

COMP ENG 4TL4:

# Digital Signal Processing

Notes for Lecture #22

Wednesday, October 29, 2003

## 6.7 FIR Filter Design

Transfer function:

$$H(z) = \sum_{k=0}^M b[k] z^{-k} = \sum_{k=0}^M h[k] z^{-k}.$$

Frequency response:

$$H(e^{j\omega}) = \sum_{n=0}^M b[n] e^{-j\omega n} = \sum_{n=0}^M h[n] e^{-j\omega n}.$$

Advantages of FIR Filters:

- FIR filters are *inherently stable*
- they can be designed to have *linear phase* or *generalized linear phase*
- they are *convenient to implement*

## Impulse response truncation method:

- Step 1:** Choose the desired amplitude response according to the filter class (LP, HP, BP, BS)
- Step 2:** Choose the filter's phase characteristics: integer or fractional group delay, initial phase
- Step 3:** Write the ideal frequency response as  $H_{\text{id}}(\omega) = A_{\text{id}}(\omega) \exp\{j(\beta_{\text{id}} - \omega\alpha_{\text{id}})\}$  and compute the ideal impulse response  $h_{\text{id}}[n]$  using the inverse DTFT
- Step 4:** Truncate the impulse response by applying a rectangular window of length  $M+1$ :

$$h[n] = \begin{cases} h_{\text{id}}[n] , & 0 \leq n \leq M, \\ 0 , & n > M. \end{cases}$$

Example: Let the *ideal lowpass frequency response* be:

$$H_{\text{id}}(e^{j\omega}) = \begin{cases} e^{-j\omega M/2}, & 0 \leq |\omega| \leq \pi/4, \\ 0, & \text{otherwise,} \end{cases}$$

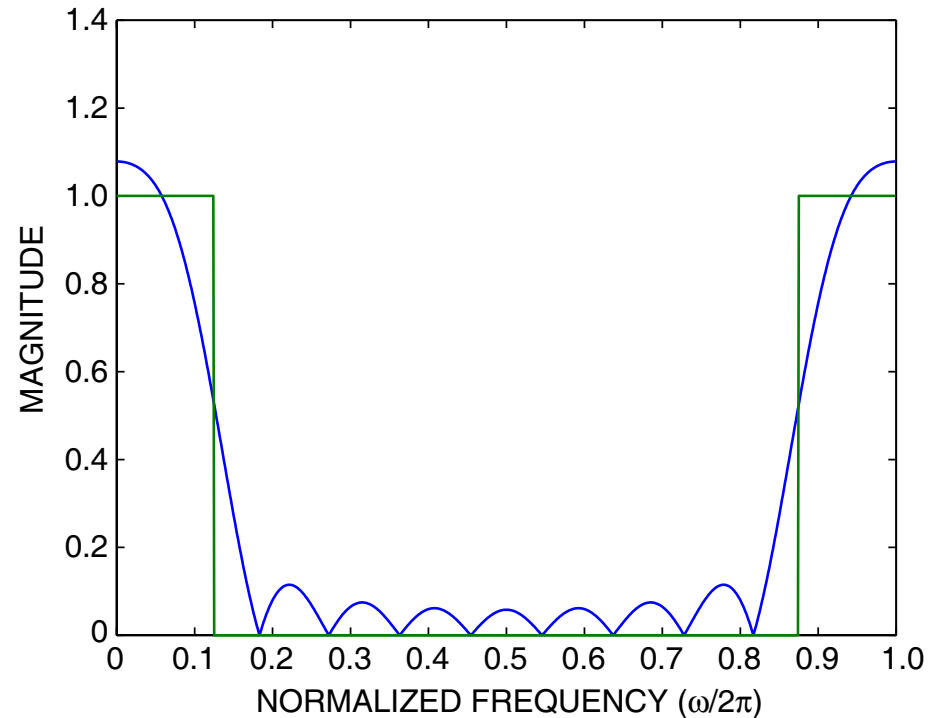
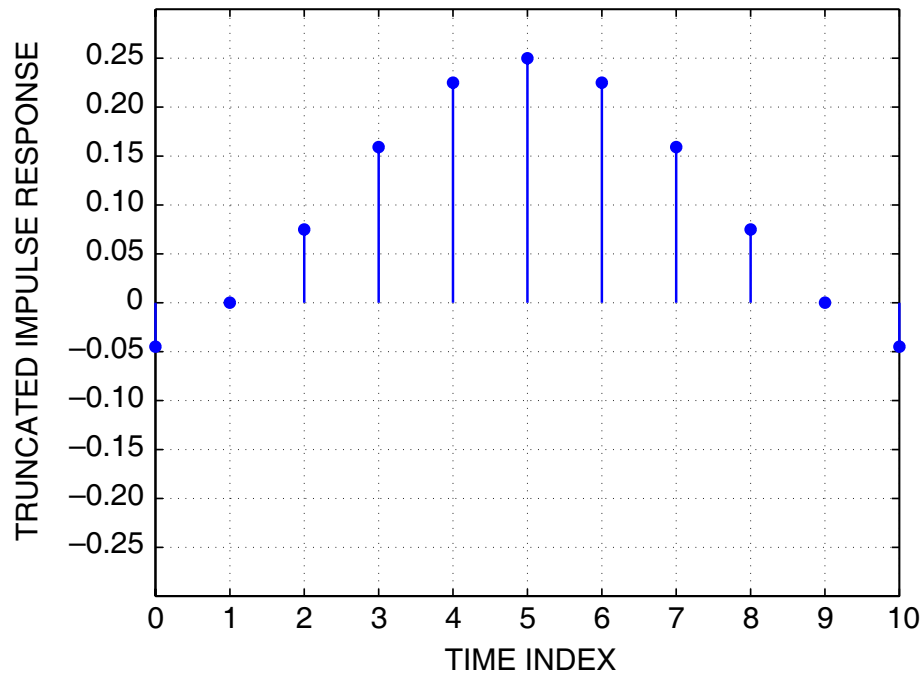
where the linear phase factor has been incorporated into the desired frequency response.

