COMP ENG 4TL4 – Digital Signal Processing

Homework Assignment #5

Submission deadline: 12 noon on Monday, December 8, 2003, in the designated drop box in CRL-101B (the CRL photocopying room).

1. Four values of a stationary random process $x[n]$ have been observed, giving the discrete-time sequence $v[n] = \{0.05, –0.3, 0.5, –1\}$.

   a. Use the *autocorrelation-based method* of spectral estimation to estimate the PSD of $x[n]$, using a triangular correlation window $w_c[m] = \{0.5, 1, 0.5\}$.

   b. Does $x[n]$ appear to be lowpass, bandpass or highpass? How confident should you be of this signal characterization, given the number of values observed and the length of the correlation window used? **(20 pts)**

2. Consider an LTI system with the transfer function:

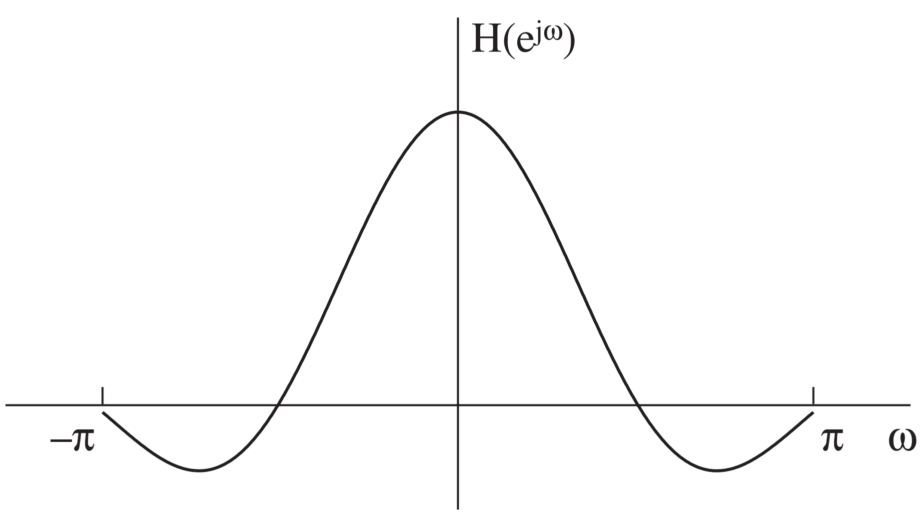
$$H(z) = \frac{z^{-2}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-3z^{-1}\right)}.$$

   a. Suppose that the system is known to be stable. Determine the output $y[n]$ when the input $x[n]$ is the unit step sequence.

   b. Suppose that the ROC of $H(z)$ includes $z = \infty$. Determine $y[n]$ evaluated at $n = 2$ when $x[n]$ is the sequence $\{1, 2, 3\}$ for $n = 0$, 1 and 2, and is zero otherwise.

   c. Suppose we wish to recover $x[n]$ from $y[n]$ by processing $y[n]$ with a LTI system whose impulse response is given by $h_i[n]$. Determine $h_i[n]$. Does $h_i[n]$ depend of the ROC of $H(z)$? **(30 pts)**

3. The Fourier transform of a stable LTI system is purely real-valued and is shown in the figure below. Determine whether this system has a stable inverse system. **(10 pts)**



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4. Consider the finite sequence $x[n] = \{0, 0.5878, 0.9511, 0.9511, 0.5878, 0\}$.

   a. Calculate the DFT of this sequence $X[k],\ \ 0 \le k \le N-1$. Apply lossy compression to the vector of DFT coefficients $X[k]$ by ignoring (i.e., setting to zero) the coefficients for $k = 2$, 3 and 4, to obtain the compressed vector $\tilde{X}[k]$. Reconstruct the sequence $\tilde{x}[n]$ by taking the inverse DFT of $\tilde{X}[k]$.

   b. Calculate the root mean squared (RMS) error between $\tilde{x}[n]$ and $x[n]$. Express this RMS error as a percentage of the RMS of $x[n]$. **(20 pts)**

5. Consider the causal LTI system:

   $$H(z) = 1 - z^{-1}.$$

   a. Show that $H(z)$ has generalized linear phase by expressing the system's frequency response in the form $H(e^{j\omega}) = A(e^{j\omega})e^{j(\beta-\omega\alpha)}$, where $A(e^{j\omega})$ is a real-valued function and β and α are constants.

   b. Determine which standard type of FIR generalized linear-phase filter $H(z)$ is:

      Type I, II, III, or IV. **(20 pts)**