4$ [6, 12]. In wired fibre networks, the amplifying effect of APDs improves the sensitivity of the receiver allowing for longer distances between repeaters in the transmission network [9].
The disadvantage of this scheme is that the avalanche process generates excess shot noise due to the current flowing in the device. This excess noise can degrade the operation of free space links since a majority of the noise present in the system is due to high intensity ambient light. These noise sources are discussed in more detail in Section 2.4.

The avalanche gain is a strong non-linear function of bias voltage and temperature. The primary use of these devices is in digital systems due to their poor linearity. Additional circuitry is required to stabilize the operation of these devices. As a result of the overhead required to use these devices, the system reliability is also degraded [5].

Comparison

APDs provide a gain in the generated photocurrent while p-i-n diodes generate at most one electron-hole pair per photon. It is not clear that this gain produces an improvement in the signal-to-noise ratio (SNR) in every case. Indeed, for the case of a free space optical link operating in ambient light, APDs can actually provide a decrease in SNR [23], as described in Section 2.4.

Due to the non-linear dependence of avalanche gain on the supply voltage and temperature, APDs exhibit non-linear behaviour throughout their operating regime. The addition of extra circuitry to improve this situation increases cost and lowers system reliability. Additional circuitry is also necessary to generate the high bias voltages necessary for high field APDs. Typical supply voltages range from 30 V for InGaAs APDs to 300 V for silicon APDs. Since these devices are destined for portable devices with limited supplies, APDs are not appropriate for this application.

p-i-n diodes are available at relatively low cost and at a variety of wavelengths. They have nearly linear optoelectronic characteristics over many decades of input level. p-i-n photodiodes can be biased from lower supplies with the penalty of increasing junction capacitance.

Due to the issues discussed in the preceding section, a p-i-n photodiode was chosen as the photodetector in this application. The characteristics of both p-i-n photodiodes as well as APDs are summarised in Table 2.4.
Table 2.4: Comparison of p-i-n photodiodes versus avalanche photodiodes for wireless optical links (based on [5, 6])

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>p-i-n Photodiode</th>
<th>Avalanche Photodiode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulation Bandwidth</td>
<td>Tens of MHz to</td>
<td>Hundreds of MHz to</td>
</tr>
<tr>
<td>(ignoring circuit)</td>
<td>Tens of GHz</td>
<td>Tens of GHz</td>
</tr>
<tr>
<td>Photocurrent Gain</td>
<td>1</td>
<td>$10^2 - 10^4$</td>
</tr>
<tr>
<td>Special Circuitry Required</td>
<td>None</td>
<td>High Bias Voltages and Temperature Compensation Circuitry</td>
</tr>
<tr>
<td>Linearity</td>
<td>High</td>
<td>Low – suited to digital applications</td>
</tr>
<tr>
<td>Cost</td>
<td>Low</td>
<td>Moderate to High</td>
</tr>
</tbody>
</table>

2.3 Experimental Channel

The use of LEDs and p-i-n photodiodes offer distinct advantages in implementation ease over other optoelectronic components. However, to have a wireless infrared solution viable for wide spread use, inexpensive optical components must be used. Additionally, since most optical links are destined for pulsed operation, few analog measurements are available in the literature. Notably, the linearity of optical components is typically unreported.

An experimental channel has been constructed using commercially available optoelectronic components. The purpose of creating an experimental channel is to determine the impact of inexpensive optical components on system bandwidth and linearity. This section presents the design methodology used to create the components of the link as well as the results of measurements on the entire channel.

2.3.1 Circuit Design

As shown in Figure 2.1 on page 7, a wireless optical link consists of transmit and receive electronics in addition to optoelectronic components. Since the goal of the experimental link is to provide information about the linearity and bandwidth of the optical components, care must be taken to ensure that the electronics do not limit or corrupt the measurements.

Transmit and receive side electronics were constructed using discrete Motorola MRF581 high-frequency bipolar transistors. The circuits were implemented on custom designed circuit boards. The issues involved in the design of these circuits along with measurements of the constructed circuits is presented in the subsequent sections.
Transmitter

The transmitter circuit converts the incoming voltage signal from a signal source into a current signal to drive the LED. A simple common-emitter transconductance amplifier is well suited to this application and is shown in Figure 2.7. A 50 Ω matching resistor is used to provide a broadband match to the signal source. Degeneration is added to improve the linearity of the driver and provide bias stability. The bias point of the transistor is set through the 40 kΩ resistor from the supply to the base. This biasing scheme is sensitive to variations in the current gain of the transistor, however the supply voltage was adjusted to provide the desired quiescent collector current. The quiescent current is set by supply level, and was set to 50 mA. This level was chosen since the $f_T$ of the transistor peaks to 5 GHz at this bias level.

To allow for characterisation of the transmit transconductor alone, the LED was replaced with a 50 Ω resistor, and the signal ac-coupled to measurement equipment, as shown in Figure 2.7. Frequency response measurements of the circuit were obtained using a Hewlett-Packard HP8595E spectrum analyzer. The transconductance was determined by dividing the measured voltage characteristic by the apparent output resistance of 25 Ω, and
the plot in Figure 2.8 was generated. The harmonic distortion of the circuit was measured by applying a 0 dBm tone at 100 MHz at the input of the transconductor from a Rhode & Schwartz RF generator (5 kHz-3 GHz). The resulting spectrum at the output was captured by the HP8595E, and is also illustrated in Figure 2.8. The measured harmonic content of the RF generator output alone showed that the second harmonic was 35 dB below the fundamental for the test tone applied.

The measurements indicate that the circuit provides a nearly constant transconductance of -40 dBΩ in a frequency range from 1 MHz to 300 MHz. The harmonic distortion of the circuit is better than approximately 35 dB and measurement is limited by the distortion of the signal source.

**Receiver**

The receive electronics amplify the received photocurrent signal and buffer the output to drive measurement equipment. The pre-amplifiers of the receive stage are critical components in any system design. As is the case in any receive chain, the front-end architecture is most important in determining the noise performance of the entire chain. A variety of amplifier techniques and topologies are available to provide low noise operation [35, 33]. The goal of the pre-amplifier for this application is to emphasize frequency response and linearity over noise performance of the circuit to allow for unhindered characterisation of these two parameters.
The receive amplifier, as shown in Figure 2.9, ac-couples the incoming photocurrent, allowing only the signal portion to enter the amplifier. The front end stage, consisting of $Q_1$ and feedback network, forms a transimpedance input stage. This type of topology provides a good compromise between low noise characteristic and high bandwidth implementation. Transimpedance amplifiers are commonly used in optical receiver design [36, 37, 33]. Placing decoupling around $R_{f1}$ allows the DC bias of $Q_1$ to be specified independently of its AC gain. This stage can be thought of as converting the current signal to a voltage signal and provides a gain of approximately 700 $\Omega$ at midband frequencies. $Q_2$ forms the heart of a common emitter amplifier which provides a modest voltage gain. The frequency response of the amplifier is improved by the addition of compensation elements $R_{e2}$ and $C_{e2}$. It is possible to show that the compensation components add a zero in the amplifier transfer characteristic at $\omega_z = 1/R_{e2}C_{e2}$ rad/s. By careful design, this zero can be used to cancel the dominant pole of the first stage determined by $R_{f2}$ and the $C_\mu$ of $Q_1$. For the final design, $R_{e2}$ was set by bias and linearity issues and $C_{e2}$ was estimated and optimised iteratively in hardware. $Q_3$ buffers the voltage output of the previous stage and presents a matched load to the measurement device.

As in the transmitter, circuit techniques were employed to allow for the measurement of the circuit parameters without optoelectronic components. A small signal current source was constructed on the same circuit board as the transimpedance amplifier, and is also shown in Figure 2.9. This circuit provides a matched load to the signal source and uses the 20 k$\Omega$ resistor to approximate voltage independence with current.

The same test equipment was used to characterise the receive amplifier as were used for the transmit amplifier. The supply voltage was set to 20 V, to improve the linearity of the amplifier (due to increased $V_{CB}$) and to allow for reasonably sized resistors for the bias currents required for maximum $f_T$. Using the same input signals as in the transmitter case, the frequency response and harmonic distortion were measured and are presented in Figure 2.10. The transimpedance gain is obtained from the voltage gain of the amplifier by approximating the input current to be $i_{in} \approx v_{in}/40k\Omega$.

The measured values indicate that the transimpedance receiver provides a gain of approximately 61 dB$\Omega$ over a range from 1 MHz to 350 MHz. The linearity of the amplifier is better than 35 dB and is limited by the harmonic distortion present in the source, as was the case in the transmitter.
Figure 2.9: Schematic of receive transimpedance amplifier and test setup.
2.3.2 Channel Measurements

The test circuits used to characterise receiver and transmitter were replaced with optoelectronic components to determine their linearity and frequency response. A Mitel 1A301 LED [38] and a Temic BPV10NF [39] silicon p-i-n photodiode were chosen as test subjects representative of current optoelectronics. The Mitel LED is designed for 266 Mbps fibre links, and has a reported bandwidth of 350 MHz. The Temic photodiode is reported as having a bandwidth of 100 MHz, however, the measurement method is not well documented. The suggested applications for this photodiode are for 450 kHz/1.3 MHz FSK remote control purposes as well as 4 Mbps IrDA links (as described in Chapter 1).

The optoelectronic devices were soldered onto the circuit boards and the test circuits were removed from the signal path. The boards were mounted in a test fixture, as shown in Figure 2.11, the LED and photodiode where aligned by hand and the distance between then adjusted to 1.5 cm.

At the transmitter, the bias for the LED was set at 50 mA, since this is in the middle of the range of allowed currents for the Mitel component. The reverse bias across the photodiode was set at 20 V. According to the data sheet [39], this should place the depletion capacitance near 2.5 pF. The reverse bias was set to a high level to limit the depletion capacitance. In this manner, the time constant due to photodiode capacitance and the input impedance of the receive amplifier is pushed to a higher frequency. This was done
Figure 2.11: Photographs of the transmit and receive electronics with test optoelectronics mounted in test fixture.

to allow for the characterisation of the photodiode alone apart from the properties of the electronics. In a real implementation, this input time constant dominates the frequency performance of the receive side, and hence the results obtained here represent a best case at the receiver.

The frequency response and harmonic distortion were measured using the instruments discussed in Section 2.3.1, and the results are presented in Figure 2.12. The distortion measurements were performed with a tone at +3 dBm from the signal generator at 20 MHz. The second harmonic output power from the generator was 40 dB below the fundamental at the test frequency.

The results indicate that the channel has a 3 dB bandwidth of 35 MHz. The response exhibits a single pole drop off up to approximately 100 MHz after which point the drop off falls more abruptly. The second harmonic falls 39 dB below the fundamental. It was only possible to determine the linearity of the electronics to $-35$ dBc; however, simulations indicate that the linearity of the electronics is at least $-55$ dBc. These measurements show that the optical components are at least linear to 35 dB below the fundamental power.

The *spurious free dynamic range* (SFDR) is a measure of the range of output power over which the channel can be used as a linear one. At low output levels, the noise floor limits performance, while at high output levels distortion products exceed the noise floor and become significant at the output. Over the bandwidth of the channel the noise floor was determined from measurement to be $-45$ dBm. With a $-33$ dBm output fundamental power, the second order distortion component was found to be at $-72$ dBm. The noise
Section 2.3. Experimental Channel

Figure 2.12: Frequency response (a) and harmonic distortion (20 MHz input tone) (b) measurements of link.

floor is 12 dB below the output signal level, while distortion is 27 dB below the noise floor. Since an increase of the fundamental by a factor of $k$ causes the second order component to increase by $2k$, an output power of -19.5 dBm will have a second order distortion component at the noise floor power of -45 dBm. So, the output power can vary from a minimum of -45 dBm to a maximum of -19.5 dBm while the distortion products sit below the noise floor. Therefore, the SFDR is 25.5 dB. Since the frequency response is not flat between 20 and 40 MHz, a correction factor of 2.5 dB is applied to the distortion component. The corrected SFDR of the channel is 23 dB.

The channel attenuation due to the optoelectronic components alone can be determined by combining the results of measurements on the electronics with the channel measurements. The channel gain from the signal current input to the LED to the photocurrent from the photodiode was found from measurement to be

$$\frac{i_{\text{photodiode}}}{i_{\text{LED}}} = -55 \text{ dB}.$$

Typical values for channel attenuation on short distance point-to-point infrared links have been measured from 20 dB for tightly focused beams to an excess of 40 dB for widely diverging beams [3]. The primary cause of the large attenuation on the measured channel was due to the integrated optics in the LED. As mentioned, the output radiation of the LED is designed to be coupled into a fibre. To improve coupling, optics are integrated into the LED package which focus the light to a point approximately 1.5 mm in front of the package.
### 2.4 Noise

Along with specifications regarding the frequency and distortion performance, the noise sources of a wireless optical link are critical factors in determining performance. The determination of noise sources at the input of the receiver is necessary since this is the location where the incoming signal contains the least power.

As was justified in Section 2.2.2, p-i-n photodiodes are commonly used as photodetectors for wireless infrared links. The two primary sources of noise at the receiver front end are due to noise from the receive electronics and shot noise from the received DC photocurrent.

As is the case with all electronics, noise is generated due to the random motion of carriers in resistive and active devices. A major source of noise is thermal noise due to resistive elements in the pre-amplifier. If a low resistance is used in the front end to improve the frequency response, an excessive amount of thermal noise is added to the photocurrent signal. Transimpedance pre-amplifiers provide a low impedance front end through negative feedback and represent a compromise between these constraints [33]. Thermal noise is generated independently of the received signal and can be modelled as having a Gaussian

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 dB Bandwidth</td>
<td>35</td>
<td>MHz</td>
</tr>
<tr>
<td>Second Order Harmonic Distortion</td>
<td>-39</td>
<td>dBC</td>
</tr>
<tr>
<td>SFDR</td>
<td>23</td>
<td>dB</td>
</tr>
<tr>
<td>Channel Attenuation</td>
<td>55</td>
<td>dB</td>
</tr>
</tbody>
</table>

Table 2.5: Measurement results of experimental free-space optical link.

[38]. As the distance increases beyond this point, the radiation pattern diverges rapidly. As a result, most the the launched power from the LED is dispersed to the environment before arriving at the photodiode. This LED was chosen since most present-day high-speed optoelectronic devices are destined for fibre applications. With the expansion of the wireless optical communication market, this trend is expected to change.

Table 2.5 presents a summary of the results of channel measurements as well as derived quantities based on those measurements.
distribution. This noise is shaped by a transfer function dependent on the topology of the pre-amplifier once the noise power is referred to the input of the amplifier. As a result, the noise due to the circuit follows a Gaussian distribution, but it is generally non-white [4].

Photogenerated shot noise is a major noise source in the wireless optical link. This noise is fundamentally due to the discrete nature of energy and charge in the photodiode. Carrier pairs are generated randomly in the space charge region due to the incident photons. Furthermore, carriers traverse the potential barrier of the p-n junction in a random fashion dependent on their energy. The probabilistic generation and transport of carriers due to quantum effects in the photodiode gives rise to shot noise in the photocurrent. This random process can be modelled as having a Poisson distribution with a white power spectral density [33, 40].

Using these two sources of noise, the signal-to-noise ratio for a wireless optical link can be approximated in a simple example (based on [35]). The input signal to the receiver is a time varying optical intensity signal. Let the transmitted intensity signal, $x(t)$, be a fixed sinusoid of the form

$$x(t) = P_t(1 + m \sin \omega t)$$

where $P_t$ is the average transmitted power and $m$ is the amplitude of the sinusoid. To ensure transmission of this signal requires $|m| < 1$, since negative intensity values are not possible as discussed in Section 2.1.

This intensity signal is attenuated as it propagates through free-space to the receive side. The received signal also consists of ambient, background light that is present in the channel and detected by the photodiode. This ambient light consists of incandescent light, natural light and other illumination in the environment. The received intensity, $r(t)$, can be written as

$$r(t) = P_r(1 + m \sin \omega t) + P_{amb},$$

where $P_r$ is the average power at the receive side and $P_{amb}$ is the power of the ambient light incident on the photodiode. The photodiode converts this incident optical intensity into a photocurrent in accordance with the responsivity relationship in (2.4). The signal and DC quantities of the photocurrent can be isolated in the following form:

$$i_{\text{photo}}(t) = R \cdot r(t) = R(P_r + P_{amb}) + (RP_r m \sin \omega t).$$

The electrical signal power at the receive side is contained entirely in the time varying
component and can be written as

\[ P_{\text{signal}} = \frac{1}{2} m^2 (RP_r)^2. \]

The noise power due to the pre-amplifier and due to shot noise is uncorrelated, and so the total noise power is simply the sum of the individual noise source powers. If the noise power due to the circuit is \( i_{\text{circ}}^2 \), the total noise power takes the form

\[ P_{\text{noise}} = i_{\text{circ}}^2 + i_{\text{shot}}^2 = i_{\text{circ}}^2 + 2qR(P_r + P_{\text{amb}})B_{\text{eff}}, \]

where \( q \) is the electronic charge and \( B_{\text{eff}} \) is the equivalent noise bandwidth of the system. Combining the results, an estimate of the signal-to-noise ratio of the system can be formed as

\[ \text{SNR}_{\text{link}} = \frac{P_{\text{signal}}}{P_{\text{noise}}} = \frac{1}{2} \cdot \frac{m^2 (RP_r)^2}{i_{\text{circ}}^2 + 2qR(P_r + P_{\text{amb}})B_{\text{eff}}}. \] (2.5)

The dominant source of noise in a wireless optical channel is due to the ambient background light. To reduce the impact of ambient light, optical low-pass filtering can be used to attenuate visible and higher frequency light sources with little added cost [24]. In some links, this ambient light may be as much as 25 dB greater than the signal power, even after optical filtering [23]. The high ambient light levels cause the ambient light shot noise component to dominate the circuit noise, allowing (2.5) to be simplified to

\[ \text{SNR}_{\text{link}} \approx \frac{m^2 (RP_r)^2}{4qRP_{\text{amb}}B_{\text{eff}}}. \]

Using this assumption, the resulting noise of the channel is signal independent, white shot noise following a Poisson distribution. This high intensity shot noise is the result of the summation of many independent, Poisson distributed random variables. In the limit, as the number of random variables summed approaches infinity, the cumulative distribution function approaches a Gaussian distribution by the central limit theorem. Thus, the dominant noise source can be modelled as being white, signal independent and having a Gaussian distribution [41, 40, 4].

This situation is in contrast to optical fibres where the ambient light is essentially zero, and circuit noise is the dominant noise factor [35, 41]. The use of an APD is advantageous in fibre applications as long as the circuit noise is much greater than the added shot noise of the APD. In this manner, APDs can provide a gain to the signal portion of the received power while keeping the noise power essentially constant. The net effect is to allow for wider repeater spacing in a fibre network, reducing system cost [9].
Emissions from fluorescent lighting create a noise source unique to wireless optical channels. Fluorescent lamps have strong emissions at the spectral lines of argon in the 780–950 nm near infrared band. Although economical narrow band optical filters have been used for some time in such links [24], significant energies are still detected by the photodiode. The detected output of fluorescent lamps is nearly deterministic and periodic with components at multiples of the ballast drive frequency. Most fluorescent ballasts drive the lamps at the line frequency of 50–60 Hz, with harmonics up to tens of kilohertz. Modern ballasts modulate the lamp at higher frequencies to improve power efficiency and reduce unit size. Typical modulation rates are 22 kHz and 45 kHz. The harmonics generated by these sources, and detected by the photodiode extend into the hundreds of kilohertz and can present an impediment to wireless optical data transmission. The impact of periodic interference from high frequency modulated fluorescent light sources has only recently been investigated for wireless optical links [42, 4, 23]. In this work it is assumed that the additive white noise is dominant over fluorescent light interference. However, the robustness of certain key modulation schemes against fluorescent light sources is briefly discussed in Chapter 3.

2.5 Summary of Characteristics and Conclusions

LEDs and p-i-n diodes are economical and practical optoelectronic components for implementing wireless optical links. They remain reliable over a long lifetime, and require no additional support circuitry to guarantee operation. The disadvantages of these devices over other optoelectronic components is their slower frequency response as well as lower efficiency.