The corresponding ideal impulse response is:

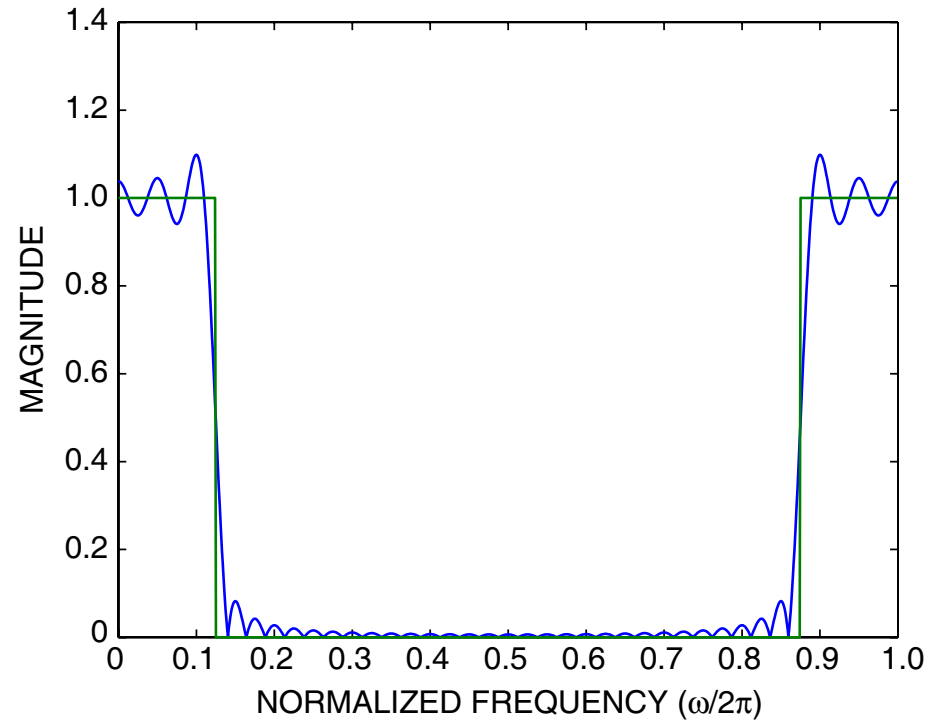
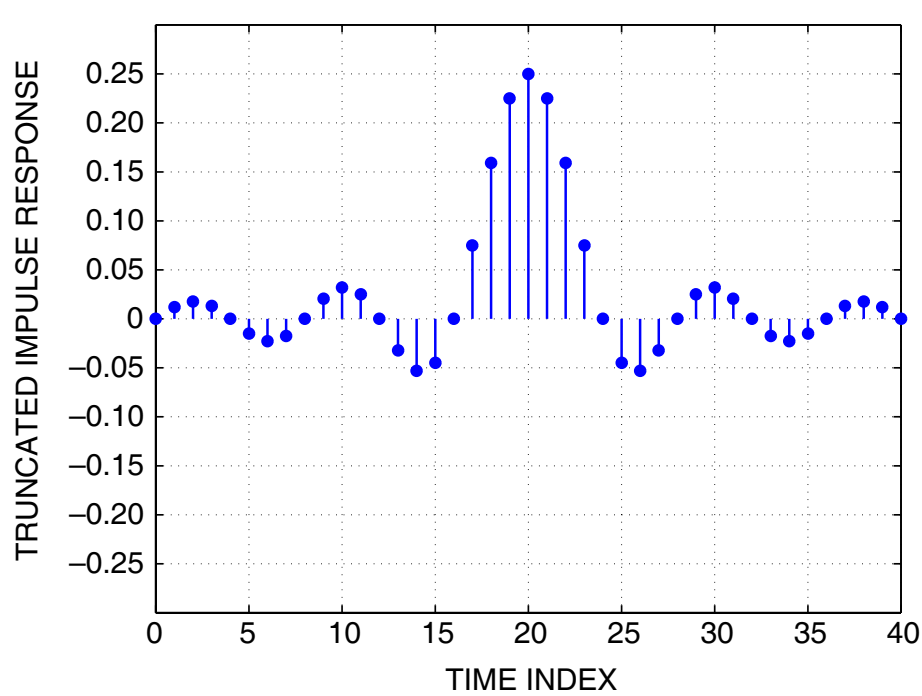
$$\begin{aligned} h_{\text{id}}[n] &= \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} e^{-j\omega M/2} e^{j\omega n} d\omega \\ &= \frac{\sin(\pi(n - M/2)/4)}{\pi(n - M/2)}. \end{aligned}$$

