COMP ENG 4TL4 – Digital Signal Processing

Homework Assignment #1

Submission deadline: 12 noon on Friday, October 3, 2003, in the designated drop box in CRL-101B (the CRL photocopying room).

1. Using the definition of *linearity* given in Lecture #5, show that the following two systems are both linear.

   a. The ideal-delay system, i.e., $y[n] = \mathcal{T}_a\{x[n]\} = x[n-n_d]$, where n_d is a fixed positive integer called the delay of the system.

   b. The moving-average system described by $y[n] = \mathcal{T}_b\{x[n]\} = \frac{1}{M_1 + M_2 + 1}\sum_{k=-M_1}^{M_2} x[n-k]$. **(20 pts)**

2. Determine which of the following signals are periodic. If a signal is periodic, determine its period.

   a. $x[n] = \mathrm{e}^{\mathrm{j}(2\pi n/5)}$

   b. $x[n] = \sin(\pi n/19)$

   c. $x[n] = n\,\mathrm{e}^{\mathrm{j}\pi n}$

   d. $x[n] = \mathrm{e}^{\mathrm{j}n}$ **(20 pts)**

3. The sequence:

   $$x[n] = \sin\left(\frac{\pi}{2}n\right), \qquad -\infty < n < \infty,$$

   was obtained by sampling a continuous-time signal:

   $$x_c(t) = \sin(\Omega_0 t), \qquad -\infty < t < \infty,$$

   at a sampling frequency f_s = 2 kHz.

   a. What are two possible *positive* values of Ω_0 that could have resulted in the sequence $x[n]$?

   b. Explain these frequency values in light of the derivation of the Nyquist sampling theorem given in Lecture #2. **(20 pts)**

4. An ideal lowpass filter has been implemented via the cascade of an A/D converter, a discrete-time ideal lowpass filter, and a D/A converter. The discrete-time ideal lowpass filter is known to have a discrete-time cutoff frequency $\omega_c = \pi/5$ radians.

   a. If $x_c(t)$ is bandlimited to 3 kHz, what is the minimum sampling frequency f_s required by the A/D converter to avoid aliasing?

   b. If f_s = 10 kHz, what will be the effective continuous-time cutoff frequency Ω_c of the ideal lowpass filter? **(20 pts)**

   Continued on the next page!

5. A particular digital communication channel is capable of transmitting 19200 bits per second. We wish to use the channel to transmit a band-limited analog signal $x_c(t)$, by sampling and digitizing. The magnitude of the analog signal is limited to $|x_c(t)| \leq X_m$. The error between the digitized signal and $x_c(t)$ must not exceed $\pm 10^{-4} X_m$.

   a. What is the required number of bits in the A/D, assuming a uniform rounding quantizer?

   b. What is the maximum bandwidth of the analog signal for which the channel can be used? **(20 pts)**