Measurements on an experimental link were performed to characterise linearity and frequency response using typical optical components. The measured second harmonic distortion was better than 35 dB, and measurement was limited by the test setup. The linear range of operation can be extended using circuit techniques [27, 28, 29], or by using more expensive higher linearity components. These results tend to suggest that sufficient linearity is present to allow distortion-free analog transmission over a practical range. As a result, transmission need not be constrained to baseband digital techniques. The bandwidth of the link was measured to be approximately 35 MHz. In order to support higher data rates, bandwidth efficient techniques must be investigated to exploit the available bandwidth.
The noise corrupting the input signal is due to high intensity shot noise due to ambient light impinging on the photodiode. This noise can be modelled as signal independent, additive, white Gaussian noise. Low pass interference is also present due to the modulation of ballasts in fluorescent lighting fixtures. The impact of this low pass interferer depends heavily on system architecture and on the modulation scheme used.
Chapter 3

Modulation Schemes

The development of a high data rate wireless optical data link using inexpensive components requires the use of efficient system level concepts to overcome the limitations of the optoelectronics. The modulation scheme selected for such a link must satisfy all the channel constraints while providing a high bandwidth and power efficiency. In order to reduce the cost of the system, the scheme must be highly efficient for a relatively modest complexity.

This chapter reviews the constraints of the wireless optical channel, and casts them in a more formal light. After a channel model is described for the analysis of modulation schemes, critical measures of performance are defined. These tools are then used to analyse popular binary level modulation schemes for wireless optical channels. The need for higher bandwidth efficiency leads to an investigation of multilevel modulation schemes. We propose a new modulation scheme, adaptively biased QAM, which improves upon the power efficiency of other multilevel schemes, while providing greater bandwidth efficiency than binary level techniques. The chapter concludes with a comparison of the modulation schemes considered and a discussion of the appropriateness of each scheme.

3.1 Definitions

This section reviews the characteristics of the wireless optical channel described in earlier chapters. A simple model for the communication system is outlined, and used in the balance of the chapter as a reference for the analysis of candidate modulation schemes. The measures used to determine the performance of the modulation scheme are also defined.
and contrasted to similar measures for conventional channels.

### 3.1.1 Channel Characteristics

Wireless optical links transmit information in the form of an optical intensity signal. Optoelectrical conversion is ideally linear, and produces an intensity waveform proportional to the electrical signal. Due to the limitations of the optoelectronics, direct detection of the radiant energy is performed at the receiver, rather than heterodyning techniques as in microwave links. Since the information transmitted in the free-space channel is a power signal, negative amplitude values are impossible. If $x(t)$ is taken to represent the time varying optical intensity signal in the free-space channel, this non-negativity constraint can be compactly written as

$$x(t) \geq 0, \forall t \in \mathbb{R}.$$  \hspace{1cm} (3.1)

This implies that any intensity modulation scheme for the wireless infrared channel must have a DC component which transmits no information, but consumes power.

Since the wireless infrared channel shares the same space as the user environment, eye safety requirements impose further constraints. For time varying signals, the average optical power transmitted must be limited. Since the signal is a time-varying intensity, eye safety imposes a limit on the average amplitude of all signals in the channel. If the average optical power allowed in the channel is $P$, the constraint due to eye safety considerations is

$$\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t) dt \leq P.$$ \hspace{1cm} (3.2)

The channel measurements in Chapter 2, indicate that inexpensive optoelectronic devices for free-space communication have bandwidths in the 35 MHz range. Optoelectronics destined for wireless communications are soon expected to provide bandwidths on the order of several hundred megahertz. In order to implement high data rate links, approaching 1 Gbps, bandwidth efficient modulation schemes must be investigated. Unlike fibre systems, in which higher cost devices are used and in which the underlying medium has thousands of gigahertz of available bandwidth, wireless optical links are constrained both by low cost and by low bandwidth.
3.1.2 System Model

In order to compare various modulation schemes for wireless optical links, a simplified model of the communication system was formed. Figure 3.1 graphically shows the transmitter, channel and receiver representations, as well as the signals at various points.

At the transmitter, an information source outputs a message at a rate of $R_s = 1/T_s$ symbols per second. The message sent during a symbol interval is chosen as one of the $M$ elements in the alphabet $\mathcal{M} = \{m_1, m_2, \ldots, m_M\}$. Messages are sent independently of each other, and the messages are selected from the alphabet with an equal probability of $1/M$.

The vector transmitter accepts the generated message and performs a one-to-one mapping between the set of messages and a set of vectors, $\Omega$, in an $N$-dimensional Euclidean space. The set of all vectors corresponding to transmitted messages, $\Omega$, is known as the constellation of the modulation scheme. Thus, message $m_i$ input to the vector transmitter generates an $N$-dimensional constellation vector $s_i$, where

$$s_i = \begin{bmatrix} a_{i1} \\ a_{i2} \\ \vdots \\ a_{iN} \end{bmatrix}.$$  \hfill (3.3)

The modulator converts the constellation point $s_i$ into a continuous time waveform. In the case of a wireless optical link, the vector is converted into a time varying optical intensity. This operation is accomplished by taking the inner product of $s_i$ with the set of
orthonormal basis functions \( \Phi \), where
\[
\Phi = \begin{bmatrix}
\phi_1(t) \\
\phi_2(t) \\
\vdots \\
\phi_N(t)
\end{bmatrix}.
\]
The \( \phi_i(t) \) are a set of real valued functions, time limited to \( t = [0, T_s] \). They are determined by the modulation scheme and must be set so that the constraints of the channel are met. The set of \( \phi_i(t) \) basis functions are conventionally chosen to be orthonormal, satisfying the constraint,
\[
\int_0^{T_s} \phi_k(t) \cdot \phi_l(t) \, dt = \begin{cases} 
1 & : \ k = l \\
0 & : \ k \neq l
\end{cases}.
\]
For message \( m_i \), the modulator outputs a time-limited optical intensity pulse, \( s_i(t) \), determined as
\[
s_i(t) = \langle s_i, \Phi \rangle = \sum_{k=1}^{N} a_{ik} \phi_k(t),
\]
where the operation \( \langle \cdot, \cdot \rangle \) is the inner product between two vectors. The collection of the individual \( s_i(t) \) symbol pulses sent over all symbol periods form the transmitted signal on the link, defined earlier as \( x(t) \). Explicitly, \( x(t) \) takes the form
\[
x(t) = \sum_{k=-\infty}^{\infty} s_{ik} (t - kT_s)
\]
where \( s_{ik}(t) \) is the symbol pulse at symbol interval \( k \), as chosen by the information source.

The symbol pulse then propagates through the free-space channel. The channel is modelled as having a very wide bandwidth, to simplify the comparison of various schemes, as is customary in other analyses of wireless optical modulation schemes \([4, 23, 43]\). The optical intensity signal is converted to the electrical domain by the factor \( R \) which represents the responsivity of the photodiode, defined in Chapter 2. The signal is corrupted by additive white Gaussian noise of zero mean and two sided power spectral density of \( \sigma_n^2 \) at the front-end of the receiver.

The corrupted symbol pulse at the receiver, \( z(t) \), is detected using a bank of \( N \) unit energy matched filters. These filters are matched to the \( \phi_i(t) \) basis functions. Assuming perfect timing synchronisation, the symbol rate sampled output of the matched filters produces an \( N \)-dimensional vector \( \mathbf{z} = \mathbf{s}_i + \mathbf{n} \) where the noise vector, \( \mathbf{n} \), is composed
of $N$ independent discrete Gaussian random variables of zero mean and variance $\sigma_n^2$. Unit energy matched filters are used to ensure that for all modulation schemes investigated, the variance of the discrete Gaussian random variables is the same. It is well known, that the received vector contains sufficient statistics to form an estimate of the sent message \[13\].

An estimate of the sent symbol, $\hat{m}$, is formed based on the received vector $z$. Since the source outputs all the symbols with equal probability, maximum likelihood (ML) rules can be used to detect the sent symbol. Since the likelihood function

$$f_{z|m_i, \text{sent}} = (2\pi\sigma_n^2)^{-N/2} \exp\left(\frac{\|z - s_i\|^2}{2\sigma_n^2}\right),$$

the ML detector assigns $\hat{m} = m_i$, for the the $i$ which minimises the Euclidean distance criterion $\|z - s_i\|^2$. In this manner, maximum likelihood decoding is equivalent to minimum-distance decoding \[13\].

### 3.1.3 Relating Channel Constraints to the Signal Space

The model of the wireless optical channel in Section 3.1.2 is useful in determining the error performance of a given modulation scheme in the presence of additive white Gaussian noise. The model, however, does not represent the fundamental constraints of the wireless optical channel described in (3.1) and (3.2). The essential trade-offs in the design of modulation schemes for wireless optical channels depend on the relationship between the optical channel constraints and the properties of the signal space, which determine the probability of symbol error.

The decoding process involves searching the constellation and computing the Euclidean distance between the received message point and each constellation point. The constellation point closest to the received point is taken as the estimate of the received symbol. An error in decoding occurs if $\hat{m} \neq m_i$, when $m_i$ is sent. This decoding process suggests that the distance between constellation points in the signal space determines the probability of error for a given modulation scheme. The squared Euclidean distance, $d^2$, between any two constellation points in the signal space, $s_i$ and $s_j$, can be written as,

$$d^2 = \|s_i - s_j\|^2 = \sum_{k=1}^{N} (a_{ik} - a_{jk})^2. \quad (3.6)$$

where the $a_{ik}$ are the vectorial components as defined in (3.3).
Equation 3.4 demonstrates the link between the coordinates in the signal space and the time domain intensity functions, \( s_i(t) \). The channel imposes constraints on these time domain functions, which can then be related to the signal space model.

The non-negativity constraint in (3.1) is imposed on the infinite extent intensity function, \( x(t) \) defined in (3.5). According to this definition of \( x(t) \), each of the \( s_i(t) \) must be transmittable in isolation which requires \( s_i(t) \geq 0 \) within the symbol period \( T_s \). Using (3.4), the non-negativity constraint can be written as

\[
s_i(t) = \sum_{k=1}^{N} a_{ik} \phi_k(t) \geq 0
\]

for \( t \in [0, T_s] \).

In a similar fashion, the average optical power constraint of (3.2) can also be written in terms of the vectorial components of the constellation points. \( P \) is related to the transmitted symbols through the set of functions \( s_i(t) \) defined in (3.4). Since the symbols are equiprobable, the average optical power is the expected value over all of the time averages of the \( s_i(t) \). Formally,

\[
P = \frac{1}{M} \sum_{i=1}^{M} \frac{1}{T_s} \int_0^{T_s} s_i(t) \, dt = \frac{1}{M} \sum_{i=1}^{M} \sum_{k=1}^{N} a_{ik} \frac{1}{T_s} \int_0^{T_s} \phi_k(t) \, dt
\]

\[
= \frac{1}{M} \sum_{i=1}^{M} \sum_{k=1}^{N} a_{ik} \overline{\phi_k(t)},
\]

where \( \overline{\phi_k(t)} \) is the average value of the basis function within a symbol period. The implication of (3.8) can be demonstrated by considering the variation of the \( a_{ik} \) with \( P \) for a fixed set of basis functions. Following (3.8), the average optical power \( P \) is proportional to every \( a_{ik} \) with the same proportionality constant. For example, an increase in \( P \) by a factor of \( K \), implies that each \( a_{ik} \) also increases by a factor of \( K \), since the \( \overline{\phi_k(t)} \) are fixed. Applying this fact to (3.6), shows that \( d^2 \propto P^2 \). Therefore, \( d \propto P \), where \( P \) is the average optical power transmitted on the link.

The channel constraints in signal space domain provide a set of relationships between the coordinates of the constellation points for any intensity modulated channel. These

\[\text{Chapter 4 addresses the case when the } s_i(t) \text{ overlap in time and are not necessarily transmittable in isolation.} \]
relationships can be simplified by noticing that the function $\phi_{\text{avg}}(t)$, defined as

$$
\phi_{\text{avg}}(t) = \sqrt{\frac{1}{T_s}} \text{rect} \left( \frac{t - T_s/2}{T_s} \right)
$$

(3.9)

where,

$$
\text{rect}(t) = \begin{cases} 
1 & : |t| \leq 1/2 \\
0 & : \text{otherwise}
\end{cases}
$$

can be chosen as a basis function for every intensity modulation technique. This basis function represents the average value added to each symbol. The non-negativity constraint ensures that all non-zero symbols transmitted must have a DC component. Defining $\phi_{\text{avg}}(t)$ as a basis function, ensures the average of all other basis functions in $[0, T_s]$, is zero due to the orthogonality of the basis functions.

For notation’s sake, let $\phi_1(t) = \phi_{\text{avg}}(t)$. The non-negativity constraint in (3.7) can be simplified as

$$
ad_1 \frac{1}{\sqrt{T_s}} \geq - \sum_{k=2}^{N} a_{ik} \phi_k(t)
$$

for $t \in [0, T_s]$. It is enough to have this expression satisfied at the value of $t$ where the summation is minimised, since by definition all other points are more positive. Additionally, since the right hand side of the inequality consists of functions with zero average value, the minimum value will always be less than or equal to zero. As a result, we can write

$$
ad_1 \frac{1}{\sqrt{T_s}} \geq - \min_{t \in [0, T_s]} \left\{ \sum_{k=2}^{N} a_{ik} \phi_k(t) \right\},
$$

(3.10)

which states that the DC bias added to each symbol must at least be equal in magnitude to the maximum negative excursion of the signal components which vary inside the symbol period.

The average power constraint in (3.8) can also be simplified by noting that

$$
\overline{\phi_i(t)} = 0, \ i > 1.
$$

The simplified expression takes the form,

$$
P = \frac{1}{M} \sum_{i=1}^{M} a_{i1} \overline{\phi_1(t)}
$$

$$
= \frac{1}{M} \sum_{i=1}^{M} a_{i1} \frac{1}{\sqrt{T_s}}.
$$

(3.11)
Substituting (3.10) gives,

\[ P \geq -\frac{1}{M} \sum_{i=1}^{M} \min_{t \in [0, T_s]} \left\{ \sum_{k=2}^{N} a_{ik} \phi_k(t) \right\} . \]

Let \( P_{ex, i} \geq 0 \) be the excess power added to symbol \( i \) above the minimum required by (3.10).

The expression for the average power constraint can then be written as an equality in the form

\[ P - \frac{1}{M} \sum_{j=1}^{M} P_{ex, j} = -\frac{1}{M} \sum_{i=1}^{M} \min_{t \in [0, T_s]} \left\{ \sum_{k=2}^{N} a_{ik} \phi_k(t) \right\} . \] (3.12)

For a given set of \( \phi_i(t) \) and \( P \), the average value is set by the choice of excess power and vector coordinates \( a_{ik} \). The power term on the left hand side of (3.12) fixes the negative peak values of the symbol functions represented on the right hand side. Minimisation of the excess power allows larger negative amplitude values for the symbols, or equivalently larger \( a_{ik} \) values. Although large values for the \( a_{ik} \) do not guarantee better separation of signal points, they increase the size of the region in signal space in which constellation points can be placed while satisfying the channel constraints. Thus, for a given modulation scheme defined by a set of \( \phi_i(t) \), the minimisation of excess optical power increases the the region in signal space where constellation points can exist for a given \( P \), which in turn can potentially lead to an increase in spacing between signal points.

This coincides with intuition, since the excess power transmitted does not transmit any information and consumes a larger than necessary part of the average power budget. Power efficient schemes should then transmit zero excess power per symbol to allow for larger values of \( a_{ik} \) which in turn provides a greater degree of flexibility in choosing points from the signal space.

It must be noted that (3.12) is highly dependent on the structure of the \( \phi_i(t) \) through the minimisation term. Further generalisation is difficult since the \( a_{ik} \) are constrained not only by the channel but also by the choice of basis functions. As a result, the expression for a distance in the signal space in (3.6) is not only dependent on the average power \( P \), but is dependent on the pulse shapes of the \( \phi_i(t) \) due to the channel constraints.

### 3.1.4 Definition of Measures

In order to compare modulation schemes on the intensity modulated channel, figures of merit must be specified which encapsulate the performance of each technique. Key
figures of merit are measures that illustrate the power efficiency and bandwidth efficiency of the modulation scheme in light of the channel constraints.

**Probability of Error**

The estimation of the probability of a symbol error occurring in the detection process is a key parameter of any modulation scheme. The probability of symbol error is defined as, \( P_e(\text{sym}) = \Pr\{\hat{m} \neq m_i \mid m_i \text{ sent} \} \). In practice, this quantity is difficult to calculate exactly, and bounds are used to estimate its value. Where the probability of symbol error cannot be calculated exactly, the following union bound approximation on the probability of error is used

\[
P_e(\text{sym}) \approx K_{\text{min}} \cdot Q\left(\frac{d_{\text{min}}}{2\sigma_n}\right),
\]

where \( d_{\text{min}} \) is the minimum Euclidean distance between two points in the signal constellation, \( K_{\text{min}} \) is the average number of constellation points \( d_{\text{min}} \) away from any point, and \( Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-z^2/2)dz \). In the comparison of various modulation schemes, the probability of bit error, \( P_e \), is a figure of merit. \( P_e \) can be determined from \( P_e(\text{sym}) \) exactly in some cases, while approximations must be made in others. A common approximation is that Gray coding is used between adjacent symbols in the constellation. In this way, a symbol error is with high probability associated with a single bit error. Using this approximation, for an alphabet size of \( M \), \( P_e \approx P_e(\text{sym})/\log_2 M \).

**Constellation Figure of Merit**

In a conventional scheme, the amount of electrical power that can be sent on the channel is limited. It is possible to show that in this case \( d_{\text{min}}^2 \) is proportional to the electrical power of the symbols sent \([41]\). Improvements in \( P_e \) can be made by increasing the transmitted electrical power, thereby increasing \( d_{\text{min}} \). However, in a wireless optical system, the limitation is not on the electrical power, but on the average optical power. Since the relationship between optical power (or intensity) and electrical amplitude is linear, the channel limitation translates to a constraint on the signal amplitude. Section 3.1.3 shows that the distance between signal points in the constellation is proportional to the average optical power. Therefore by extension, in an optical intensity channel \( d_{\text{min}} \propto P \).

In conventional channels, a key measure is the constellation figure of merit (CFM) \([44]\). The constellation figure of merit is used to categorise the energy efficiency of a mod-
ulation scheme, and is defined as the ratio $\Delta \triangleq \frac{d_{\text{min}}^2}{E}$, where $E$ is the average energy of the constellation. This quantity is dimensionless, and independent of the pulse shape or scaling.

In an intensity modulated channel a similar measure can be established. The constellation figure of merit in an optical channel can be defined as

$$\text{CFM} = \frac{d_{\text{min}}}{P}.$$ (3.14)

The CFM is scale invariant in terms of the transmitted optical power, since $d_{\text{min}} \propto P$. However, unlike the CFM defined for the conventional channel, the optical CFM is not unitless, but suffers from a mismatch of electrical and optical quantities. The optical CFM depends on the structure of the pulses used for a given modulation scheme as well as the average power as discussed in Section 3.1.3.

Using the definition of CFM in (3.14), the probability of symbol error defined in (3.13) can then be re-written as

$$P_e(\text{sym}) \approx K_{\text{min}} \cdot Q\left(\frac{\text{CFM} \cdot P}{2\sigma_n}\right).$$

For conventional channels corrupted by Gaussian white noise, the ratio of the CFMs for different modulation schemes is used to compare their relative electrical energy efficiency. It can be shown that in a conventional channel for two modulation schemes, denoted 1 and 2, that $\frac{\text{CFM}_1}{\text{CFM}_2}$ is the factor improvement in electrical signal-to-noise ratio due to the use of modulation scheme 1 over 2, for the same $P_e(\text{sym})$ and noise power [45].

An analogous quantity can be computed for optical intensity channels corrupted by Gaussian white noise. Using the definition of optical CFM in (3.14), the gain of optical modulation scheme 1 over 2 is given as

$$\Gamma_{1/2} = 10 \log_{10} \frac{\text{CFM}_1}{\text{CFM}_2} \text{ dB},$$ (3.15)

where $\Gamma_{1/2}$ is the gain for using modulation scheme 1 over 2. The meaning of this expression can be simplified using (3.13), which contains an expression for the probability of symbol error. For cases where the $P_e(\text{sym})$ approaches zero, the $K_{\text{min}}$ term becomes less significant than the exponential $Q(\cdot)$ function, and the probability of error can be thought to depend on $d_{\text{min}}$ and $\sigma_n^2$ alone. So, in this case, if the two modulation schemes have the same
probability of error and noise variance, then the $d_{\text{min}}$ for both schemes is approximately the same. Applying this simplification to (3.15) gives,

$$
\Gamma_{1/2} = 10 \log_{10} \frac{P_2}{P_1} \text{ dB} \quad (3.16)
$$

where $P_1, P_2$ are the average optical power transmitted in modulation scheme 1 and 2 respectively. Therefore the gain $\Gamma$ is equal to the number of decibels savings in average optical transmit power required for scheme 1 over scheme 2 for the same noise power and probability of symbol error.