$h_{\text{id}}[n]$  is symmetric around  $M/2$ , so applying a rectangular window of length  $M+1$  will result in a linear-phase system of order  $M$  and length  $M+1$  with the impulse response  $h[n] = h_{\text{id}}[n]$ ,  $0 \leq n \leq M$ .

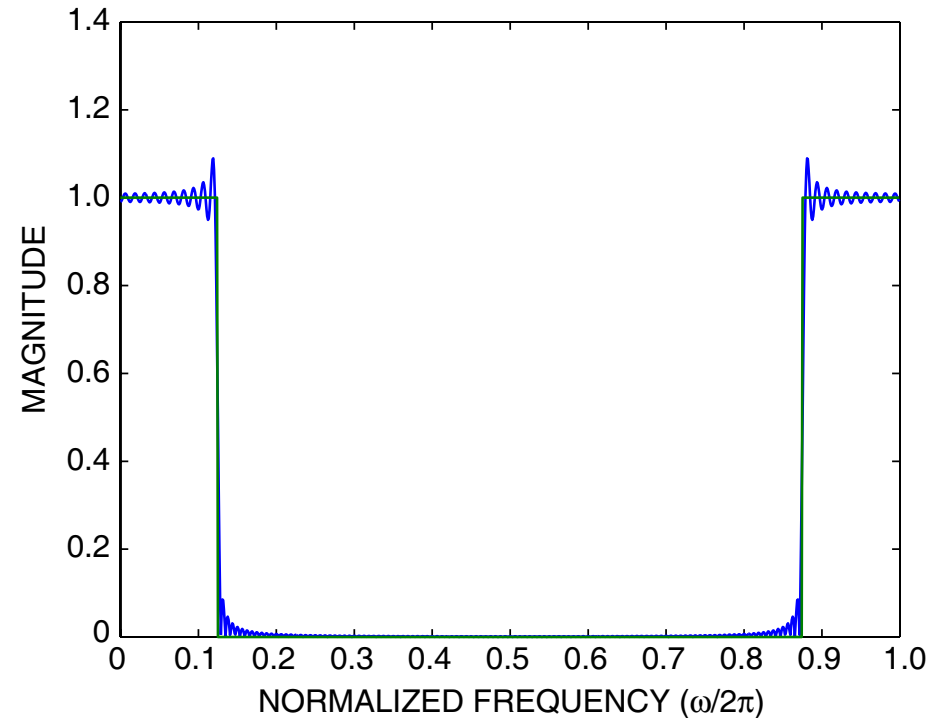
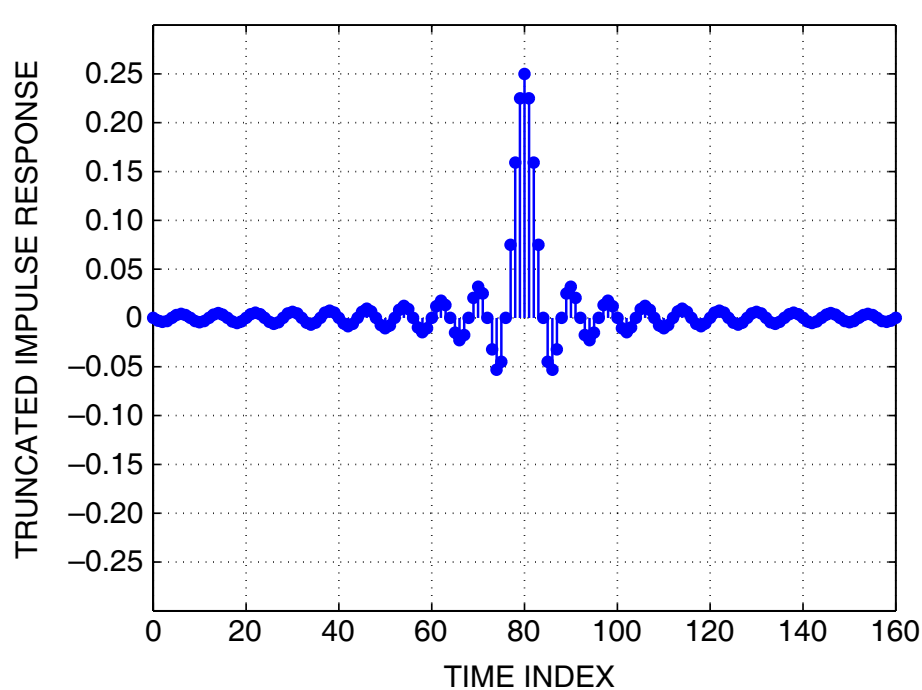
Truncated impulse response ( $M = 10$ ) and corresponding approximation of the ideal lowpass frequency response:



Truncated impulse response ( $M = 40$ ) and corresponding approximation of the ideal lowpass frequency response:



Truncated impulse response ( $M = 160$ ) and corresponding approximation of the ideal lowpass frequency response:

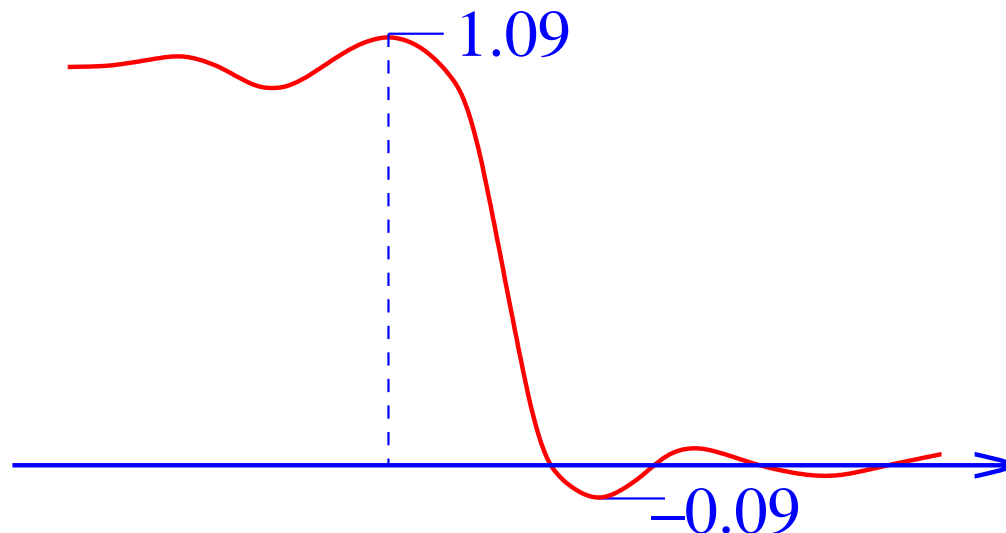


## Gibbs phenomenon:

Oscillations at the edges of the pass- and stopband **cannot be reduced** by the increase of  $M$ ! For any  $M$ , these oscillations correspond to about 0.09 passband ripple and stopband attenuation parameters!

⇒ The impulse response truncation method is a simple but not very good method of filter design.

### The Gibbs phenomenon





## The effects of windowing on the actual frequency response:

In the preceding slides it was shown that truncation of an ideal impulse response by application of a rectangular window produces non-zero passband ripple and stopband attenuation parameters.