It must be noted that the gain values are in terms of the received average optical power and not in terms of electrical quantities. As noted earlier, $d_{\text{min}} \propto P$ in optical channels and $d_{\text{min}}^2 \propto E$ in electrical channels, where $E$ is the average pulse electrical energy for the constellation. Therefore, the gain in (3.16), in decibels, is a factor of 0.5 less than the comparable measure in the electrical domain. For example, a $\Gamma = 3$ dB corresponds to an increase in the electrical signal-to-noise ratio (SNR) by 6 dB.

The optical CFM is calculated for each modulation scheme considered here and is used to perform comparisons on the relative optical power efficiency of the schemes.

**Spectral Characteristics**

The frequency characteristics of the transmitted modulation scheme must also be considered. For a wide-sense stationary or wide-sense cyclostationary random process it is possible to define a quantity called the power spectral density (psd). The psd is an averaged second order statistic of a random process, and gives insight on the distribution of power over frequency. As discussed in Chapter 2, the electrical characteristics of the channel impose the bandwidth penalty to the data bearing signal. The psd of the transmitted waveform in electrical domain, $Rx(t)$, is used to determine the frequency characteristics of a modulation scheme. The power spectral density of $Rx(t)$ can be calculated for a modulation scheme of the form of (3.5), assuming that all symbols are sent independently and equally likely using the expression

$$
G_x(f) = \frac{R^2}{M^2 T_s^2} \sum_{n=-\infty}^{\infty} \left| \sum_{i=1}^{M} S_i \left( \frac{n}{T_s} \right) \right|^2 \delta \left( f - \frac{n}{T_s} \right) + \frac{R^2}{T_s} \left[ \frac{1}{M} \left| S_i(f) \right|^2 - \left| \frac{1}{M} \sum_{i=1}^{M} S_i(f) \right|^2 \right], \quad (3.17)
$$
where $G_x(f)$ is the power spectral density, $S_i(f)$ is the Fourier transform of $s_i(t)$, $R$ is the responsivity and $\delta(\cdot)$ is the Dirac delta function \cite{46}. The power spectral density consists of two components: a line spectrum and a continuous spectrum. The line spectrum consists of a series of $\delta(\cdot)$ functions at various frequencies. These impulses carry no information, although they are often used for timing recovery. All modulation schemes destined for the wireless optical channel have an impulse in their psd at DC, due to the non-negativity constraint. The continuous spectrum is shaped by the distribution of the data symbols as well as by the pulse shape itself. Line coding can be used to introduce nulls in the spectrum, or to tailor the spectral response of the modulated signal to the channel in question. In this chapter, the data distribution is taken to be independent and equiprobable. The power spectral density is calculated for each modulation scheme considered in the following sections.

The bandwidth occupied by a modulation scheme is a measure of the amount of spectral support needed for the transmission of the signal. Since all real signals have have even magnitude spectra, only positive frequencies are considered. There are several definitions of bandwidth which are popular in literature. In the electronics domain, the $-3$ dB bandwidth occurs at the frequency when the psd is a factor $1/\sqrt{2}$ lower than the peak value. The fractional power bandwidth is defined as the positive frequency which contains a certain fraction of the total signal power. Typical values are 90-95\% of total signal power \cite{47}. The bandwidth measure used in this chapter is the null-to-null bandwidth. The bandwidth of the signal is defined as the width in positive frequencies of the main lobe of the signal. Although most of the energy of the pulse is contained in the main lobe, this measure does not penalise schemes with large side-lobe power. This choice of bandwidth definition is in keeping with previous work done in the field \cite{4, 23, 43}. For example, for the power spectral density $G(f) = T\sin^2(\pi fT)/(\pi fT)^2$, the $-3$ dB bandwidth is $f \approx 0.44/T$, the 90\% fractional bandwidth is $f \approx 0.9/T$, and the null-to-null bandwidth is $f = 1/T$  

The bandwidth efficiency of a modulation scheme is given as the ratio of the bit rate $R_b$ in bits/second and bandwidth $B$ in Hz, and is used as a figure of merit in the comparison of modulation schemes.
Optical Signal-to-Noise Ratio

In conventional systems the electrical signal-to-noise power ratio is a key parameter in determining the reliability of transmission on a channel. It is most often reported in decibels, and is a measure of the amount of signal power in excess of the noise floor.

In wireless optical systems a similar measure can be defined using the average optical power and electrical noise in the system. The average optical power is constrained in the optical system and is a function of the amplitude of the intensity signal. The noise at the front-end is Gaussian, and exists in the electrical domain. Combining optical and electrical power expressions defines the optical signal-to-noise ratio as,

\[
\text{OSNR} \Delta = 10 \log_{10} \left( \frac{P}{2\sigma_n^2 B} \right)
\]

where, \( B \) is the bandwidth of the system.

3.2 Binary Level Modulation Schemes

Most popular schemes in use for optical networks rely on binary level modulation schemes. These modulation techniques transmit information in each symbol period through the variation of two intensity levels. An advantage of these schemes is that they typically have simple and inexpensive implementations. This section presents an analysis of two well known modulation schemes on optical links, on-off keying (OOK) and pulse position modulation (PPM).

3.2.1 On-Off Keying

On-off keying is a popular modulation scheme not only in wireless infrared links, but also in a wide variety of data communication applications. In many conventional channels, this scheme is also known as non-return-to-zero (NRZ) encoding.

On-Off keying is a binary level modulation scheme consisting of two symbols. In each symbol interval one of the two symbols is chosen with equal probability. The transmitted symbols consist of constant intensities of zero or \( 2P \) through the symbol time. The signal can be represented by the basis function for OOK, \( \phi_{\text{OOK}}(t) \), illustrated in Figure 3.2. This basis function is defined as

\[
\phi_{\text{OOK}}(t) = \sqrt{\frac{1}{T_s}} \text{rect} \left( \frac{t - T_s/2}{T_s} \right).
\]
Using this basis function, an expression for the time varying optical intensity is

\[ x(t) = \sum_{k=-\infty}^{\infty} 2P\sqrt{T_s}a_k\phi_{OOK}(t - kT_s) \]  

(3.20)

where \( a_k \in \{0, 1\} \). Since the basis function is non-negative in the symbol period, and since only non-negative multipliers are used, \( x(t) \) satisfies the non-negativity constraint. The average amplitude of \( x(t) \) is set at \( P \) due to the distribution of the data symbols and the scaling of the multipliers.

The constellation for OOK consists of two points in a one dimensional space as illustrated in Figure 3.2. The probability of a bit error can be determined using the previously defined framework as

\[ P_e = Q\left( \frac{RP}{\sqrt{R_b}\sigma_n^2} \right) \]

since the bit rate \( R_b = 1/T_s \) and \( P_e(\text{sym}) = P_e \) in this case. The constellation figure of merit can be calculated, following (3.14) as

\[ \text{CFM}_{OOM} = \frac{2R}{\sqrt{R_b}}. \]

The power spectral density of OOK was calculated to determine the bandwidth of the system. Using (3.17), the power spectral density of OOK was found to be

\[ G_{OOK}(f) = (RP)^2\delta(f) + (RP)^2T_s\text{sinc}^2(\pi f T_s), \]

where \( \text{sinc}(x) = \sin(x)/x \). Figure 3.3 contains a plot of the continuous portion of power spectral density of OOK, indicating that the bandwidth, as defined earlier, is \( 1/T_s \) Hz.
Section 3.2. Binary Level Modulation Schemes

Figure 3.3: The continuous portion of the power spectral density of on-off keying, for $RP = 1$ and $T_s = 1$.

This corresponds to a 100% excess bandwidth over the minimum specified by the Nyquist criterion. The bandwidth efficiency of this scheme is thus $R_b/B = 1$ bit/s/Hz.

3.2.2 Pulse Position Modulation

Pulse position modulation (PPM) is a standard modulation technique used in optical communications. The IrDA specification for the 4 Mbps short distance wireless infrared link uses a 4-PPM modulation scheme \[1\].

$L$-PPM is an $L$-ary modulation scheme using two distinct intensity levels. Each symbol interval is divided into a series of $L$ subintervals, or chips. Information is sent by transmitting a non-zero optical intensity in a single chip, while other chip intervals remain dark. Each of the chips is non-overlapping in time, and so each symbol is orthogonal to all the others and is represented by its own basis function. The basis functions, $\phi_i(t)$ for $i \in \{1, 2, \ldots, L\}$, take the form

$$\phi_i(t) = \sqrt{\frac{L}{T_s}} \text{rect} \left( \frac{t - (T_s/L)(i - 1/2)}{T_s/L} \right)$$

(3.21)

Figure 3.4 shows an example of the basis functions for 4-PPM.

The signal space of $L$-PPM is an $L$ dimensional Euclidean space with a single constellation point on each of the $L$ axes. A time domain representation of the intensity
Section 3.2. Binary Level Modulation Schemes

The waveform as sent on the channel is

\[ x(t) = \sum_{k=-\infty}^{\infty} LP \sqrt{\frac{T_s}{L}} \phi_{a_k}(t - kT_s) \]

where \( a_k \) chooses the basis function according to the symbol to be sent with \( a_k \in \{1, 2, \ldots, L\} \).

The pulses remain non-negative for all time due to their construction. The average optical power of each symbol is fixed at \( P \) by setting the peak value of each symbol to \( LP \). The information in this system is transmitted in the position of the pulse within the symbol interval.

The probability of error for this modulation scheme can be calculated by noting that each constellation point is orthogonal to all others and that each constellation point is equidistant to all the other points. Based on this geometry the probability of symbol error can be found as

\[ P_e(\text{sym}) \approx (L - 1) \cdot Q \left( RP \sqrt{\frac{L}{2R_s\sigma_n^2}} \right). \]

Due to the orthogonality of all the points in the space, and the fact they are equiprobable, the probability of symbol error can be converted to a probability of bit error by multiplying...
by a factor of $\frac{L}{2}/(L - 1)$ [48]. Combining these results gives the probability of bit error as

$$P_e \approx \frac{L}{2} \cdot Q \left( R P \sqrt{\frac{L \log_2 L}{2R_b \sigma_n^2}} \right)$$

since the bit rate $R_b = R_s \log_2 L$ in this case. The CFM for PPM can be written directly from the probability of symbol error expression as,

$$\text{CFM}_{PPM} = R \sqrt{\frac{2L \log_2 L}{R_b}}.$$

The characteristics of PPM are well suited to fibre applications. Fibre optical links are limited by the thermal noise of the electronics, as noted in Chapter 2. However, insight into the fundamental workings of optical transmission are gained by considering the optical channel apart from the electronics. In this regime, the main impediment to communication is due to the discrete nature of photons and electrons. The random arrival times of photons, the random motion of electrons across the potential barrier include a random, yet signal dependent noise source in the link. In these types of links, the receiver is modelled as an electron counter over discrete intervals of time. If $K$ is a random variable indicating the number of electrons received in a given interval, the probability of receiving $k$ electrons is modelled as the Poisson distribution $\Pr\{K = k\} = \exp(-\lambda)\lambda^k/k!$, where $\lambda$ is the average number of received electrons per interval.

Significant work has been done on the consideration of pulses transmitted on Poisson counting channels. Indeed, this scenario is merely a low intensity illumination case of the Gaussian noise limited channel considered in this chapter. For modulation schemes with only two symbols and in which the average output amplitude is limited, the optimum signal set has been derived using a signal-to-noise ratio criterion similar to a matched filter relationship. If the symbol period is broken into $J$ counting intervals, each signal can be represented as a vector of $J$ electron counts. This optimum signal set is represented by the count vectors

$$\mathbf{s}_0 = \mathbf{0},$$

$$\mathbf{s}_1 = \{ \mathbf{v} \in \mathbb{R}^J \mid v_j = S \cdot \delta_{j_0}, \mathbf{v} = \{ v_j \}, j = 1, 2, \ldots, J \}$$

where $\mathbf{s}_0$ and $\mathbf{s}_1$ represent the two symbol vectors, $S$ is a scaling factor and $\delta_{j_0}$ is 1 for some $j_0 \in \{1, 2, \ldots, J\}$ and zero otherwise [49, 50]. Appendix A contains a simple proof which confirms this result using a distance criterion.
This work has been extended to prove the optimality of $L$-ary modulation techniques as well. It has been shown that the following intensity set of $L$ waveforms is optimal for average amplitude limited signalling on a Poisson counting channel:

$$s_i = \{ v \in \mathbb{R}^L \mid v_j = S \cdot \delta_{ij}, v = \{v_j\}, j = 1, 2, \ldots, L \}$$

for $i = 1, 2, \ldots, L$ where $S$ is a scaling factor and $\delta_{ij}$ is the Kronecker delta function [51]. Some have even gone so far as to conjecture that the above modulation scheme may be optimal in a global sense for a Poisson counting receiver, although no proof was provided [40].

Therefore, for channels in which the average output amplitude is limited, significant theoretical work has been done to establish that concentrating symbol energy in disjoint short intervals is beneficial. $L$-PPM approximates these “optimal” pulses to provide a large separation in the signal space, and hence a lower $P_e$, for a given average optical power. The high power efficiency of PPM can also be explained using the channel constraints along with the signal space concepts, as demonstrated in Section 3.1.3. For a given $P$, PPM maximises the vector coordinate values in (3.8) while keeping the average of the basis function low. The average value of the basis functions is equal and set at $\sqrt{T_s/L}$, which decreases with increasing dimensionality. The magnitude of the coordinate factors increase as $L$. As a result, for a given $P$, the distance between signal points increases with $L$ to yield a lower probability of error. PPM enjoys wide application in wireless and wired optical networks due to its high power efficiency.

The price paid for this power efficiency is a reduction in the bandwidth efficiency of the scheme. Qualitatively, the concentration of power in short pulses spreads the frequency spectrum over a wide range. Using (3.17), the power spectral density of $L$-PPM is

$$G_{PPM}(f) = (RP)^2 \delta(f) + (RP)^2 T_s \text{sinc}^2 \left( \frac{\pi f T_s}{L} \right) \left[ 1 - \frac{1}{L^2} \left( L + 2 \sum_{i=1}^{L-1} (L - i) \cos \left( \frac{2\pi f T_s}{L} i \right) \right) \right].$$

Figure 3.5 contains an example of the continuous power spectral density for 4-PPM. This result can be generalised to show that the occupied bandwidth is $B = L/T_s = LR_s$. Following this expression, the bandwidth efficiency is given as

$$\frac{R_b}{B} = \frac{1}{L} \log_2 L \frac{\text{bits/s}}{\text{Hz}}$$

since $R_s = R_b / \log_2 L$. 

\[Section 3.2. Binary Level Modulation Schemes\]
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3.2.3 Comparisons and Conclusions

The operation of binary level modulation schemes can be compared versus the derived “optimal” pulses in Appendix A. Although these pulses were derived for a Poisson photon counting channel, they also represent the optimal set of two pulses where the basis function is constant during each of the $M$ subintervals of $T_s$.

The optimum pulses in (A.7) with $M = 1$ correspond to the symbols for OOK in (3.20). Therefore, OOK is the optimum pulse for a counting channel with one counting interval. Stated differently, OOK is the optimum binary modulation technique where the two symbols must be constant in the symbol period.

2-PPM can be compared against the pulses in Appendix A since it is constant over a half symbol period. Assuming that the two schemes are transmitting at the same bit rate $R_b$, the bandwidth of each scheme can be made equal by setting the parameter $M$ in (A.7) is set to 2. The gain is computed as in (3.16) to yield,

$$\Gamma_{\text{opt}/2-\text{PPM}} = 1.5 \, \text{dB}.$$ 

Therefore, the $M = 2$ pulse in (A.7) is 1.5 dB more power efficient that 2-PPM for the same bandwidth, and probability of error. Therefore, 2-PPM is sub-optimal and requires a greater complexity to implement than the optimal $M = 2$ pulse set.

A comparison of OOK and 2-PPM, shows that $\Gamma = 0 \, \text{dB}$, while 2-PPM requires...
Section 3.3. Multilevel Modulation Schemes

Twice the bandwidth of OOK for the same $R_b$. Therefore, the advantage of using PPM comes in using it with greater than two symbols.

Binary level modulation schemes provide an efficient means to implement modulation for optical links which are limited by the amount of transmitted power. In fibre applications, which are primarily loss limited, these schemes are popular due to their high average optical power efficiency.

The price paid for this advantage is that the bandwidth efficiency of the schemes is poor. For fibre links, where the medium has thousands of gigahertz of unused bandwidth, this is an acceptable trade-off. However, inexpensive, wireless optical links do not share this luxury. In order to be able to attain high speed data transmission with current optoelectronics, bandwidth efficiencies greater than 1 are required. This motivates the study of multilevel modulation schemes for wireless optical channels.

3.3 Multilevel Modulation Schemes

Traditional approaches to modulation for the optical intensity channel centre on developing power efficient schemes. The bandwidth of the transmitted symbols is not a critical issue due to the wide bandwidth available in fibre systems. However, the wireless optical channel relies on the use of inexpensive devices which are severely limited in available bandwidth, as shown in Chapter 2. As a result, for wireless optical links, the bandwidth efficiency of the modulation scheme is a parameter of critical importance.

Multilevel modulation techniques transmit symbols in which the intensity values are continuous in a range or take on a set of values. The advantage of these schemes is that they provide a higher bandwidth efficiency than binary level techniques, since data is transmitted in the amplitude level as well as the structure of the basis functions.

This section presents an analysis of two classical multilevel schemes for the intensity modulated channel: pulse amplitude modulation (PAM) and quadrature pulse amplitude modulation (QAM).

3.3.1 Pulse Amplitude Modulation

Pulse amplitude modulation is a classical modulation scheme which can be adapted to operate on an intensity modulated optical channel. PAM is a generalisation of on-off keying from a set of two symbols to a set of $L$ symbols. The basis function for $L$-PAM is
identical to the basis function for OOK in (3.19). The basis function for PAM is illustrated in Figure 3.6, and takes the form

\[ \phi_{PAM}(t) = \sqrt{\frac{1}{T_s}} \text{rect} \left( t - \frac{T_s}{2} \right) \]

In each symbol period one of \( L \) scaling factors are chosen with equal probability. These scaling factors are chosen so that for an equiprobable distribution of transmitted symbols, the average output amplitude is fixed at \( P \). Unlike PPM, where the average of the modulation scheme is independent of the symbols chosen, PAM requires prior knowledge of the data distribution in order to ensure the average optical power is limited. The non-negativity constraint of the optical intensity channel is met by choosing \( L \) non-negative scaling factors. The time varying intensity signal corresponding to PAM which satisfies the channel constraints is

\[ x(t) = \sum_{k=-\infty}^{\infty} 2P \frac{L}{L-1} \sqrt{T_s} a_k \phi_{PAM}(t - kT_s) \]

where \( a_k \in \{0, 1, 2, \ldots, (L - 1)\} \). Note that for \( L=2 \) this expression is the same as the intensity waveform specified in (3.20) for OOK. As a result, OOK can be treated as a special case of PAM.

The probability of symbol error can be determined by constructing the constellation for this modulation scheme. Since all signal points can be expressed in terms of one basis function, the constellation is one dimensional. The \( L \) constellation points are evenly spaced on the axis as is illustrated in Figure 3.6. The probability of symbol error can be
determined by assuming each symbol is sent with equal probability as

\[ P_e(\text{sym}) = \frac{2}{L} (L - 1) \cdot Q \left( \frac{RP}{L - 1} \sqrt{\frac{1}{R_n \sigma_n^2}} \right). \]

Assuming Gray coding is used where adjacent symbols differ by a single binary digit, the probability of bit error is approximately

\[ P_e \approx \frac{2}{L \log_2 L} (L - 1) \cdot Q \left( \frac{RP}{L - 1} \sqrt{\frac{\log_2 L}{R_b \sigma_n^2}} \right) \]

since the bit rate \( R_b = R_s \log_2 L \). The CFM can be determined using (3.14) as,

\[ \text{CFM}_{PAM} = \frac{2R}{L - 1} \sqrt{\frac{\log_2 L}{R_b}}. \tag{3.22} \]

The power spectrum of PAM can be calculated using (3.17). Assuming equiprobable symbol distribution the psd for \( L \)-PAM is

\[ G_{PAM}(f) = (RP)^2 \delta(f) + (RP)^2 T_s \frac{L + 1}{3(L - 1)} \text{sinc}^2(\pi f T_s). \]

The continuous portion of the power spectral density of \( 5 \)-PAM is plotted in Figure 3.7. As is the case with all modulation schemes for the optical intensity channel, the DC component is fixed at \( P \). The bandwidth of this scheme is set at \( B = R_s \), as was the case in OOK. The addition of multiple levels per symbol allows for more than 1 bit of information per transmitted symbol. The bandwidth efficiency of PAM on the intensity modulated channel is

\[ \frac{R_b}{B} = \log_2 L \frac{\text{bits/s}}{\text{Hz}}. \tag{3.23} \]

### 3.3.2 Quadrature Pulse Amplitude Modulation

Quadrature pulse amplitude modulation (QAM) is a popular modulation scheme in conventional channels such as high speed wired channels and radio frequency radio channels. The basic structure of QAM is unaltered in the wireless optical channel, however, changes must be made to ensure the channel constraints are met.