However, the actual frequency response oscillates around the ideal frequency response, so we could consider a smoothing (i.e., averaging) operation in the frequency domain to reduce these oscillations. If we convolve the frequency response with a smoothing function in the frequency domain, this is equivalent to windowing (multiplying) in the time domain.

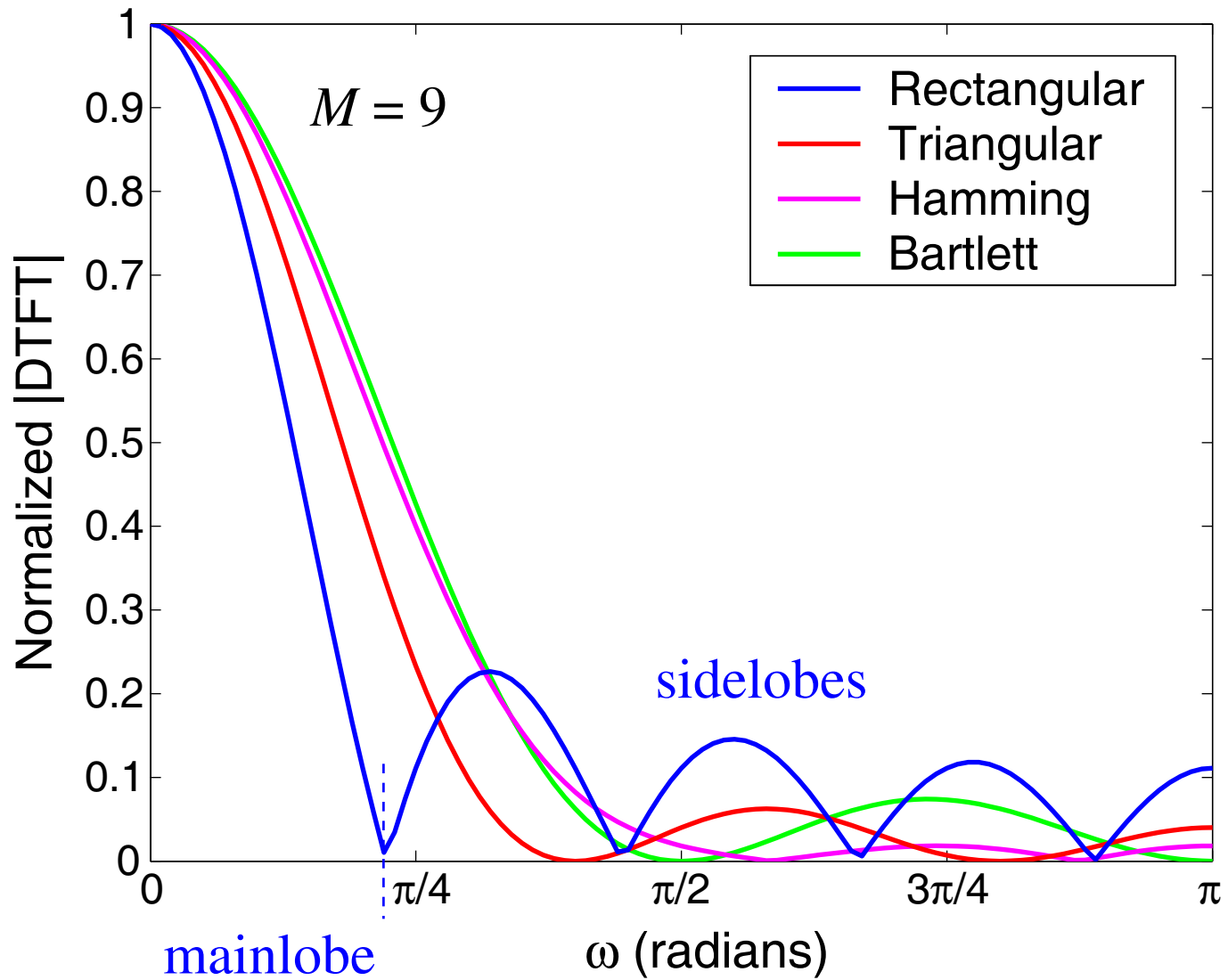
However, we must be careful with our choice of windows if we want the resulting impulse response to remain (generalized) linear-phase.