The \( L^2 \) symbols of \( L^2 \)-QAM consist of an in-phase and quadrature component basis function which are orthogonal to each other, as shown in Figure 3.8. The basis functions,
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Figure 3.7: The continuous portion of the power spectral density of 5-PAM, for $RP = 1$ and $T_s = 1$.

$\phi_I(t)$ and $\phi_Q(t)$, represent the in-phase and quadrature components of the data signal and take the form

$$
\begin{align*}
\phi_I(t) &= \sqrt{2 \over T_s} \cos (\omega_p t) \text{rect} \left( t - T_s / 2 \right) \\
\phi_Q(t) &= \sqrt{2 \over T_s} \sin (\omega_p t) \text{rect} \left( t - T_s / 2 \right)
\end{align*}
$$

where $\omega_k = 2p\pi / T_s$ for any $p \in \mathbb{N}$. In each symbol instant, independent data is used to modulate the two basis functions. As a result, each basis function is multiplied by a series of $L$ amplitude values to comprise the $L^2$ symbols. Taking into account the channel constraints, the time varying intensity function, $x(t)$ can be written as

$$
x(t) = \sum_{k=-\infty}^{\infty} {P \over 2(L-1)} \sqrt{T_s} (a_k \phi_I(t - kT_s) + b_k \phi_Q(t - kT_s)) + P \cdot \text{rect} \left( t - T_s (k + 1/2) \right).
$$

for $a_k, b_k \in \{-(L-1), -(L-3), -(L-5), \ldots, (L-1)\}$. Since the basis functions $\phi_I(t)$ and $\phi_Q(t)$ take on negative amplitudes at intervals within the symbol period, a DC bias must be added to ensure the non-negativity constraint is met. QAM adds a fixed bias of $P$ to every symbol. This added DC bias is rejected by the matched filter front end since the basis functions have an average value of zero. The addition of a fixed DC bias also ensures that the average power constraint in (3.2) is met, independent of the data distribution. The maximum amplitude value for the data is fixed at $P\sqrt{T_s}/2$, and the minimum amplitude
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Figure 3.8: Basis function (a) and $L^2 = 16$ point constellation (b) of quadrature pulse amplitude modulation (for $\omega_p = 2\pi/T_s$).

value for these symbols is 0. However, for symbols with smaller swing values the minimum value of the symbol is greater than zero. This is the condition of excess symbol power discussed in Section 3.1.3. The amount of bias optical intensity added to small swing signal is excessive to support the signal swing. As a result of this excess average optical power transmitted, less of the average power budget of (3.8) is left to support the information bearing, time varying portion of the symbol signal. This inefficiency is quantified in the following section by comparing it to PAM.

The constellation of QAM is a two dimensional regular array of points, as illustrated in Figure 3.8. The minimum spacing between the points is fixed by the amount of DC bias added due to the non-negativity constraint. It can be shown from (3.25), that

$$d_{\min} = \frac{RP}{L - 1} \sqrt{\frac{2 \log_2 L}{R_b}}.$$

The CFM is then,

$$\text{CFM}_{QAM} = \frac{R}{L - 1} \sqrt{\frac{2 \log_2 L}{R_b}}.$$
The probability of symbol error can be determined by applying the union bound approximation in (3.13). By computing the average number of neighbours \(d_{\text{min}}\) away from each constellation point, an estimate for \(P_e(\text{sym})\) can be formed as

\[
P_e(\text{sym}) \approx \frac{4(L - 1)}{L} \cdot Q \left( \frac{RP}{L - 1} \sqrt{\frac{1}{4R_s\sigma_n^2}} \right).
\]

Using the Gray coding approximation, the probability of bit error for \(L^2\)-QAM is found to be

\[
P_e \approx \frac{2(L - 1)}{L \log_2 L} \cdot Q \left( \frac{RP}{L - 1} \sqrt{\frac{\log_2 L}{2R_b\sigma_n^2}} \right),
\]

where \(R_b = R_s \log_2 L^2\).

The spectral characteristics of QAM can be investigated by computing the power spectral density via (3.17). The power spectral density for the \(L^2\)-QAM signal in (3.25) is

\[
G_{\text{QAM}}(f) = (RP)^2 \delta(f) + (RP)^2 T_s \frac{L + 1}{12(L - 1)} \left[ \sin^2 \left( \pi T_s \left( f - \frac{1}{T_s} \right) \right) + \sin^2 \left( \pi T_s \left( f + \frac{1}{T_s} \right) \right) \right].
\]

Figure 3.9 illustrates an example of the continuous portion of the psd for 25-QAM. QAM is a passband modulation scheme with little signal energy at low frequencies. This is a beneficial trait in channels which are corrupted by low frequency interferers, such as fluorescent lighting ballasts in wireless optical channels (see Section 2.4).

The bandwidth of \(L^2\)-QAM, as defined by the frequency of the first spectral null, is \(B = 2R_s\) Hz. Following this result, the bandwidth efficiency of \(L^2\)-QAM is

\[
\frac{R_b}{B} = \log_2 L \frac{\text{bits/s}}{\text{Hz}}.
\]

which is identical to the bandwidth efficiency of \(L\)-PAM in (3.23). Although the bandwidth required for \(L^2\)-QAM is double that of \(L\)-PAM, the entropy of the QAM symbols is double that of PAM, which satisfies intuition.

### 3.3.3 Comparisons and Conclusions

For the same bandwidth, bit rate and \(P_e(\text{sym})\), \(\Gamma_{L\text{-PAM}/L^2\text{-QAM}} = 1.5\) dB. This is in contrast to a conventional channel, in which \(L\)-PAM and \(L^2\)-QAM have the same energy efficiency. The loss in power efficiency in the optical QAM constellation is due to the fact that the fixed DC bias added to every symbol is excessive for the signals with small
multilevel modulation schemes

amplitude. The excessive DC content of low amplitude symbols transmits no information
and reduces the power efficiency of the scheme as shown in Section 3.1.3.

The loss of average power efficiency in QAM over PAM can also be explained by
viewing QAM as a series of two orthogonal PAM signals. Since the average power is fixed
at $P$ by the DC bias, the peak amplitude of the resulting modulated signal is $P$. According
to this constraint, the maximum value that any signal in either in-phase or quadrature
directions can take is $P/\sqrt{2}$. Therefore, each phase of $L^2$-QAM behaves as an $L$-PAM
signal with its signal points distributed evenly between $-P/\sqrt{2}$ and $P/\sqrt{2}$. It is easy to
show that in this case, the distance between constellation points in this $L$-PAM signal is a
factor of $\sqrt{2}$ smaller than in the $L$-PAM constellation in Section 3.3.1. Therefore, $L^2$-QAM
constellation is $10 \log_{10} \sqrt{2} = 1.5$ dB less power efficient than $L$-PAM for the same $P_e$ and
bandwidth efficiency.

The comparison between $L$-PAM and $L$-PPM is necessary to justify the use of a
multilevel scheme over a binary system. Computing the gain of the two schemes assuming
the same $R_b$ in both cases yields

$$\Gamma_{PPM/PAM} = 5 \log_{10} \left( L(L-1)^2 \right) - 1.5 \text{ dB},$$

$^2$Note that the use of sinusoids as basis functions improves the power efficiency of QAM, since the peak
value of the sum of sinusoids is the geometric sum of the component amplitudes, and not the linear sum. Therefore, the maximum of the sum of phases adds according to Pythagorean identity, $\sqrt{a_k^2 + b_k^2}$, as in
(3.25), which fixes the maximum value per dimension at $P/\sqrt{2}$ for a DC bias of $P$. 

Figure 3.9: The continuous portion of the power spectral density of 25-QAM, for $RP = 1$
and $T_s = 1$. 

\[ G_{25-QAM}(f) \] 

\[ T_s \cdot f \]
which is greater than or equal to zero for all $L \geq 2$. As the number of symbols increases in either constellation it is clear that $L$-PPM enjoys a significant advantage over $L$-PAM in the area of average power efficiency. For example, for 4-ary signalling 4-PPM has a $10 \log_{10} 4.24 = 6.3$ dB average optical power advantage over 4-PAM.

However, since the wireless optical channel is constrained by bandwidth considerations, the bandwidth of the two schemes must be compared as well. For the same $R_b$, $L$-PPM requires $L$ times more bandwidth than $L$-PAM. So, in the example of 4-ary signalling, although 4-PPM enjoys an average optical power advantage over 4-PAM, it requires four times the bandwidth.

Multilevel schemes achieve the goal of increased bandwidth efficiency at the price of reduced average optical power efficiency when compared to binary level schemes. Since bandwidth restrictions on the wireless optical channel are a severe limitation for high speed communications, highly bandwidth efficient schemes are required. At the same time, in order to have a highly robust link, a high average power efficiency is needed. A modulation scheme which conserves the bandwidth efficiency of $L$-PAM while improving on its bandwidth efficiency is necessary for the high speed short distance infrared link.

### 3.4 Adaptively Biased QAM

In this section we present a new modulation scheme for the wireless infrared channel called Adaptively Biased QAM (AB-QAM). It improves the average optical power efficiency of current bandwidth efficient techniques while maintaining the same bandwidth efficiency for a small increase in complexity [52].

This section introduces the basic structure of AB-QAM and shows that it meets all the constraints imposed by the optical channel. The methodology used in the development of AB-QAM is highlighted by examining a single phase of the modulation scheme. The spectral characteristics of the modulation scheme are described via the power spectral density to demonstrate the bandwidth efficiency of this scheme. The impact of fluorescent interferers as well as the use of different pulse shapes is also briefly discussed. The section concludes with comparison of AB-QAM to the multilevel techniques described earlier.
3.4.1 Modulation Scheme Definition

The operation of AB-QAM is best described by using the signal space framework of Section 3.1.2. The basis functions follow the methodology laid out in Section 3.1.3, where one basis function defines the symbol average and the others vary in the symbol period. The basis functions of AB-QAM are illustrated in Figure 3.10, and can also be represented mathematically as

\[
\phi_1(t) = \sqrt{\frac{1}{T_s}} \left( \text{rect} \left( \frac{t - T_s/2}{T_s} \right) - 2 \text{rect} \left( \frac{t - T_s/2}{T_s/2} \right) \right)
\]

\[
\phi_2(t) = \sqrt{\frac{1}{T_s}} \left( \text{rect} \left( \frac{t - T_s/2}{T_s} \right) - 2 \text{rect} \left( \frac{t - 3T_s/4}{T_s/2} \right) \right)
\]

\[
\phi_3(t) = \sqrt{\frac{1}{T_s}} \text{rect} \left( \frac{t - T_s/2}{T_s} \right).
\]

The basis function \(\phi_3(t)\) determines the average of each symbol, while \(\phi_1(t)\) and \(\phi_2(t)\) have zero average value in the symbol period.

The collection of scaled basis functions forms all of the signals of the modulation scheme. AB-QAM transmits one of \(L^2\) symbols per \(T_s\), chosen independently and with equal probability. The transmitted intensity waveform for \(L^2\)-AB-QAM is denoted \(x(t)\) and takes the form,

\[
x(t) = \sum_{k=-\infty}^{\infty} \frac{AP}{(L-1)} \sqrt{T_s} (a_k \phi_1(t - kT_s) + b_k \phi_2(t - kT_s) + c_k \phi_3(t - kT_s))
\]

for \(a_k, b_k \in \{-L-1, -(L-3), -(L-5), \ldots, (L-1)\}\), \(c_k = |a_k| + |b_k|\), and

\[
A = \begin{cases} 
\frac{L}{T+1} & \text{for } L^2 \text{ odd;} \\
\frac{L-1}{T} & \text{for } L^2 \text{ even.}
\end{cases}
\]

The information in this modulation scheme is carried in the independent choice of \(L\) values for each \(a_k\) and \(b_k\). The information sent on the \(\phi_3\) basis is not independent, but is used to satisfy the non-negativity constraint of the channel. The magnitude of the DC bias added to the intensity waveform is equal to the sum of the amplitudes of the signals sent in the \(\phi_1\) and \(\phi_2\) dimensions. This adaptive bias term added to each symbol is analogous to the general case in (3.10), since the minimum amount of DC bias is added to each intensity symbol to satisfy the non-negativity constraint.

\[3\] If the signals in (3.26) are written in terms of the 4-ary basis functions in (3.21), the co-ordinates form three rows of the \(4 \times 4\) Hadamard matrix. The fourth row of the matrix, which is orthogonal to the other three, is not used since it requires twice the bandwidth.
Section 3.4. Adaptively Biased QAM

The average power of the modulation scheme is dependent on the data distribution. The scaling term $A$ is set to ensure that for equiprobable symbol selection the average power is set at $P$. The need for different expressions for $A$ in the case of odd and even sized constellations is due to the fact that the constellation points are evenly spaced in the $\phi_1$ and $\phi_2$ directions. For even sized constellations, there is no zero symbol for either $a_k$ or $b_k$, which is reflected by the change in the scaling factor to maintain the same average optical transmitted power. The derivation of these terms is given in more detail in the following section.

3.4.2 Development

The motivation for the development of AB-QAM is to improve the optical power efficiency of multilevel modulation schemes while conserving their bandwidth efficiency. This section presents the central factors which improve the power efficiency of AB-QAM over traditional multilevel schemes. The structure of the modulation scheme is justified by considering a simple example, and then generalising it to the AB-QAM case.
Adaptively Biased PAM

The key ideas which lead to the power efficiency of AB-QAM can be analysed using a single phase of the modulation scheme. Suppose a modulation scheme, named adaptively biased PAM (AB-PAM), is defined using basis functions $\phi_2(t)$ and $\phi_3(t)$ in (3.26) and Figure 3.10. The optical intensity waveform for this modulation scheme is

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{AP}{(L-1)} \sqrt{T_s} (b_k \phi_2(t - kT_s) + c_k \phi_3(t - kT_s))$$

for $b_k \in \{-L,-(L-3),-(L-5),\ldots,(L-1)\}$, $c_k = |b_k|$, and $A$ is a scaling factor used to ensure that the average optical power constraint is met. The fundamental idea of AB-PAM is the same as that for AB-QAM: in each symbol instant, a scaled time varying pulse is sent, and the minimum amount of DC bias is added to ensure that the non-negativity constraint in (3.10) is met. It was also shown in (3.12) that this intuitive notion of sending zero excess average optical power for improved power efficiency was justified.

The scaling factor $A$ determines the average power of AB-PAM. The scaling factors for $\phi_2(t)$ are evenly spaced from $-(L-1)$ to $(L-1)$ to ensure the same $d_{\text{min}}$ between adjacent symbols. Due to this fact, schemes with an odd number of signal points will have a zero symbol, while constellations with an even number of points will not have a zero symbol. Due to this asymmetry, the value of $A$ for the AB-PAM modulation scheme with $L$ constellation points can be found to be,

$$A = \begin{cases} 
\frac{2L}{L+1} & : \text{for } L \text{ odd;} \\
\frac{2(L-1)}{L} & : \text{for } L \text{ even.}
\end{cases}$$

If the symbols are detected using a filter matched to $\phi_2(t)$ only, then the minimum distance between the constellation points is a function of the scaling factor $A$ and the number of points $L$ for a fixed $P$, and $R_b$. The CFM for AB-PAM can be written as,

$$\text{CFM}_{AB-PAM} = \frac{2RA}{L-1} \sqrt{\frac{\log_2 L}{R_b}}.$$  

The gain of AB-PAM over PAM using the information from the $\phi_2$ dimension only, $\Gamma_{AB-P}^{(1)}$, can be determined using the CFM of PAM in (3.22) and (3.30) as,

$$\Gamma_{AB-P}^{(1)} = \begin{cases} 
3 + 10 \log_{10} \left( \frac{L}{R_b} \right) \text{ dB} & : \text{for } L \text{ odd} \\
3 + 10 \log_{10} \left( \frac{L-1}{L} \right) \text{ dB} & : \text{for } L \text{ even}
\end{cases}$$
which is greater than zero for all $L > 1$. For large values of $L$, the gain $\Gamma_{AB-PAM}^{(1)}$ approaches 3 dB. The gain of AB-PAM over PAM is due to the adaptive biasing of each symbol which allows for positive as well as negative scaling factors for $\phi_2(t)$. The improvement in power efficiency can be seen by noting that the average optical power of the modulation scheme is the expected value of symbol average as defined in (3.8). As a result, for a given $d_{\text{min}}$, the power efficiency of a modulation scheme is improved by minimising the individual symbol averages.

For $L$-PAM, each symbol has a unique average value, and the $L$ constellation points are evenly spaced between 0 and $2RP\sqrt{T_s}$. In $L$-AB-PAM the constellation points are evenly distributed in the range $[-RAP\sqrt{T_s}, RAP\sqrt{T_s}]$. As a result, half of the constellation points arise due to positive scaling factors, while the other half arise from negative scaling factors (ignoring the zero symbol). The symbol average, however, is dependent on the magnitude of the scaling factor through the relation $c_k = |b_k|$. Therefore, except for the zero symbol, there are two constellation points for each unique symbol average. Figure 3.11 shows the distribution of symbol average values for a PAM and AB-PAM constellation when $d_{\text{min}}$ is fixed. As a result, for the same $d_{\text{min}}$, AB-PAM places a greater number of constellation points at a lower average optical power than does the same sized PAM constellation. In the limit, this pairing of constellation points at each unique symbol average value ideally halves the amount of average optical power required, or provides a gain of 3 dB.

Therefore, AB-PAM picks the constellation points so as to minimise the average value of each symbol. Looked at in a different fashion, for the same average optical power,
AB-PAM will have an asymptotic doubling of $d_{\text{min}}$ over PAM for large constellations. Figure 3.12 illustrates the increase in $d_{\text{min}}$ in the $\phi_2$ direction of 5-AB-PAM over 5-PAM, for a fixed symbol interval and average optical power.

Traditional approaches to modulation for wireless optical channels form a signal set at the transmitter and add a fixed DC bias to ensure the signal remains non-negative. At the receiver, the average of each symbol is removed, and that data is detected [23, 4]. In the case of AB-PAM, the symbol average provides information which can be used to improve the estimate of the sent symbol.

The gain $\Gamma_{\text{AB-P}}^{(1)}$ considers only the information transmitted in the $\phi_2$ dimension. However, unlike PAM, AB-PAM is a two-dimensional modulation scheme defined by basis vectors $\phi_2$ and $\phi_3$. Adding matched filters to both of the basis functions allows for the information in both dimensions to be extracted. The information in the $\phi_3$ dimension is a consequence of the adaptive biasing of each symbol, and as a result is not independent of the data sent in the $\phi_2$ dimension. The symbol bias information in the $\phi_3$ dimension provides a degree of redundancy or signal space diversity to the detection process, which increases the $d_{\text{min}}$ over the one-dimensional case. Figure 3.13 presents an example of the two-dimensional constellations for $L$-odd and $L$-even AB-PAM constellations.

For the $L$-odd case, the amount of DC bias added to each symbol is identical to its amplitude. As a result the minimum distance is improved by a factor of $\sqrt{2}$ over the one-dimensional case treated earlier in this section. In the case of $L$-even, the minimum distance between constellation points is unaffected by the use of information in the $\phi_3$ direction. This is due to the fact that there is no zero symbol. In the $L$-odd constellation, the zero symbol forces adjacent symbols in the $\phi_2$ direction to have differing symbol averages. For $L$-even
Section 3.4. Adaptively Biased QAM

\[
\begin{align*}
\phi_3 & \quad 5-\text{AB-PAM} \\
0 & \quad d_{\text{min}} = \sqrt{2}\Delta
\end{align*}
\]

\[
\begin{align*}
\phi_3 & \quad 6-\text{AB-PAM} \\
0 & \quad d_{\text{min}} = \Delta
\end{align*}
\]

Figure 3.13: Constellations of 5-AB-PAM and 6-AB-PAM in the \(\phi_2\) and \(\phi_3\) dimensions. Note the bold lines indicate the \(d_{\text{min}}\) of the constellation, and \(\Delta\) indicates the one-dimensional minimum distance.
Section 3.4. Adaptively Biased QAM

the absence of this zero symbol causes the minimum distance for the pair of points with lowest symbol average to be unchanged from the one dimensional case.

The gain of AB-PAM over PAM using the information contained in the two dimensions is denoted $\Gamma^{(2)}_{AB-P}$. Using the definition of $\Gamma^{(1)}_{AB-P}$ in (3.31), the gain can be written as

$$
\Gamma^{(2)}_{AB-P} = \begin{cases} 
\Gamma^{(1)}_{AB-P} + 1.5 \text{ dB} & : \text{for } L \text{ odd;} \\
\Gamma^{(1)}_{AB-P} & : \text{for } L \text{ even.}
\end{cases}
$$

(3.32)

From AB-PAM to AB-QAM

Although AB-PAM offers a significant improvement in power efficiency over PAM, it suffers from a reduction in the bandwidth efficiency. It can be shown that the bandwidth efficiency of AB-PAM is half of that for PAM. Since bandwidth efficiency is a key parameter for inexpensive wireless optical links, steps must be taken to preserve the bandwidth efficiency of PAM and maintain some of the power efficiency improvement of AB-PAM.

In order to recoup the bandwidth efficiency of AB-PAM another data bearing signal is added in quadrature to $\phi_2$. The basis function $\phi_1(t)$ in (3.26) is orthogonal to $\phi_2(t)$ and $\phi_3(t)$, and has the same bandwidth requirements as $\phi_2(t)$. As a result, AB-QAM is formed by composing two information bearing signals in quadrature, and then adding sufficient DC bias to ensure the non-negativity constraint is met. This preserves the basic AB-PAM framework while providing the same bandwidth efficiency as PAM.

The cost associated with this method is that the amplitude values allowed for each phase are halved from those allowed in AB-PAM. This is because the amplitudes of the $\phi_1$ and $\phi_2$ phases are linearly related to the DC bias which must be added to ensure non-negativity. Since the two phases send independent data, it is as if two AB-PAM signals were combined to form the AB-QAM signal. This addition of the signals, causes the average of each phase to add as well. As a result, the 3 dB improvement $\Gamma^{(1)}_{AB-P}$ of AB-PAM, is lost in AB-QAM.

3.4.3 Probability of Error Analysis

In order to determine the error performance of AB-QAM, the geometry of the signal space must be investigated. AB-QAM is a three dimensional modulation scheme defined by $\phi_1$, $\phi_2$ and $\phi_3$ in (3.26). The information contained in all three dimensions is used to form the estimate of the received symbol.
One of the basic principles behind AB-QAM is that the DC bias added per symbol is equal to the sum of the magnitudes of the data sent in the $\phi_1$ and $\phi_2$ directions. Ignoring the scaling factors, if $x$ and $y$ are taken to represent the amplitudes of the data symbols in the $\phi_2$ and $\phi_2$ directions respectively, the amount of DC bias added to each symbol is

$$z = |x| + |y|.$$  \hspace{1cm} (3.33)

This expression defines a set of four intersecting planes in $\mathbb{R}^3$ defined by the various permutations of sign for $x$ and $y$ and where $z \geq 0$. Due to the construction of the modulation scheme, all the constellation points of AB-QAM lie on the surface defined in (3.33). Suppose that only one of the bounding planes is considered, $z = x + y$, where $z, x, y \geq 0$. It can be shown that all the AB-QAM constellation points in this bounding plane can be written in terms of the $[x, y, z]^T$ basis vectors,

$$\hat{\nu}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \hat{\nu}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

This definition of basis vectors satisfies intuition since every change in the magnitude of the $x$ or $y$ signal values is accompanied by the same change in DC bias added through the $z$ coordinate. The set of constellation points in the bounding plane in question form a subset of points of the infinite lattice defined by $\hat{\nu}_1$ and $\hat{\nu}_2$. The packing density of the lattice can be found to equal $\pi/\sqrt{12}$, which is the most dense packing possible in two dimensions. This lattice is termed hexagonal, since each constellation point has six neighbours and the distance between any two points is the same throughout the lattice [53]. Generalising, the constellation points in each of the bounding planes of AB-QAM are subsets of the hexagonal lattice in two dimensions. Since the minimum distance between all points in each plane is the same, $d_{\text{min}}$ of the modulation scheme is determined by either the spacing of the lattice points or due to distances between points in adjacent planes at the interface of the four bounding planes.

Figure 3.14 shows the constellation for two AB-QAM schemes. The distance between points in the $\phi_1-\phi_2$ plane is set at $\Delta = 1$. In order to aid in the visualisation of the constellation, the constellation is drawn from two orthogonal views. Due to the symmetry of the constellations, these two views capture the positioning of points in the space. The dotted lines indicate contours which represent a unique bias level in the $\phi_3$ dimension that
is added to allow transmission in the channel. As is the case in AB-PAM, the AB-QAM constellation has different properties for $L^2$ odd and even.

For $L^2$ odd, there are constellation points on the interface between the four bounding planes. This is analogous to the case of AB-PAM where odd sized constellations have a zero symbol. Due to the geometry of the planes, the presence of a symbol at the interface ensures that the minimum distance of the constellation is determined by the lattice spacing parameter. Figure 3.14 shows a 25-AB-QAM constellation with the minimum distances indicated. The hexagonal pattern of $d_{\text{min}}$ transitions due to the underlying lattice is evident in this figure. The constellation points at the interface between the bounding planes has points equidistant from it in two planes. Since the angle formed by the intersection of the planes is $\pi/2$ radians, the distance between points in adjacent planes is always larger than the intra-plane spacing of points.

As a result, using the expression for the optical intensity function in (3.27), the constellation figure of merit for $L^2$-AB-QAM is,

$$\text{CFM}_{AB-QAM} = \frac{4LR}{L^2 - 1} \sqrt{\log_2 L}$$

where $L^2$ is odd. The probability of symbol error can be approximated using (3.13) using the CFM and the average number of neighbours as,

$$P_e(\text{sym}) \approx \frac{6L^2 - 8L + 14}{L^2} \cdot Q \left( \frac{\sqrt{2LRP}}{L^2 - 1} \sqrt{\frac{1}{R_b \sigma_n^2}} \right).$$

Using the assumption that a symbol error generates a single bit error, the probability of bit error can be approximated from (3.35) as

$$P_e \approx \frac{6L^2 - 8L + 14}{2L^2 \log_2 L} \cdot Q \left( \frac{2LRP}{L^2 - 1} \sqrt{\frac{\log_2 L}{R_b \sigma_n^2}} \right).$$

Since each constellation point can have up to six closest neighbours, the use of the Grey coding approximation is not strictly valid for moderate sized constellations, but the impact of this scaling factor is negligible in the low $P_e$ regime.

In the case that the number of constellation points $L^2$ is even, the probability of symbol error is limited by the inter-plane spacing of points. Since there are no symbols on the interface between the bounding planes, points in adjacent planes at the interface do not improve their $d_{\text{min}}$ as a result of using the redundant information in the $\phi_3$ direction.
Figure 3.14: Two views of the constellations for 16-AB-QAM and 25-AB-QAM for $\Delta = 1$, $R = 1$ and $T_s=1$ in each. Bold lines indicate minimum distance between constellation points, while the dotted lines connect points of equal symbol average.
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This is analogous to the case of an even sized AB-PAM constellation. The distance between points at the boundary in the three dimensional signal space is the same as the distance of the points in the $\phi_1-\phi_2$ plane. As a result, there is no gain in exploiting the signal space diversity inherent in AB-QAM for $L^2$-even constellations. The CFM for $L^2$-AB-QAM can be written as,

$$\text{CFM}_{AB-QAM} = \frac{2\sqrt{2}R}{L} \sqrt{\frac{\log_2 L}{R_b}}$$

(3.36)

where $L^2$ is even. The probability of symbol error can be determined approximately as,

$$P_e(\text{sym}) \approx \frac{4}{L} \cdot Q\left(\frac{RP}{L} \sqrt{\frac{1}{R_s\sigma_n^2}}\right) + 6 \cdot Q\left(\frac{\sqrt{2}RP}{L} \sqrt{\frac{1}{R_s\sigma_n^2}}\right).$$

(3.37)

The AB-QAM modulation scheme described in (3.27) was simulated using the channel model of Figure 3.1. The symbol error rates were determined for a variety of constellation sizes. Assuming that the random processes in the simulation are ergodic, the symbol error rate can be used to estimate the probability of symbol error. Figure 3.15 presents a comparison between the simulated symbol error rate and the approximates for $P_e(\text{sym})$ presented in this section. The approximations on $P_e(\text{sym})$ based on the geometry of the constellation closely approximate the simulated error rates.

The gain of AB-QAM over PAM, $\Gamma_{AB-QAM}$, can be determined using the same methodology as was used to compute the gain for AB-PAM. The two dimensional gain,

$$\Gamma_{AB-QAM} = \frac{\text{CFM}_{AB-QAM}}{\text{CFM}_{PAM}}.$$
\( \Gamma_{AB-Q}^{(2)} \), is the gain arising due to the detection of the \( \phi_1 \) and \( \phi_2 \) basis functions. Since the information in the \( \phi_3 \) direction is not independent, detection in the \( \phi_1 \) and \( \phi_2 \) dimensions provides sufficient statistics to extract the symbol. The two dimensional gain for AB-QAM over PAM is

\[
\Gamma_{AB-Q}^{(2)} = \begin{cases} 
1.5 + 10 \log_{10} \left( \frac{L}{L+1} \right) \text{ dB} & : \text{for } L^2 \text{ odd;} \\
1.5 + 10 \log_{10} \left( \frac{L}{L-1} \right) \text{ dB} & : \text{for } L^2 \text{ even.}
\end{cases}
\]

This expression is of similar form as the one dimensional gain for AB-PAM in (3.31). As discussed in Section 3.4.2, the transition from AB-PAM to AB-QAM incurs a 3 dB penalty in the gain. As a result, only the second term of (3.31) remains in the definition of \( \Gamma_{AB-Q}^{(2)} \). An additional 1.5 dB gain is apparent in AB-QAM over PAM due to the fact that, for the same bit rate, the symbol rate of AB-QAM is half that of PAM. As a result of the pulse definitions, the energy of the AB-QAM pulses is \( \sqrt{2} \) times larger than the comparable PAM pulses.

The gain due to the use of the redundant information in the \( \phi_3 \) dimension is apparent only for odd sized constellations, as was the case in AB-PAM. The three dimensional gain of AB-QAM over PAM can be found using the CFM expressions in (3.34), (3.36) and (3.22), and takes the form

\[
\Gamma_{AB-Q}^{(3)} = \begin{cases} 
\Gamma_{AB-P}^{(2)} + 1.5 \text{ dB} & : \text{for } L^2 \text{ odd;} \\
\Gamma_{AB-P}^{(2)} & : \text{for } L^2 \text{ even.}
\end{cases}
\]

Just as in the AB-PAM case, a gain due to signal space diversity is only apparent in odd sized constellations due to the boundary conditions at the intersection of the bounding planes.

### 3.4.4 Coding and Shaping Gain in the AB-QAM Framework

The gain in optical power efficiency provided of AB-QAM over PAM and QAM arrives due to the manipulation of the signal space geometry. The relative spacing between constellation points is varied by extending scaling factors to positive as well as negative values and by exploiting the signal space diversity of the constellation. The AB-QAM constellation is three dimensional, with signal points on four bounding planes. QAM is a two dimensional scheme where the signal points are arranged in a rectangular grid. The improvement of power efficiency due to the manipulation of the geometry of the constellation is termed a \textit{coding gain} [41].
Another means of improving the power efficiency of a modulation scheme is by the selection of constellation points from a lattice to minimise the power requirements. In conventional channels, a constellation is formed as a subset of points chosen from an infinite lattice of points, such that the average energy of the constellation is minimised. For an \( N \)-dimensional modulation scheme, this is done by bounding all the constellation points within an \( N \)-dimensional sphere, since this surface has the lowest average energy encompassing a given volume \([54]\). This gain is known as a *shaping gain* \([41]\).

In optical intensity channels, a similar principle can be applied to minimise the amount of average optical power. Using the framework defined in Section 3.1.3, a single basis function, \( \phi_{\text{avg}} \) in (3.9), is used to determine the average power of each symbol. In order to minimise the average optical amplitude of the modulation scheme, each individual symbol average should be minimised, following (3.8). Since the symbol averages are represented in one dimension, this suggests that all symbols should occupy the lowest average power level possible. Therefore, determining the optimum enclosing boundary for intensity channels is a one dimensional problem, and symbol points must be chosen from the underlying lattice such that they occupy the lowest possible coordinates in the \( \phi_{\text{avg}} \) dimension.

Constellation shaping can be applied to the array of points defined for odd sized AB-QAM constellations. These constellations consist of points on four bounding planes, with signal points at the interface between planes. The minimum distance for these constellations is due to the spacing of points within the underlying lattice of each plane, as described in Section 3.4.2. Figure 3.14 shows the constellation for 25-AB-QAM. In keeping with the paradigm of assigning constellation points to the lowest possible symbol averages, the four constellation points with the highest average of 4 can be moved into the lower average power level with an average of 3. This is possible by moving these points to the \((\phi_1, \phi_2)\) coordinates \((3, 0)\), \((0, 3)\), \((-3, 0)\) and \((0, -3)\) as seen in Figure 3.16. This new shaped 25-AB-QAM constellation requires less average optical power for the same minimum distance. This improvement in the CFM over AB-QAM described in Section 3.4.2 is denoted as the shaping gain \( \Gamma_s \).

The shaping gain can be quantified by noting that for an \( L^2 \) sized constellation, the points are assigned such that the lowest levels of average optical power are filled. This is equivalent to stating that points are assigned so that the \( \phi_3 \) coordinate value of each constellation point is minimised. Therefore, the assignment of points can proceed from the point of view of starting at the \( \phi_3 = 0 \) average plane, and filling points in lower
average planes before a higher average plane is used. For an $L^2$-AB-QAM constellation, the minimum number of unique average levels required other than zero is denoted $N$, and takes the form

$$N = \left\lceil \frac{\sqrt{2L^2 - 1}}{2} \right\rceil$$

where $\lceil y \rceil$ denotes the smallest integer not smaller than $y$. This expression is derived assuming that the points are assigned so that the zero and the first $N - 1$ average levels are filled with the $N^{th}$ level filled to satisfy the $L^2$ points of the constellation. It can be shown that the shaping gain over the specified AB-QAM constellation in Section 3.4.2 takes the form

$$\Gamma_s = \begin{cases} 
10 \log_{10} \left( \frac{3L^2}{N(3L^2 - 1 - 2N^2)} \frac{L^2 - 1}{2L} \right) & \text{dB : for } L \text{ odd} \\
10 \log_{10} \left( \frac{3L^2}{N(3L^2 - 1 - 2N^2)} \frac{L^2 - 1}{2L} \right) + 1.5 & \text{dB : for } L \text{ even} 
\end{cases}$$

(3.38)

The fixed gain of 1.5 dB is added to even sized constellations since in the case of unshaped AB-QAM no gains are derived from the signal space diversity of the constellation. Using the array of points contained in all odd sized AB-QAM constellations, and choosing a subset with an even number of points ensures that this coding gain is present for even sized constellations as well.
Using the shaping gain expression in (3.38), the shaping gain available for 25-AB-QAM is \( \Gamma_s = 0.3 \) dB. Figure 3.17 shows the constellation for 64-AB-QAM after shaping. The gain due to shaping in this case is \( \Gamma_s = 1.75 \) dB. A majority of this gain is due to the availability of coding gain using the information in the \( \phi_3 \) basis.

The complexity of the receiver for a multilevel modulation scheme can be measured by the number of distinct output amplitudes which are generated. For any AB-QAM constellation, the number of discrete levels in the output is equal to the number of unique symbol averages possible. This is due to the fact that in AB-QAM the symbol average is the sum of the individual phase magnitudes. Therefore, every possible amplitude which can be generated by the scheme must have a corresponding DC bias term to ensure its non-negativity. For the \( \text{L}^2\)-AB-QAM modulation scheme of Section 3.4.2 the number of symbol averages possible is \( \text{L} \) for odd size constellations and \( \text{L} - 1 \) for even sized constellations. With shaping gain, \( \text{N} + 1 \) distinct average values are present in the constellation. The shaped constellation has a smaller number of output levels, and hence complexity, for \( \text{L} \geq 3 \) except \( \text{L} = 4 \).

Therefore, a shaping gain is possible for AB-QAM by using the set of all points contained in odd sized AB-QAM constellations and choosing constellation points with the minimum average optical power requirements. Constellations with an even number of points benefit from this techniques not only through the efficiency of the shaping but also due to the availability of coding gain.

### 3.4.5 Spectral Characteristics

As was done for the other modulation schemes, the frequency characteristics of the AB-QAM, as defined in Section 3.4.1, were investigated using the power spectral density. Using (3.17) the psd of \( \text{L}^2\)-AB-QAM was found to be

\[
G_{\text{AB-QAM}}(f) = (RP)^2 \delta(f) - \frac{T_s(RP)^2}{2} \text{sinc}^2(\pi f T_s) \\
+ \frac{2T_s A^2 (RP)^2 (L + 1)}{3(L - 1)} \left[ 2 \text{sinc}^2(\pi f T_s) + \text{sinc}^2\left(\frac{\pi}{2} f T_s\right) \\
- (1 + \cos\left(\frac{\pi}{2} f T_s\right)) \text{sinc}(\pi f T_s) \text{sinc}\left(\frac{\pi}{2} f T_s\right) \right],
\]

where \( A \) is defined in (3.28). Figure 3.18 illustrates the power spectral density of 25-AB-QAM. The bandwidth, as defined as the first spectral null, can be shown to be at \( f = 2/T_s \).
Figure 3.17: Shaped constellation for 64-AB-QAM for $\Delta = 1$, $R = 1$ and $T_s=1$ in each.
Section 3.4. Adaptively Biased QAM

Figure 3.18: The continuous portion of the power spectral density of 25-AB-QAM, for $RP = 1$ and $Ts = 1$.

Since two independent data streams are sent per symbol period the bandwidth efficiency is,

$$\frac{R_b}{B} = \log_2 L \frac{\text{bits/s}}{\text{Hz}}.$$  

This is the same expression that was determined for $L$-PAM and $L^2$-QAM in Section 3.3. Therefore, using the measures defined in Section 3.1.4, AB-QAM preserves the bandwidth efficiency of other multilevel schemes while providing an improvement in the power efficiency.

Although the bandwidth measure defined in Section 3.1.4 verifies that AB-QAM has the same bandwidth efficiency of other multilevel schemes, it suffers from greater side-lobe power. The current bandwidth measure does not penalise AB-QAM for this fact. The issue of side-lobe power and means to reduce it are addressed in Section 3.4.7 and as well as in Chapter 4.

**Fluorescent Light Interference**

As was discussed in Chapter 2, fluorescent light fixtures introduce a low frequency interference into the received signal of wireless optical channels. The channel model in Section 3.1.2 does not take this into account and considers only the Gaussian noise introduced by background lighting. Passband modulation schemes with little signal energy at low frequencies, such as PPM and QAM, have been shown to be effective means of combatting fluorescent light interference [23].
Section 3.4. Adaptively Biased QAM

AB-QAM exploits the redundant information contained in the DC bias of each symbol. The data transmitted in the $\phi_1$ and $\phi_2$ directions has no energy at DC and minimises the low frequency component of the symbol. The data transmitted in these phases provides sufficient statistics to make an estimate of the sent symbol. Therefore, fluorescent light interference will impact the low frequency content of AB-QAM, dominated by the $\phi_3$ component. Knowledge of the low frequency interferer statistics would allow for a change in the decoding rules in the $\phi_3$ direction to adhere to MAP rules, but would necessitate an increase in decoder complexity. An asymptotic coding gain of 1.5 dB is still achievable with AB-QAM even if the redundant information in the $\phi_3$ direction is unused.

3.4.6 Extension of AB-QAM to Different Pulse Shapes

In an effort to reduce the side-lobe power of AB-QAM, the shape of the pulses $\phi_1$ and $\phi_2$ can be altered to be more spectrally efficient. The fundamental idea of adaptive biasing can be used on a variety of pulse shapes to ensure non-negativity is maintained. However, the the structure of the constellation will vary greatly, and the simple lattice structure of Section 3.4.2 does not generalise to other pulse shapes.