## Alternative interpretation:

We may also view the truncation procedure as multiplication by a rectangular window in the time domain  $\Rightarrow$  convolution in the frequency domain of the ideal frequency response with the DTFT of the rectangular window.

You will recall from Lecture # 15 that the rectangular window has the narrowest mainlobe of the common window types. This narrow mainlobe produces a fairly sharp transition region in the achieved frequency response.

However, the rectangular window has the largest sidelobe magnitudes – these sidelobes are the cause of the non-zero passband ripple and stopband attenuation in the truncation method. Consequently, we may wish to use a window with smaller magnitude sidelobes to reduce the passband ripple and stopband attenuation, at the expense of have a less sharp transition region.



## FIR filter design using windows:

**Step 1:** Define the ideal frequency response as in the impulse response truncation method

**Step 2:** Obtain the impulse response  $h_{\text{id}}[n]$  of this ideal filter as in the impulse response truncation method

**Step 3:** Compute the coefficients of the filter by:

$$h[n] = \begin{cases} h_{\text{id}}[n] w[n] , & 0 \leq n \leq M, \\ 0 , & n > M, \end{cases}$$

where  $w[n]$  is some chosen window function.

## Window properties:

- windows are always *real and symmetric*, i.e., they must satisfy:

$$w[n] = w[M - n], \quad n = 0, \dots, M,$$

for either even or odd  $M$ . If the ideal FIR filter has linear phase, symmetric windows do not affect this property.

- windows must be *positive* to avoid any changes of sequence polarities
- a “good” window must satisfy the tradeoff between the *width of the mainlobe* and the *magnitudes of the sidelobes* in the frequency domain
- a “good” window must have a *smooth transition* at the ends in order to reduce the Gibbs phenomenon
- the *window length* must correspond to the sequence length

Classical window types (have been derived based on intuition and educated guesses):