For example, consider the case when the $\phi_1$ and $\phi_2$ basis functions of AB-QAM are replaced by the QAM basis functions $\phi_I$ and $\phi_Q$ in (3.24). The adaptive biasing technique stipulates that the minimum amount of bias required to ensure non-negativity should be added to the symbol. If the amplitudes sent on $\phi_I$ and $\phi_Q$ are $a_k$ and $b_k$ respectively, the amount of bias added to each symbol is $c_k = \sqrt{a_k^2 + b_k^2}$. In this case, the bounding surface is a paraboloid opening around $\phi_3$ on which all constellation points lie. This is due to the fact that the relationship between the amplitude values and the symbol average is not linear, as it is in AB-QAM. This increases the decoding complexity since the spacing of the constellation points does not follow a regular pattern, but are determined by the shape of the bounding surface.

The issue of pulse shaping for intensity channels is highly dependent on the channel constraints as well as on the pulse shape used. Simple extension of adaptive biasing principles does not guarantee the coding gains available in AB-QAM. Decoding complexity also becomes an issue, since the bounding surfaces are no longer planes. Additional study is required to investigate the impact of the channel constraints on possible pulse shapes in intensity modulated channels. The pulse shapes chosen for the AB-QAM constellation
in Section 3.4.1 were chosen since they require a relatively low complexity receiver and require a simpler transmitter while providing a gain in the power efficiency of the modulation scheme.

### 3.4.7 Comparison with other Multilevel Schemes

AB-QAM provides a coding gain over PAM and QAM through the modification of the signal space geometry. The signal space is modified by extending scaling values of the basis functions to positive and negative values. As well, the redundant information transmitted to satisfy the non-negativity constraint provides a degree of signal space diversity.

The geometry of the AB-QAM constellation depends on whether there are an even or odd number of constellation points. For an odd number of constellation points, the minimum distance is limited by the spacing the the hexagonal lattice in each bounding plane. As a result, for large constellations with an odd number of points, $L^2\text{AB-QAM}$ provides a 3 dB gain in power efficiency over $L\text{-PAM}$ and a 4.5 dB gain over $L^2\text{-QAM}$ for the same bandwidth and bit rate. Figure 3.19 shows the variation of bit error rate versus optical SNR, as defined in (3.18), for a 49 point constellation. The figure demonstrates that 49-AB-QAM provides a gain of approximately 2.4 dB over 7-PAM.

Even sized constellations do not have any signal points on the boundary between signal planes, and as a result do not benefit from coding gain to the same extent as odd sized AB-QAM constellations. Even sized AB-QAM constellations provide an asymptotic 1.5 dB gain over PAM, and 3 dB gain over QAM schemes. Figure 3.20 illustrates the change in bit error rate versus optical SNR for 64-AB-QAM and 8-PAM. In this case, AB-QAM provides a gain of nearly 1 dB in power efficiency over PAM.

Constellation shaping of $L^2\text{-AB-QAM}$ constellations further improves the power efficiency. Figures 3.19 and 3.20 show that even sized constellations benefit from shaping gain to a greater extent. This is because, the underlying array of points used for the shaping was taken to be the set of all points in all odd sized AB-QAM constellations. The shaping gain available for 49-QAM is 0.2 dB, while for 64-AB-QAM the shaping gain is 1.75 dB.

The underlying assumption of $L^2\text{-AB-QAM}$ is that the bandwidth efficiency is identical to that of $L\text{-PAM}$ and $L^2\text{-QAM}$. The measure of bandwidth, however, neglects the significant spectral side-lobes present in AB-QAM. Although, AB-QAM still remains more spectrally efficient than binary level techniques, the side-lobe power needs to be controlled.
Section 3.4. Adaptively Biased QAM

Figure 3.19: Comparison of bit error rates versus optical signal-to-noise ratio for 49-QAM, 7-PAM, 49-AB-QAM and shaped 49-AB-QAM ($R = 1\, \text{A/W}$, $\sigma_n^2 = 10^{-2}\, \text{W/Hz}$, $B = 2\, \text{Hz}$ and $R_b = \log_2(49)\, \text{bits/s}$).

Figure 3.20: Comparison of bit error rates versus optical signal-to-noise ratio for 64-QAM, 8-PAM, 64-AB-QAM and shaped 64-AB-QAM ($R = 1\, \text{A/W}$, $\sigma_n^2 = 10^{-2}\, \text{W/Hz}$, $B = 2\, \text{Hz}$ and $R_b = 6\, \text{bits/s}$).
3.5 Conclusions and Comparison

The need for bandwidth efficient modulation schemes is motivated by the use of inexpensive optoelectronic devices for wireless optical links. Binary level schemes, although power efficient, do not provide a high degree of bandwidth efficiency. As a result, multilevel schemes, which trade-off power efficiency for bandwidth efficiency are investigated.

A multilevel modulation scheme, adaptively biased QAM, is proposed which preserves the bandwidth of PAM and QAM while providing a gain in power efficiency. The use of shaping gain to further improve the power efficiency has also been discussed in the framework established for intensity modulated channels in this chapter.

In order to compare the various modulation schemes discussed in the chapter, the constellation figure of merit is used. The gain of each modulation scheme relative to on-off keying was calculated as a metric of comparison. OOK was chosen as a reference point due to its simplicity of implementation, and pervasiveness in conventional communications. Formally, the gain $\Gamma_{OOK}$ is defined as,

$$\Gamma_{OOK} = 10 \log_{10} \left( \frac{\text{CFM}_x}{\text{CFM}_{OOK}} \right)$$

where CFM$_x$ is the constellation figure of merit for modulation scheme under consideration. In order to capture the spectral performance of each scheme as well, $\Gamma_{OOK}$ is plotted versus the bandwidth efficiency for various sized constellations in Figure 3.21.

The optimal point for any modulation scheme destined for wireless optical links would be in the upper right hand corner of the figure, where bandwidth efficiency and power efficiency are maximised. Binary level techniques offer the best power efficiency at the cost of reduced bandwidth efficiency. Multilevel schemes provide the necessary bandwidth efficiency for wireless optical links at the cost of power efficiency. AB-QAM maintains the bandwidth efficiency of other multilevel techniques while improving on the power efficiency. The use of constellation shaping techniques provides significant gains in the power efficiency of AB-QAM, especially for even sized constellations. The use of even sized constellations which are powers of 2 is commonly used in binary data communication systems.
Figure 3.21: Comparison of power efficiency gain of modulation schemes over OOK plotted versus bandwidth efficiency.
Section 3.5. Conclusions and Comparison

The focus of this chapter has been to develop means of improving the power efficiency of existing multilevel techniques. Although AB-QAM does provide an improvement in the power efficiency over other multilevel techniques, significant spectral side-lobes are present. In order to achieve the high bandwidth efficiencies required by high speed wireless optical links, spectrally efficient pulses must be used on intensity modulated channels.
Chapter 4

Constrained Coding Techniques for Intensity Modulated Channels

The optical intensity channel imposes constraints on the type of signals which may be transmitted. Chapter 3 discusses the physical channel constraints imposed by the nature of information transmission using an optical intensity signal. The use of inexpensive optical components along with the requirement for high data rates imposes an added constraint of high bandwidth efficiency. Conventional schemes satisfy the channel constraints through the careful selection of the pulse shapes used. As a result, the choice of pulse shapes which are bandwidth efficient cannot be made independently of the channel constraints.

This chapter introduces a novel means of satisfying the channel constraints of the optical intensity channel. Well-developed techniques of constrained coding are applied to ensure that the non-negativity constraint is met when using bandwidth efficient pulses in the optical intensity channel. Basic elements of constrained coding are reviewed and the mathematical constructs of symbolic dynamics are introduced. The general concepts are applied to an intensity modulated channel using an example and compared to a simple baseline. A coder is developed using well known techniques, and issues involved in the construction of practical encoders and decoders are briefly discussed.

4.1 Motivation

Coding techniques have been used for some time to surmount physical constraints imposed by a communications channel. In magnetic disk channels, runlength-limited codes
are used to ensure the minimum and maximum number of consecutive “1” or “0” symbols recorded. If the transitions between symbols are too close together, intersymbol interference will occur due to the proximity of the magnetic domains on the disk. However, transitions between the symbol levels must occur with a guaranteed periodicity to ensure timing synchronisation is maintained [55, 56]. In channels where the signals are capacitor or transformer coupled to the receiver, baseline wander is introduced. This effect is due to the fact that AC coupling introduces a zero at DC which causes intersymbol interference to be introduced into the signal. So called charge-constrained codes ensure that the running digital sum is bounded, minimising the baseline wander interference [41, 57].

The goal of this chapter is to propose and demonstrate the use of similar constrained coding techniques for the intensity modulated channel. The non-negativity constraint (3.1) specifies that the channel constraint applies to the transmitted intensity waveform $x(t)$. The definition of $x(t)$ is contained in (3.5), which states simply that the intensity waveform over all time is composed of the addition of symbol functions for each interval. The previous chapter assumed that each symbol was time limited and non-negative throughout a symbol period in order to ensure that the non-negativity constraint was met. However, this is an overly conservative assumption since the non-negativity constraint applies to the sum of the symbol pulses and not to each individual symbol function. If a PAM type modulation scheme is considered, the non-negativity constraint can be satisfied by choosing a bandwidth efficient pulse which is non-zero outside the symbol interval and constraining the scaling factors so that the sum of the symbol pulses remains non-negative. The average optical power transmitted can then be controlled by a scaling factor dependent on the output data distribution.

The impact of using coding techniques to satisfy the non-negativity constraint can be illustrated by looking at the case of Nyquist pulses. Nyquist pulses are a set of functions which introduce no intersymbol interference to the transmission. In a conventional channel, the minimum bandwidth Nyquist pulse occupies a bandwidth of $B = 1/T_s$ Hz, where $T_s$ is the symbol period. However, Appendix B shows that in an intensity channel the minimum bandwidth of a non-negative Nyquist pulse occupies twice this bandwidth or $B = 2/T_s$ Hz. Therefore, the imposition of the non-negativity constraint increased the minimum bandwidth required for Nyquist pulses. On the other hand, $1/T_s$ Hz bandwidth Nyquist pulses can be used on an intensity modulated channel if a coding scheme is developed so that the choice of scaling factors ensures that the summation of all symbols remains non-negative.
for all time.

The remainder of the chapter describes the development of constrained codes for a pulse amplitude modulated type scheme with a family of pulse shapes. Before discussing the application of constrained coding techniques to the intensity modulated channel, the basic framework for the development of these codes is presented.

### 4.2 Constrained Coding and Symbolic Dynamics

Channels which do not permit certain sequences of symbols to be sent due to physical or practical limitations are referred to as constrained channels or input-restricted channels. The channel constraints are taken to be deterministic and invariant in time. The role of the channel coder, or constrained coder, is to take an unrestricted input sequence and transform it into a constrained sequence which satisfies all of the channel restrictions.

The development of constrained codes can be analysed using the mathematics of symbolic dynamics. Symbolic dynamics deals with the transformation and analysis of fundamental units known as shift spaces. Shift spaces are composed of bi-infinite sequences of symbols chosen from a finite alphabet $A$. A block is a finite length sequence of symbols chosen over $A$. Constraints are introduced by specifying a set $F$ of forbidden blocks, none of which may appear as a sub-block of any sequence in the shift space. The forbidden blocks, in the case of an optical intensity channel, correspond to sequences of symbols which are not transmittable in the channel. A shift space, $X$, is defined as a bi-infinite sequence of symbols chosen over a finite alphabet $A$ such that no sub-block is contained in the set $F$. For the systems considered in this chapter, the set $F$ is finite, and hence the shift spaces considered are called shifts of finite type.

Shifts of finite type in which every block in $F$ is of length $M + 1$ are called $M$-step shifts. In this type of shift space, it is only necessary to look at the past $M$ symbols to detect whether a forbidden block was sent. In other words, the system has a memory of $M$ symbols. It can be shown that any shift of finite type, $X$, can be represented as a directed graph, $G$. If the labelling of the edges in $G$ is taken from $A$, the resulting shift is known in literature as a sofic shift [55]. The connection of the vertices in the graph is determined so that the sequence of edges traversed in any bi-infinite walk through the graph form $X$. Explicitly, every element of the shift space $X$ corresponds to some infinite walk in the graph $G$. In this manner, analysis of the shift space $X$ can be performed using the
Section 4.2. Constrained Coding and Symbolic Dynamics

model represented by the graph $G$.

The graph $G$, which represents the shift space is composed of a set of $L$ vertices from the vertex set $\mathcal{V}$ interconnected by a set of edges. The topology of the graph can be represented in the form of an $N \times N$ non-negative, integer matrix known as the adjacency matrix. Without loss of generality we assume that the vertices are numbered from the set of natural numbers such that $\mathcal{V} = \{1, 2, 3, \ldots, N\}$. If $I, J \in \mathcal{V}$ are two vertices in the graph, let $a_{IJ}$ denote the number of edges originating at vertex $I$ and terminating at vertex $J$. The adjacency matrix of a graph is composed of the set of all $a_{IJ}$ in the form $A = [a_{IJ}]$. The adjacency matrix determines the structure of the graph, but neglects the labelling of the edges. The graphs considered in this chapter have the property that no two edges leaving a given vertex carry the same labelling. This type of graph is known as a right-resolving presentation, and it can be shown that every sofic shift has this type of presentation [55].

The entropy of the shift space is a measure of the maximum amount of information that can be represented in the messages coded according to the rules of the space. For noiseless, discrete channels, this measure can be thought of as the capacity of the channel, since it determines the upper bound on the amount of information which can be represented in the shift. The entropy of a shift space $X$ is defined as

$$h(X) \triangleq \lim_{n \to \infty} \frac{1}{n} \log_2 |B_n(X)|,$$  \hspace{1cm} (4.1)

where $|B_n(X)|$ is the number of blocks of length $n$ which occur in the shift $X$ [55]. This definition is analogous to the definition for the capacity of a noiseless discrete channel given by Shannon [58], and can be thought of as the capacity of the channel as imposed by the structure of the shift space.

The notion of entropy can be related to the directed graph representation of the shift space. The fundamental definition of the entropy of the shift space can be written using the adjacency matrix of the graph. The entropy of a shift $X$ is

$$h(X) = \log_2 \lambda$$  \hspace{1cm} (4.2)

where $\lambda$ is the largest, real eigenvalue of the adjacency matrix $A$, satisfying

$$Av = \lambda v$$  \hspace{1cm} (4.3)

for the corresponding eigenvector $v$. The existence of $\lambda > 0$ for the adjacency matrices considered in this chapter is guaranteed by the Perron-Frobenius Theorem [55]. The calculated
Section 4.2. Constrained Coding and Symbolic Dynamics

The entropy of \( X \) in (4.1) and (4.2) is often referred to as the topological entropy of the shift space [59]. This terminology is more precise since conventional definitions of entropy rely on probabilistic distributions while the entropy of the shift space, as defined, depends only on the structure of the shift space itself. As a result, the topological entropy is determined by the structure of the shift space and sets the upper bound on the entropy of all messages arising from the space.

The probabilistic nature of data transmission can be incorporated into the graph model of the shift space. If the vertices of the graph are viewed as a set of states, \( \mathcal{S} \), and if transition probabilities are assigned to each edge, the resulting structure is a discrete-time Markov process. The probability of making a transition to state \( j \) given that the current state is \( i \) is denoted \( q_{ij} \), where \( i, j \in \mathcal{S} \). Assuming that \( \mathcal{S} \) is chosen in the same way as \( \mathcal{V} \), the collection of all transition probabilities forms an \( N \times N \) state transition matrix \( Q = [q_{ij}] \). The matrix \( Q \) is a stochastic matrix since its row sums are all unity, or formally,

\[
\sum_{j=1}^{N} q_{ij} = 1
\]

for every \( i = 1, 2, 3, \ldots, N \). Given a starting state, \( s_o \in \mathcal{S} \), the process chooses one of the outgoing edges according to the transition probabilities. At each transition, a symbol value is output corresponding to the edge labelling. The maximum entropy of the output messages is the topological entropy of the underlying graph. Assuming no multiple edges between pairs of states, as is the case in the graphs considered in this chapter, the maxentropic state transition matrix, \( Q \), can be determined in terms of its elements \( q_{ij} \) as,

\[
q_{ij} = \Pr\{ \text{next state} = j \mid \text{current state} = i \} = a_{ij} \frac{1}{\lambda} v_j v_i \tag{4.4}
\]

where \( a_{ij} \) are the elements of \( A \) and \( \mathbf{v} = [v_1, v_2, \ldots, v_N]^T \), \( \lambda \) are defined in (4.3) [56]. To summarise, if the state transition probabilities in (4.4) are used, the entropy of the resulting Markov chain will be equal to the topological entropy of the shift space defined in (4.2).

The Markov chain derived from the graphs presented in this chapter is homogeneous since the channel constraints are fixed and deterministic. Additionally, it can be shown that the Markov chain is fully regular [55, 47]. In this case, the stationary distribution vector, \( \Pi \), is independent of the starting state, and illustrates the probability of being in a given state. The stationary distribution vector, \( \Pi \), can be determined from the
expressions,

\[
\Pi Q = \Pi \quad \sum_i \Pi_i = 1
\]  

(4.5)

where \(\Pi = [\Pi_1, \Pi_2, \ldots, \Pi_N]\).

The concepts presented in this section can be illustrated using an example. For the alphabet \(A = \{0, 1\}\), a \((d, k)\) run-length limited code is defined by the property that between each “1” symbol there are between \(d\) and \(k\) “0” symbols, where \(d \leq k\) [55]. Consider a \((1,2)\) run-length limited shift space. The set of forbidden blocks is determined by the fact that no two “1” symbols can be adjacent, and no more that two “0” symbols can be adjacent. The set of forbidden blocks representing this relation can then be written as \(F = \{11, 000\}\).

Say the shift space defined by \(F\) is denoted \(X\). \(X\) can be represented as a directed graph \(G\) in Figure 4.1. The vertices are labelled according to the convention in this section. The edges are labelled from \(A\), and the set of bi-infinite walks on \(G\) forms the shift space \(X\).

The adjacency matrix, \(A\) for this graph is

\[
A = \begin{bmatrix}
0 & 1 & 0 \\
1 & 0 & 1 \\
1 & 0 & 0
\end{bmatrix}
\]

The topological entropy of this graph can be calculated through (4.2) to be approximately 0.45 bits/symbol. The state transition matrix corresponding to maximum entropy conditions is determined using (4.4) to be approximately,

\[
Q = \begin{bmatrix}
0 & 1 & 0 \\
0.57 & 0 & 0.43 \\
1 & 0 & 0
\end{bmatrix}
\]
The stationary distribution vector for this $Q$ is $\Pi = [0.41 \ 0.41 \ 0.18]$.

This section aims to provide a brief overview of some central concepts in symbolic dynamics as applied to the examples in this chapter. More complete references cataloguing the links between symbolic dynamics and coding can be found elsewhere in the literature [55, 59, 60, 57].

4.3 A Constrained Code for Intensity Modulated Channels

4.3.1 Introduction

In an effort to control the spectral characteristics of a modulation scheme bandwidth efficient pulses are employed. In PAM, spectrally efficient pulses such as raised-cosine pulses are used to control the amount of bandwidth occupied by each pulse and minimise the out of band power.

The goal of the constrained coding technique in this chapter is to exploit similar bandwidth efficient pulses on an intensity modulated channel. Indeed, the same raised-cosine pulses could be employed on an intensity modulated channel if a suitable constrained coder could be constructed to ensure that the resulting collection of pulses remains non-negative for all time.

In order to initiate study into the applicability of constrained coding for intensity modulated channels, a simplified discrete time channel model is employed. The channel is set as being discrete in order to simplify the computation requirements required for the example, while preserving some meaning in the results. This discrete case can be looked upon as a bounding case for a set of continuous functions where non-negativity at the sample points of the output implies global non-negativity of the function. Additionally, the channel is taken as being noiseless to determine the maximum possible information transfer rate.

This section presents a discrete time example of constrained coding applied to the non-negativity constraint for intensity modulated channels. We present the structure of the pulses along with the procedure used to determine the performance of the coding scheme. Comparisons to a low complexity baseline scheme are presented in the same framework to determine the possible gains available from the constrained coding technique.
4.3.2 Structure and Methods

The modulation scheme considered in this example is an $L$-PAM scheme in a discrete, noiseless intensity channel. The choice of PAM is due to the fact that it is the simplest multilevel scheme in terms of implementation and computation.

The pulse chosen as the basis of the modulation scheme is taken from the family of pulses shown in Figure 4.2. The value of $k_1$ is chosen as $k_1 = 3, 4, 5, \ldots, 10$ to allow for characterisation of the constrained coding scheme for a variety of pulses. As discussed in Chapter 3, the non-negativity constraint of the channel is dependent upon the shape of the pulses defined through the choice of $k_1$. At each sample instant, a scaled version of the pulse is sent. The form of the transmitted discrete-time intensity signal takes the form,

$$x[n] = \sum_{k=-\infty}^{\infty} b_k p[n-k]$$

where $b_k \in \{1, 2, 3, \ldots, L\}$ such that $x[n] \geq 0$ for all $n$. Unlike the PAM case, $b_k \neq 0$ since this would imply a negative transition. This constraint on the scaling factor can be determined by examining the structure of the pulses. Since the pulses are limited to a length of 3, at each sample instant only the previous two symbols must be taken into account to constrain the current symbol. This implies that the system requires a memory of two symbols. Therefore, for each $n = n_o$, the amplitude value is

$$x[n_o] = -b_{n_o} + k_1 b_{n_o+1} - b_{n_o+2}. \quad (4.6)$$

If the scaling factor $b_{n_o+1} = 0$, the the value of $x[n_o]$ would certainly be negative. Therefore, the alphabet of scaling factors for $p[n]$ is taken from the set $\{1, 2, 3, \ldots, L\}$.

The operation of this scheme can be represented as a constrained data series which is filtered by a finite impulse response discrete-time filter. Figure 4.3 shows a description of the coder along with the pulse shape used in this example. The data input to the coder is an unconstrained sequence, chosen from some symbol set. Based on this input, the coder produces an output data stream of integer symbols, $s[n]$, in the range $[1, L]$, so that the output of the digital filter remains non-negative. The exclusion of $k_1 = 1$ and $k_1 = 2$ can be justified based on this filter definition of the pulse. For these cases, the gain of the filter transfer function at DC is less than or equal to zero. Since the DC content of the input signal, $s[n]$, is always greater than zero, this implies that the average value of the output
Section 4.3. A Constrained Code for Intensity Modulated Channels

Figure 4.2: The family of pulse shapes used for the example PAM scheme.

Figure 4.3: Description of the coder and the pulse shape used in the example, where D indicates a delay of one interval.
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is zero or negative. A zero average value implies that zero is sent at every instant, while a negative average is impermissible.

The parameters $L$ and $k_1$ completely describe a modulation scheme in the form of that in Figure 4.3. In order to determine the maximum amount of information that a given scheme can send, the techniques in Section 4.2 must be employed.

Define a shift space, $X$, which is determined by the set of forbidden blocks $\mathcal{F}$, where symbols are chosen from the alphabet $A = \{1, 2, 3, \ldots, L\}$. The bi-infinite sequence generated through $X$ is used to define the signal $s[n]$ at the input of the filter in Figure 4.3. The blocks in $\mathcal{F}$ can be seen as the set of inputs which cause negative transitions in $x[n]$. Since the system has a memory of two symbols, each of the forbidden blocks can be represented as length three blocks. The number of blocks in $\mathcal{F}$ is finite since both the alphabet and memory of the system are finite. Therefore, $X$ is a 2-step shift of finite type and can be represented by a directed graph $G$.

The graph $G$ can be determined by specifying a set of $L^2$ vertices, $V$, corresponding to the set of all length 2 blocks over $A$. These 2-blocks correspond to possible values for the filter state and are represented as the ordered pair $\Psi_i = [\psi_{1i}, \psi_{2i}]$ where $\Psi_i \in V$ and $i \in \{1, 2, 3, \ldots, L^2\}$. The edges interconnecting the vertices are determined according to the non-negativity constraint of the channel as determined by the filter transfer function. An edge exists between state $\Psi_i$ and $\Psi_j$ if and only if the inequality

$$-\psi_{1j} + k_1\psi_{1i} - \psi_{2i} \geq 0$$

(4.7)

is satisfied. Let $e_{ij} = \psi_{1j}$ for those edges which exist. Due to this construction, for a given state there is at most one edge leading to any other state since each of the $\Psi_i$ are unique. The adjacency matrix corresponding to $G$, $A$, can be determined based on the above structure.

Once the adjacency matrix is formed, $A$ is simplified by removing unnecessary states. All vertices with zero indegree or outdegree can be pruned from the graph. These states represent discontinuities in the bi-infinite shift space represented by the graph, and are not permitted. An adjacency matrix which has vertices with outdegree and indegree of at least one is termed essential. Similarly, all vertices with an outdegree of one with self loops were eliminated. These states are called “black hole” states since once entered, there is no sequence of inputs to escape the state. These states can be removed since they provide
no information to the output message. After pruning, let the number of states remaining be $N \leq L^2$.

Using the adjacency matrix, the topological entropy of the graph can be calculated via (4.2) for each $L$ and $k_1$.

The average power constraint in (3.2), fixes the distribution of the amplitude values of the output. In this discrete channel, it is necessary to determine the expected value of the sequence $x[n]$ for some probability distribution. Using the maxentropic transition probabilities in (4.4) to form $Q$, ensures that the average amplitude calculated corresponds to the case where the entropy of the output message is equal to the topological entropy of the graph. Using the state transition matrix, $Q$, the stationary distribution vector in (4.5) is calculated. The $\Pi_i$ are the steady-state probability of being in a given state. The transition between states $i$ and $j$ in $Q$ represents the output of a symbol, and occurs with probability $\Pi_i q_{ij}$. Visiting every element of $Q$, the expected value of $x[n]$ can then be written in the form,

$$E\{x[n]\} = \sum_{i=1}^{N} \sum_{j=1}^{N} (-e_{ij} + k_1 \psi_{1i} - \psi_{2i}) \cdot \Pi_i q_{ij}. \quad (4.8)$$

Note that $q_{ij} = 0$ if there is no edge between states $\Psi_i$ and $\Psi_j$, and so the inconsistency in the $e_{ij}$ definition is removed.

### 4.3.3 Results

The topological entropy and the average of each modulation scheme defined by $k_1$ and $L$ are calculated according to the guidelines in Section 4.3.2.

Figure 4.4 shows a plot of the topological entropy of each modulation scheme over $k_1$ and $L$. The function $h(X)_{\text{max}} = \log_2 L$ plotted in the figure represents the maximum capacity of a noiseless channel using $L$ symbols. The distance from every curve in Figure 4.4 to the $h(X)_{\text{max}}$ curve indicates the number of bits/symbol lost due to the channel constraints. This loss of capacity is due to the sequences of symbols which must be excluded due to the non-negativity constraint. The figure indicates that the penalty reduces as the $k_1$ term of the pulse increases. For a fixed $L$, the alphabet $\mathcal{A}$ of all the schemes is the same. The inequality in (4.7), demonstrates that for a fixed alphabet increasing $k_1$ increases the number of points which satisfy the inequality. Qualitatively, the amount of DC bias of each symbol is increased as $k_1$ increases. This added bias term allows more flexibility in the choice of symbols since the negative components of each pulse become less significant.
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Figure 4.4: Topological entropy, \( h(X) \) in bits/symbol, of each modulation scheme over \( k_1 \) and \( L \).

with respect to the \( k_1 \) term. As a result, the amount of information represented in the modulation scheme can increase. Taken to a limit, the plots of entropy versus \( L \) should approach \( h(X)_{\text{max}} \) for fixed \( L \) as \( k_1 \) approaches infinity.

Figure 4.5 illustrates a plot of the average amplitude of each modulation scheme considered. This data on the figure is calculated using (4.8). As is expected the average of \( x[n] \) increases as does \( L \). However, it should be noted that increases in \( k_1 \) also provide substantial increases in the average of the output. So, although increases in \( k_1 \) and \( L \) improve the topological entropy of the system, they are accompanied by increases in the average output level.

The trade-offs between entropy, average output amplitude and system complexity can be viewed more easily with the aid of Figure 4.6. This figure plots the same data as in Figure 4.5, however points of equal entropy are connected with dashed lines. In the wireless optical channel, the bandwidth is a parameter which is fixed by the choice of hardware. Thus, in order to meet a target specification, the amount of entropy, in
Figure 4.5: Average output amplitude of each modulation scheme over $k_1$ and $L$. 
Section 4.3. A Constrained Code for Intensity Modulated Channels

Figure 4.6: Same data as figure 4.5, where dashed lines connect points of equal entropy \( h(X) \) bits/symbol.

bits/symbol, is a fixed quantity. The scheme chosen for a particular channel will rest on one of the equientropic dashed lines in Figure 4.6. Power efficiency demands that the lowest average output amplitude should be selected. This suggests that \( k_1 \) should be minimised and \( L \) increased. An increase in \( L \) however, suggests an increase in the complexity of the system. As is often the case in conventional coders, added power efficiency can be achieved by increasing the complexity of the coder and the receiver structures.

4.3.4 Comparison to Baseline

In order to determine if the constrained coding approach is viable, comparisons versus a baseline are required. A low complexity alternative to the constrained coder is proposed in this section using the pulse shapes in Figure 4.2.

Figure 4.7 illustrates the baseline model scheme. In this scheme the data symbols sent into the digital filter are chosen from the set \( \{0, 1, 2, 3, \ldots, L\} \). The symbols are chosen equiprobably, and without constraints. At the output of the filter, the minimum amplitude
value possible is $-2L$. As a result, a fixed bias added to the output ensure that the non-negativity constraint is met.

The maximum entropy available from this scheme is equal to the entropy of the data symbols entering the filter. Since the alphabet of input sources consists of $L + 1$ symbols, the entropy of the messages arising from the baseline case is $H = \log_2 (L + 1)$. Due to the fact that the amplitude values are chosen equiprobably, it can be shown that the expected value of the output amplitude for the baseline case is

$$E\{x[n]\} = \frac{k_1 + 2}{2} L.$$

The comparison between the baseline case and the constrained coding technique of Section 4.3.2 is done using the fact that the output discrete signals represent an intensity function. Hence the expected value of the output amplitude is really a measure of the average optical power transmitted. For a given entropy, the gain $G$ is defined as,

$$G \Delta = 10 \log_{10} \frac{P_{\text{baseline}}}{P_{\text{coded}}}$$

where $P$ represents the average optical power required for the baseline and coded schemes. Figure 4.8 plots this comparison for $k_1 = 3, 4, 5, \ldots, 10$.

The figure demonstrates that constrained coding has the greatest benefits over the baseline as the entropy required increases and as $k_1$ decreases. The essential trade-off of the constrained coder is that it takes a given pulse shape and a number of levels $L$ such that $\log_2 L$ is greater than the entropy of the data to be coded. Through the structure of the constrained coder, the excess entropy of system is traded to ensure that the channel constraints are met. The addition of excess $L$ in large $k_1$ schemes is costly to the average
Figure 4.8: Power gain $G$ (dB) of constrained coding technique over baseline case versus topological entropy, $h(X)$ in bits/symbol.
optical power budget due to their inherent high DC content. In the low $k_1$ case, the addition of levels requires less average optical power than adding a bias to each symbol ensure non-negativity. As a result, Figure 4.8 indicates that constrained coding is advantageous over the baseline for small $k_1$, at the cost of increased number of levels and complexity.

### 4.3.5 Code Construction

The constrained coder accepts an unconstrained input sequence and outputs a sequence which meets the channel constraints. In order to implement the constrained systems discussed in Section 4.3.2, a coder must be designed. In the example presented in Figure 4.3, the constrained coder is designed so that the output sequence remains non-negative once filtered by the digital filter corresponding to the pulse shape. This section presents an example of a code which satisfies the non-negativity constraint for a given pulse shape and $L$.

Let a modulation scheme be defined by the pulse shape in Figure 4.2 with $k_1 = 3$ and where the alphabet is set by $L = 4$. Assume that the input symbols are chosen from the binary alphabet $\mathcal{A}_{\text{in}} = \{0, 1\}$ with equal probability. This corresponds to the types of signals commonly used in data communications. It is also assumed that the input symbols are grouped in blocks of length $p$, while the output symbols are grouped in blocks of length $q$. The constrained coder performs a mapping from the set of $p$ length binary input blocks to the set of $q$ length blocks over $\mathcal{A} = \{1, 2, 3, \ldots, L\}$.

The topological entropy of the defined system is $h(X) \approx 1.46$ bits/symbol, as shown in Figure 4.4. The amount of information that can be coded in the output sequence must be less than the topological entropy of the shift space. This follows directly from Shannon’s channel coding theorem, where the data transfer rate must be less than or equal to the channel capacity for lossless transmission of data. The constraint on the amount of information which can be transmitted in the $k_1 = 3, L = 4$ system can be written in terms of the input and output block sizes as,

$$h(X) \geq \frac{p}{q}.$$  

Stated in words, the topological entropy of the underlying graph must be greater than or equal to the number of bits/symbol encoded in the output sequence. Integer values for $p$ and $q$ are chosen so that their ratio satisfies the inequality. The choice of $p$ and $q$ affects the
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complexity of the coder design, and determines the coder latency. In this example, \( p = 4 \) and \( q = 3 \) yielding a 4b-3s coder: four input bits map to a three symbol output sequence.

The formulation of the structure of the coder was performed using a well know technique known as the ACH algorithm [59, 55]. This algorithm provides a set of guidelines which direct a series of state splittings of a higher order edge shift. At each step of the algorithm decisions are made which can affect the final form of the coder. Heuristic techniques are used to ensure that like states are merged at every step to minimise the complexity of the implementation.

In the end, the algorithm produces a state transition diagram, where the outdegree of every state is equal to \( 2^p \). Figure 4.9 presents the state transition diagram for the example 4b-3s code after applying the ACH algorithm. The branches of the state transition diagram are labelled with the input sequence and output sequence corresponding to each transition. For compactness, the sequences of four input bits are represented by the integer set \( \{0, 1, 2, 3, \ldots, 15\} \) through the traditional binary to decimal number system relationship. At each transition one of the 16 edges leaving each state is selected by the input sequence. The assignment of four bit input sequences to each edge, also known as the input road-colouring, determines the implementation complexity of the coder and is discussed in Section 4.3.6.

Using the 4b-3s coder defined in this section, Figure 4.10 shows a series of sequences at various point in the model presented in Figure 4.3. The input sequence is an unconstrained equiprobable sequence of binary symbols represented in decimal format. The output of the coder consists of the symbols taken from \( \mathcal{A} \) according to the channel constraints. The output sequence remains non-negative over the period of the simulation, as stipulated by the coder.

4.3.6 Decoder Structure

The implementation of the constrained code as given by the state transition diagram in Figure 4.9 depends on the assignment of the input symbol values to each edge. The set of labellings corresponding to the input values which choose an edge to be followed is known as the input road-colouring of the coder.

The input road-colouring has no impact on whether the channel constraints are satisfied. The code is structured so that any walk through the state transition diagram
Figure 4.9: State transition diagram of the example 4b-3s coder (Note: edges are labelled with the pairs input/output).
Figure 4.10: Critical signals in the 4b-3s code example: (a) the input sequence (in decimal format), (b) the output of the coder, (c) the output intensity signal of the filter.
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results in output sequences which are transmittable.

At the receiver a structure is necessary to decode the received symbols. The decoder must perform the inverse mapping of the coder to arrive at the binary symbols transmitted in the channel. A straightforward way in which to accomplish this decoding is to use the same state machine in the decoder as was used in the coder. The output of the decoder in this scheme depends on the input as well as an internal state variable. If some of the transmitted symbols are corrupted by noise, the internal state variable may also become corrupted. In theory, one incorrect symbol detected may permanently corrupt the internal state, causing every input sequence to be decoded incorrectly. This situation is known as infinite error propagation.

A class of decoders known as sliding block decoders alleviate this infinite error propagation problem. The data input to the decoder is broken into a series of blocks of length $q$ symbols, where the block and instant $n$ is denoted $x_n$. At instant $n$, the decoder mapping function $f$ maps the input blocks to a decoded binary block of length $p$, which is denoted $b_n$. This mapping can be represented as

$$ b_n = f(x_{n-a}, \ldots, x_n, \ldots, x_{n+m}) $$

where $m$ is known as the memory of the decoder and $a$ is the anticipation of the decoder. Therefore, the data is decoded using only the input sequence of symbol blocks. An error in decoding a given block propagates a distance of $m + a + 1$ blocks which is known as the window length of the decoder. This sliding block decoder can be implemented with the prudent choice of input road-colouring.

Table 4.1 shows the relationships between received and decoded symbol blocks which define a sliding block decoder for the 4b-3s code of Section 4.3.5. The input to the decoder is a set of blocks of length $q = 3$, chosen over $\mathcal{A}$. The memory of the decoder is one block and the anticipation is zero. Therefore an error in detecting a block will propagate for two input blocks to the decoder, or for eight output bits. The entries in table 4.1 are grouped according to the states from which they originate. There are 22 unique edge labellings for the coded data, which originate from the 3 states. No two edges coming from a given state can share the same input or output labellings. The input labellings assigned to each unique edge value depend on the states from which these edges can be generated. For the edge “334”, the input labelling depends on the value of the previous block. Analysis of the graph in Figure 4.9 shows that in this case, the previous block received at the decoder
## Section 4.3. A Constrained Code for Intensity Modulated Channels

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<th>$r$</th>
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Table 4.1: Sliding block decoder for 4b-3s code considered in the example. $u$ is the current input block and $v$ is the previous input block. $r$ is the decoded block of binary data represented in decimal format. The entries denoted by “*” indicate that any block may be received in this interval.
gives information about the previous state of the decoder. This fact is used to differentiate the two possible decoded values for this state.

The final structure of the sliding block decoder is heavily dependent on the choice of actions during the code formation and decoder determination. Decisions at every step of code construction process can lead to different structures for the coder structure. The size of the decoding window as well dependent on the process used to derive the code.

4.4 Conclusions

This chapter has introduced the concept of using constrained coding techniques to satisfy the non-negativity constraint in wireless optical channels. The use of constrained coding eliminates the assumption that non-negative pulses must be used, and applies the constraint directly to the intensity function. This scheme has applications in the use of bandwidth efficient pulses, which do not remain non-negative, but whose sum can be constrained to be non-negative.

The example of a constrained code applied to a discrete optical channel shows that constrained coding techniques are able to satisfy channel constraints over a family of pulses. The inherent trade-off in using constrained coding is an increase in complexity over simpler baseline cases. The formulation of coder and decoder structures using the ACH algorithm is highly dependent on decisions made during at each step of the algorithm. Heuristic techniques are used to achieve lower complexity implementations.

The use of a discrete channel as an example was adopted to reduce the complexity of the calculations required for the formulation of the coder. The results of this example apply directly to the set of continuous time functions of length three symbol periods where non-negativity at the sampling instants of the output signal ensures that the signal remains non-negative between instants. If this set of functions is used to bound the amplitudes of practical bandwidth efficient pulses, conservative estimates on performance are possible from this framework. Additional study is necessary to apply this work to a more general class of continuous time pulses.

This chapter is intended as an introduction to the concept of using constrained coding to satisfy the constraints of an optical channel. To the knowledge of the author, this is the first suggestion of using constrained coding to implement modulation schemes on wireless optical channels. Further work is required to extend these results to a set of bandwidth efficient pulses suitable for implementation.
Chapter 5

Conclusions and Future Directions

5.1 Conclusions

Wireless optical links are a viable solution to the problem of short distance, high data rate connections. These links have advantages over mechanical and RF wireless techniques due to their flexibility and low cost. The channel characteristics of a wireless optical channel differ significantly from those in conventional communications channels. Since the data is sent as an intensity waveform, negative amplitudes are not possible. Additionally, eye-safety constraints limit the average amplitude level of the signalling scheme.

In order to develop signalling schemes for the wireless optical channel, the channel characteristics were investigated. The optoelectronic components used to generate and detect the launched optical intensity signal are key determinants in the channel characteristics. Light emitting diodes and p-i-n photodiodes are well suited to wireless optical applications due to their low cost, high linearity and modest support circuitry requirements. Measurements on an experimental channel were performed using typical optoelectronics indicating that the wireless optical channel is limited by poor bandwidth and moderate linearity. These effects are primarily due to the fact that inexpensive optical components are needed to ensure that the proposed link has the widest field of application possible.

To surmount the channel constraints and limitations, the modulation scheme chosen for the wireless optical channel must simultaneously provide good bandwidth and power efficiency. Conventional modulation schemes for optical channels rely on binary level pulse techniques which sacrifice bandwidth efficiency for power efficiency. Multilevel schemes provide high bandwidth efficiencies at the price of reduced power efficiency. Adaptively biased
QAM is developed as a modulation scheme which preserves the high bandwidth efficiency of other multilevel techniques while improving on the power efficiency. An asymptotic gain of 3 dB is possible over PAM for large constellations. Constellation shaping was also shown to provide added power savings, especially for even sized constellations.

The non-negativity constraint of the optical channel applies to the transmitted intensity signal. Symbol pulses can be chosen so that they are spectrally efficient, so long as the resulting sum of signals remains non-negative. The use of constrained coding techniques allows for the selection of pulses which assume both positive and negative amplitudes while satisfying the non-negativity constraint. This allows a degree of freedom in the choice of pulse shape giving the system designer the ability to tradeoff of system complexity for bandwidth efficiency or power efficiency.

5.2 Future Directions

The selection of optoelectronics determines the severity of the channel limitations. The development of inexpensive, high bandwidth optoelectronics for the 1.55 µm infrared wavelength would improve the channel constraints, allowing for simpler implementations. The longer wavelength lessens the impact of the average optical power constraint as seen in Chapter 2. Higher bandwidth devices would more easily accommodate the high data rates required, and device structures which deliver these characteristics at low cost are required.

In addition to the development of advanced optoelectronics, models are required to allow system designers to more accurately predict link performance. In order to formulate a model for the optoelectronics, a sensitive design bench must be constructed to extract all relevant properties of the devices. Calibrated light emitters along with receivers are required to test the linear as well as non-linear performance of the LEDs as well as photodiodes. The model for the optoelectronics would simplify the design of linearised transmitters and receivers for future high speed links.

The development of pulse shaping techniques for intensity channels is also a necessary next step. In an effort to improve the spectral performance of existing modulation schemes, pulse shaping may be employed. The links between individual pulse shapes and the channel constraints on the amplitude must be generalised in order to determine what gains are possible with this technique. The concept of signal space diversity, introduced in Chapter 3, fits in with the pulse shaping idea since the amount of bias for each symbol
transmitted is intimately linked to the pulse shape. As a result, the use of redundant information in the average of each pulse transmitted can be used to improve the estimate of the received symbol, improving power efficiency.

This thesis has introduced the notion of employing constrained coding techniques to satisfy the non-negativity constraint of optical channels. Further generalisation of this concept to a set of continuous time pulses is required, as well as a study of candidate pulse shapes that optimise performance on the link. Simulation with an accurate channel model of the link is required to explore the practicality of using constrained coding techniques in optical channels.
Appendix A

Optimum Binary Pulse Set for Average Constrained Channels

A.1 Introduction

Intensity modulated channels are limited by the fact that all signals must remain positive for all time and that the average amplitude is limited. This appendix presents a squared distance argument which is used to develop an optimum set of two signals for binary signalling on an intensity channel.

A.2 Problem Definition

Let $\gamma_1(t)$ and $\gamma_2(t)$ be the two time domain representations of the intensity signal set which are time limited to $t \in [0,T_s]$, where $T_s$ is the symbol interval. Furthermore, assume that each symbol interval is divided into $M$ disjoint time intervals of equal duration. If the intensity is taken not to vary in each sub-interval, then the set of time functions in which only one interval is non-zero may be taken as a set of basis functions of the functions $\gamma_1(t)$ and $\gamma_2(t)$. More formally, the basis functions can be written as

$$\phi_i(t) = \sqrt{\frac{M}{T_s}} \text{rect} \left( \frac{t - T_s/M(i - 1/2)}{T_s/M} \right)$$  \hspace{1cm} (A.1)

where $i = 1, 2, \ldots, M$ and $\text{rect}(\cdot)$ is the rectangular window functions as defined in Section 3.2.
Section A.2. Problem Definition

The signals can be represented as a pair of \( L \) dimensional vectors, \( \hat{\gamma}_1 \) and \( \hat{\gamma}_2 \), in the signal space defined by the \( \phi_i(t) \) basis functions. These signal vectors can be broken into a set of components to give

\[
\hat{\gamma}_1 = \{\gamma_{11}, \gamma_{12}, \ldots, \gamma_{1M}\}
\]
\[
\hat{\gamma}_2 = \{\gamma_{21}, \gamma_{22}, \ldots, \gamma_{2M}\}.
\] (A.2)

The channel constraints can now be applied to the components of \( \hat{\gamma}_1 \) and \( \hat{\gamma}_2 \).

The non-negativity constraint forces all of the components of the signal vectors to be non-negative. Since the basis functions are always non-negative, the vector components must satisfy

\[ \gamma_{ij} \geq 0, \quad i = 1, 2 \quad j = 1, 2, 3, \ldots, M. \]

The average amplitude constraint forces the average of the output signal to be fixed. The average value of each signal, \( \text{avg}(\gamma_i) \) is specified as

\[ \text{avg}(\gamma_i) = \frac{1}{T_s} \int_0^{T_s} \gamma_i(t) \, dt. \]

The expression for each symbol average can be simplified by re-writing \( \gamma_i \) in terms of its basis functions, giving

\[ \text{avg}(\gamma_i) = \frac{1}{T_s} \int_0^{T_s} \sum_{m=1}^{M} \gamma_{im} \phi_m(t) \, dt. \]

Since both the summation and integration are over finite intervals, they can be exchanged yielding,

\[ \text{avg}(\gamma_i) = \sum_{m=1}^{M} \gamma_{im} \frac{1}{T_s} \int_0^{T_s} \phi_m(t) \, dt. \]

The average value of the amplitude of each basis function is the same, and is represented as the constant \( K_\phi \). Simplifying the expression for symbol average gives,

\[ \text{avg}(\gamma_i) = K_\phi \sum_{m=1}^{M} \gamma_{im}. \] (A.3)

Assuming that the two signals are sent independently and equiprobably, the expected value of the symbol averages is proportional to the sum of the symbol averages. If the average amplitude of the modulation scheme is fixed at \( P \), then the sum of the symbol averages can be related to \( P \) through (A.3) as

\[ \sum_{i=1}^{2} \sum_{m=1}^{M} \gamma_{im} \propto P. \]
So, the average amplitude constraint reduces to the following constraint on the sum of the components of $\gamma_1$ and $\gamma_2$

$$\sum_{i=1}^{2} \sum_{m=1}^{M} \gamma_{im} = K$$

for some constant $K$.

Chapter 3 discusses how the distance between constellation points is indicative of the reliability of transmission for a given modulation scheme. The figure of merit of a set of signal points is taken as the square distance between them in the signal space. The optimal scheme in this sense can be determined by maximising the distance between constellation points subject to the channel constraints.

The non-negativity constraint and the limited average amplitude constraint are then just limitations on the values and sum of the components of the signal vectors in the defined signal space. Since the constraints and the objective function to maximise are vector quantities, the problem definition is formulated as a generic vector problem.

**Find** :

$\gamma_1$ and $\gamma_2$ are two $L$ dimensional vectors in a Euclidean $L$ space. The components of these vectors are given as in (A.2).

**Find** $\hat{\gamma}_1$ and $\hat{\gamma}_1$ such that $\|\gamma_1 - \gamma_2\|^2$ is maximised subject to the constraints :

1. $\gamma_{ij} \geq 0, \ i = 1, 2 \ j = 1, 2, 3, \ldots, M$

2. $\sum_{i=1}^{2} \sum_{m=1}^{M} \gamma_{im} = K$

### A.3 Development

Since the channel constraints are written in terms of the co-ordinate values of the vectors, the objective function is rewritten in terms of components values as,

$$\|\gamma_1 - \gamma_2\|^2 = \sum_{m=1}^{M} (\gamma_1m - \gamma_2m)^2$$

Re-arranging the terms,

$$\|\gamma_1 - \gamma_2\|^2 = \sum_{m=1}^{M} \gamma_1^2m + \gamma_2^2m - 2 \sum_{m=1}^{M} \gamma_1m \gamma_2m$$

$$= \|\gamma_1 + \gamma_2\|^2 - 4 \langle \gamma_1, \gamma_2 \rangle$$  \hspace{1cm} (A.4)
where $\langle \cdot, \cdot \rangle$ is defined as the inner product between two vectors. This new form of the problem re-casts the original problem of maximising the square distance between them to one of simultaneously maximising the magnitude of the sum vector, while minimising the inner product. Figure A.1 illustrates this re-arrangement of the problem in the $M$ dimensional space by the two dimensional plane containing the vectors. Along with the vectors in question, the hyperplane defined by constraint 2 due to the average limitation is also drawn. Note that the vector $\hat{\gamma}_1 + \hat{\gamma}_2$ will always reside on this hyperplane.

The individual terms in the re-arrangement in (A.4) can be examined to determine the extrema and the conditions under which they occur.

Let $\hat{\theta} = \hat{\gamma}_1 + \hat{\gamma}_2$. The goal is to maximise $||\hat{\theta}||^2$ subject to the constraints outlined earlier. Constraint 2 applied to the co-ordinate values of $\hat{\theta}$, $\theta_i$, gives the following relation:

$$\sum_{i=1}^{M} \theta_i = K.$$  \hspace{1cm} (A.5)

Due to this relation, the vector $\hat{\theta}$ can be re-written as,

$$\hat{\theta} = K \cdot \{\alpha_1, \alpha_2, \ldots, \alpha_M\}$$

where the $\alpha_i$ are real, non-negative constants which satisfy the relations $\alpha_i \leq 1$ and
\[ \sum_{i=1}^{M} \alpha_i = 1 \]. An expression for \( \| \hat{\theta} \|_2 \) can be given by performing the operation \( \langle \hat{\theta}, \hat{\theta} \rangle \) to give,

\[ \| \hat{\theta} \|_2^2 = K^2 \cdot \sum_{i=1}^{M} \alpha_i^2. \]

Since \( 0 \leq \alpha_i \leq 1 \) then \( \alpha_i^2 \leq \alpha_i \). So, if \( \sum_{i=1}^{M} \alpha_i = 1 \), then \( \sum_{i=1}^{M} \alpha_i^2 \leq 1 \). Applying this inequality gives,

\[ \| \hat{\theta} \|_2^2 = K^2 \cdot \sum_{i=1}^{M} \alpha_i^2 \leq K^2. \]  

(A.6)

Therefore, \( \max \{ \| \hat{\gamma}_1 + \hat{\gamma}_2 \|_2^2 \} = K^2. \)

The second term of the objective function in (A.4) consists of the inner product between signal vectors \( \hat{\gamma}_1 \) and \( \hat{\gamma}_2 \). This inner product is the sum of a series of products of corresponding co-ordinate values in the two vectors. Since all co-ordinate values are constrained to be non-negative by constraint 1, the minimum sum possible is 0.

Using Figure A.1 and geometric intuition, the choice of \( \hat{\gamma}_1 \) and \( \hat{\gamma}_2 \) can be directed by the expansion in (A.4). To achieve the minimum possible inner product, \( \hat{\gamma}_1 \) and \( \hat{\gamma}_2 \) must be orthogonal. Additionally, to have the maximum possible magnitude for \( \hat{\theta} = \hat{\gamma}_1 + \hat{\gamma}_2 \), from (A.6),

\[ \sum_{i=1}^{M} \alpha_i^2 = \sum_{i=1}^{M} \alpha_i = 1. \]

This equality holds in the case that \( \alpha_i^2 = \alpha_i \). In this case, the \( \alpha_i \) must be chosen from the set \( \{0,1\} \), where \( \alpha_i^2 = \alpha_i \). Equation (A.5) implies that all the \( \theta_i \) co-ordinates are zero except for at one co-ordinate value, \( \theta_{i_o} \), where \( \theta_{i_o} = K \). From the definition, \( \theta_{i_o} = \gamma_{1i_o} + \gamma_{2i_o} = K \). If the inner product \( \langle \hat{\gamma}_1, \hat{\gamma}_2 \rangle = 0 \), which is the minimum possible, this implies that \( \gamma_{1i_o} \gamma_{2i_o} = 0 \), since all other co-ordinate values are zero. The two constraints on the \( i_o \) co-ordinate values can only be met if one co-ordinate is set to 0 while the other is set at \( K \).

Therefore, setting the signal vectors \( \hat{\gamma}_1 \) and \( \hat{\gamma}_2 \) as

\[ \begin{align*}
\hat{\gamma}_1 &= 0 \\
\hat{\gamma}_2 &= \{ \mathbf{v} \in \mathbb{R}^M \mid v_m = K \cdot \delta_{m_o}, \mathbf{v} = \{v_m\}, m = 1, 2, \ldots, M, m_o \in \{1, 2, \ldots, M\} \}
\end{align*} \]

simultaneously maximises the term \( \| \hat{\gamma}_1 + \hat{\gamma}_2 \|^2 \) while minimising \( \langle \hat{\gamma}_1, \hat{\gamma}_2 \rangle \) of (A.4). This implies that the above choice for signal vectors maximises the square distance between the signal points while satisfying constraints 1 and 2.
Section A.3. Development

Using the basis functions in (A.1), a time domain representation for the signal points can be given for \( t \in [0, T_s] \) as

\[
\begin{align*}
\gamma_1(t) &= 0 \\
\gamma_2(t) &= 2MP \cdot \text{rect}\left(\frac{t - T_s/M(i - 1/2)}{T_s/M}\right)
\end{align*}
\]

(A.7)

for some \( i \in \{1, 2, \ldots, M\} \).

If these pulses were transmitted on the linear, Gaussian white noise limited channel described in Section 3.1.2, it is possible to determine key metrics to allow for comparison with other schemes. For this optimal signalling set, the constellation figure of merit, described in (3.14), can be found to be,

\[
\text{CFM}_{\text{opt}} = \frac{2MR}{\sqrt{MR_b}}
\]

(A.8)

where \( R_b = R_s = 1/T_s \), as defined in Chapter 3. Similarly, the bandwidth efficiency can be shown to be

\[
\frac{R_b}{B} = \frac{1}{M} \text{ bits/s Hz}.
\]

(A.9)
Appendix B

Minimum Bandwidth, Non-Negative Nyquist Pulse

B.1 Purpose

Optical intensity modulated channels impose the constraint that transmitted signals must remain non-negative for all time. This is due to the fact that optoelectronic components can only modulate or detect the presence of an optical power signal. Since data is transmitted as a power signal on the channel, no negative values are physically possible.

In a conventional channel, the minimum bandwidth required for zero ISI transmission is $\pi/T$ rad/s (where $T$ is the sampling period). The purpose of this appendix is to identify the pulse which occupies the minimum bandwidth required to meet the Nyquist I criterion while satisfying the optical channel non-negativity constraint.

B.2 Problem Definition

Let $p(\cdot)$ be a real valued, smooth, continuous function of time $t$, $t \in \mathbb{R}$. Let $p(t)$ have a Fourier transform $P(\omega)$. Additionally, let $p(t)$ satisfy the following constraints:

1. $P(\omega) = 0$, for $|\omega| \geq B$, $B > \pi/T$.

2. $p(kT) = \delta_k$, where $T$ is the sampling period, $k \in \mathbb{I}$.

3. $p(t) \geq 0 \forall t \in \mathbb{R}$. 

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Find $p(t)$ satisfying above constraints such that $B$ is minimised.

## B.3 Development

Constraint (2) is the well known Nyquist I constraint for zero ISI. It can be cast in frequency domain by taking the Fourier transform, resulting in

$$\sum_{m=-\infty}^{+\infty} P\left(\omega - m \frac{2\pi}{T}\right) = T. \tag{B.1}$$

Constraint (2) along with (3) ensures that $p(t)$ will attain its minimum value of 0 at all non-zero integer sampling instants (i.e., at $t = kT, k \neq 0$). Since $p(t)$ is a smooth, continuous function, the extrema at all sampling instants, other than $t = 0$, are accompanied by the first derivative of $p(t)$ becoming zero. At $t = 0$, $p(0)$ is fixed by (2), however, the value of the first derivative is not specified explicitly. The observations above can be summarised as

$$\left.\frac{dp}{dt}\right|_{t=kT} = c \cdot \delta_k, \tag{B.2}$$

for some $c \in \mathbb{R}$. Figure B.1 illustrates the above observation along with (2) together in the time domain.

As was done in (B.1), (B.2) can be represented in the frequency domain as

$$\sum_{m=-\infty}^{+\infty} \left(\omega - m \frac{2\pi}{T}\right) P\left(\omega - m \frac{2\pi}{T}\right) = cT. \tag{B.3}$$

This infinite sum can be simplified by applying bounds on $B$ and $\omega$. Constraint (1) stipulates that $P(\omega)$ is bandlimited to $B$ rad/s. Since $B$ is being minimised no loss in generality is incurred by letting $B \in (\pi/T, 2\pi/T]$. The relation in (B.3) must be satisfied over every interval of $\omega$. Due to the bounds on $B$, in the interval $\omega \in (-2\pi/T, 2\pi/T]$, (B.3) reduces to

$$\left(\omega + \frac{2\pi}{T}\right) P\left(\omega + \frac{2\pi}{T}\right) + \omega P(\omega) + \left(\omega - \frac{2\pi}{T}\right) P\left(\omega - \frac{2\pi}{T}\right) = -jcT.$$

Expanding and collecting like terms gives

$$\omega \sum_{n=-1}^{1} P\left(\omega - n \frac{2\pi}{T}\right) + \frac{2\pi}{T} \left( P\left(\omega + \frac{2\pi}{T}\right) - P\left(\omega - \frac{2\pi}{T}\right) \right) = -jcT. \tag{B.4}$$

Equation (B.1) can be used to simplify the sum in the first term of (B.4). Since $B$ is restricted to less than $2\pi/T$, over the interval $\omega \in (-2\pi/T, 2\pi/T]$, (B.1) reduces to the sum
in the first term of (B.4). Applying this further simplification to (B.4) yields
\[ \omega T + \frac{2\pi}{T} \left( P \left( \omega + \frac{2\pi}{T} \right) - P \left( \omega - \frac{2\pi}{T} \right) \right) = -j c T. \]

The resulting sum of two shifted version of \( P(\omega) \) can be written over two distinct ranges of \( \omega \) as
\[ P \left( \omega + \frac{2\pi}{T} \right) = -\frac{T^2}{2\pi} (\omega + j c), \quad \omega \in (-2\pi/T, 0] \]  
\[ \text{(B.5)} \]
and
\[ P \left( \omega - \frac{2\pi}{T} \right) = \frac{T^2}{2\pi} (\omega + j c), \quad \omega \in (0, 2\pi/T]. \]  
\[ \text{(B.6)} \]
Furthermore, the above expressions show that (B.5) and (B.6) are not satisfied for \( B < 2\pi/T \) in \( \omega \in \{(0, \frac{2\pi}{T} - B] \cup (-\frac{2\pi}{T} + B, 0)\}. \) For \( B < 2\pi/T, \) the left hand side of (B.5) and (B.6) go to zero over a finite interval where the right hand sides remain non-zero. Thus, the minimum bandwidth under which these constraints are met is \( B = 2\pi/T \) rad/s.

Expressions (B.5) and (B.6) provide enough information to get an expression for \( P(\omega) \). Looking at real and imaginary components of \( P(\omega) \) separately,
\[ \text{Re}\{P(\omega)\} = \begin{cases} 
\frac{T^2}{2\pi} (\omega + \frac{2\pi}{T}) & : -\frac{2\pi}{T} < \omega \leq 0 \\
-\frac{T^2}{2\pi} (\omega - \frac{2\pi}{T}) & : 0 < \omega \leq \frac{2\pi}{T} \\
0 & : \text{otherwise} 
\end{cases} \]  
\[ \text{(B.7)} \]
The expressions for the real and imaginary components of $P(\omega)$ are plotted in Figure B.2.

Since the Fourier transform is linear, the inverse transform of the real and imaginary components can be calculated separately and added in time domain. After some simplification, the resulting expression for $p(t)$ is:

$$p(t) = (1 + ct) \cdot \frac{\sin^2(\pi t/T)}{(\pi t/T)^2}$$  \hspace{1cm} (B.9)

The value of $c$ can be constrained by noting that the condition in (B.2) is a necessary but not sufficient condition of (3). That is, the set of all non-zero, Nyquist pulses are a subset of the set of functions described in the $p(t)$ expression in (B.9). Applying (3) to (B.9) directly,

$$(1 + ct) \cdot \frac{\sin^2(\pi t/T)}{(\pi t/T)^2} \geq 0 \ \forall t, c \in \mathbb{R} \implies c = 0$$  \hspace{1cm} (B.10)
Therefore, the minimum bandwidth Nyquist pulse which remains non-negative occupies a bandwidth of $2\pi/T$ rad/s and takes the form:

$$p(t) = \frac{\sin^2(\pi t/T)}{(\pi t/T)^2}$$

(B.11)
Bibliography


Bibliography