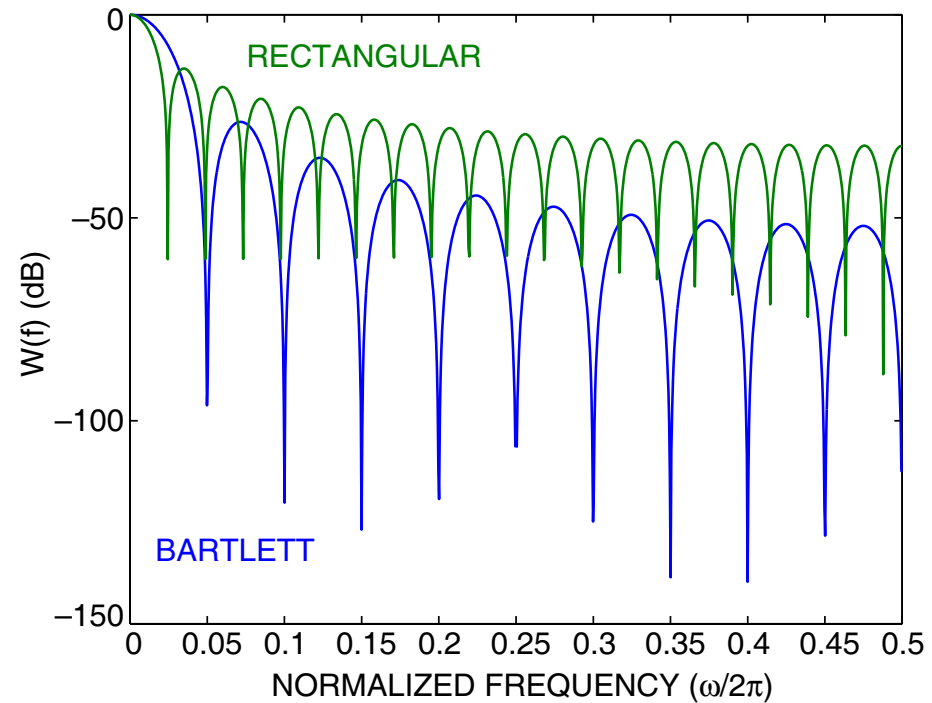
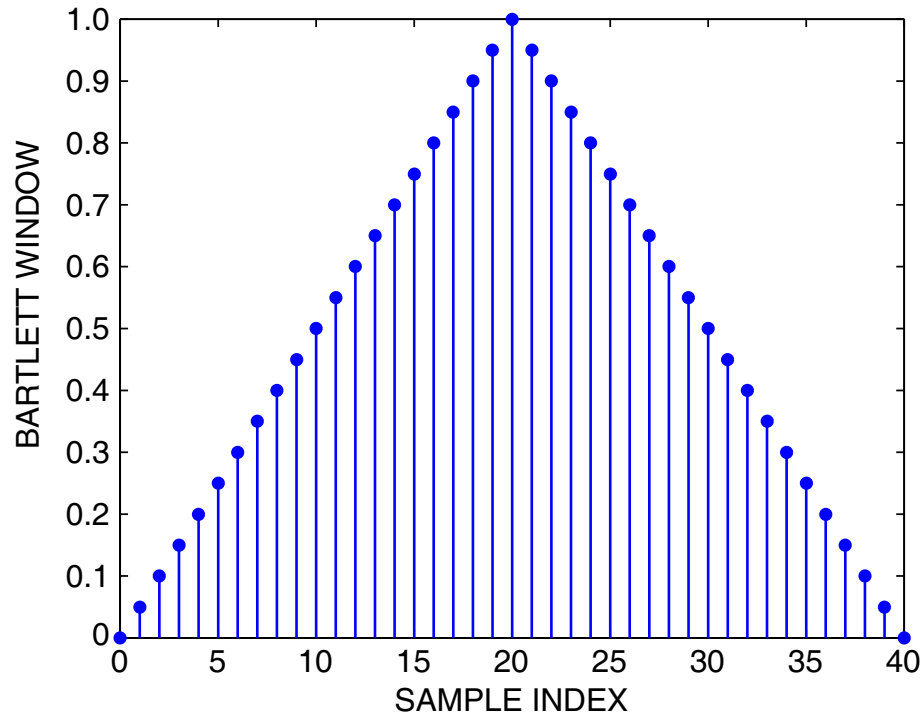
- Rectangular window: (corresponds to natural truncation like in the impulse response truncation method!)

$$w[n] = \begin{cases} 1, & 0 \leq n \leq M, \\ 0, & \text{otherwise.} \end{cases}$$

- Triangular (or Bartlett) window:

$$w[n] = \begin{cases} 2n/M, & 0 \leq n \leq M/2, \\ 2 - 2n/M, & M/2 < n \leq M, \\ 0, & \text{otherwise.} \end{cases}$$

# Bartlett window compared to the rectangular window in the frequency domain: ( $M = 40$ )



– Hanning (or Hann) window:

$$w[n] = \begin{cases} 0.5 - 0.5 \cos(2\pi n/M), & 0 \leq n \leq M, \\ 0, & \text{otherwise.} \end{cases}$$

– Hamming window:

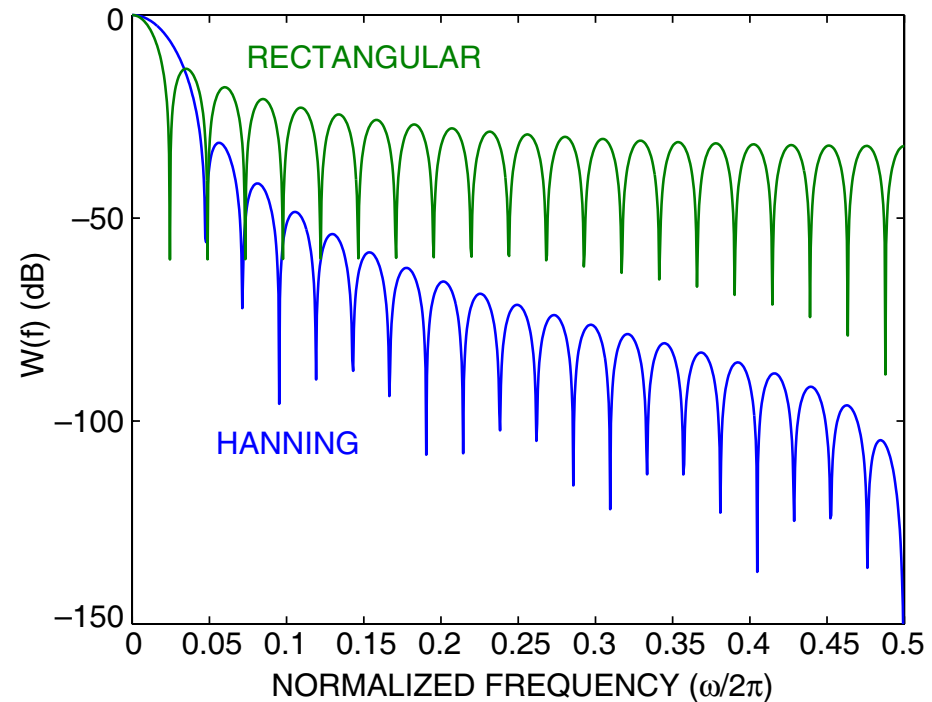
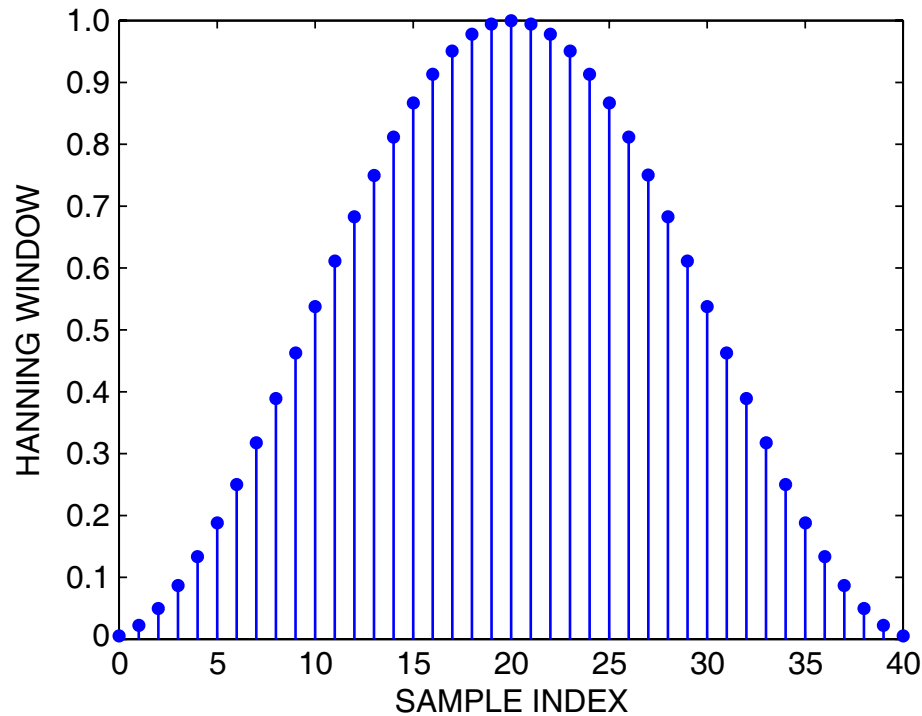
$$w[n] = \begin{cases} 0.54 - 0.46 \cos(2\pi n/M), & 0 \leq n \leq M, \\ 0, & \text{otherwise.} \end{cases}$$

– Blackman window:

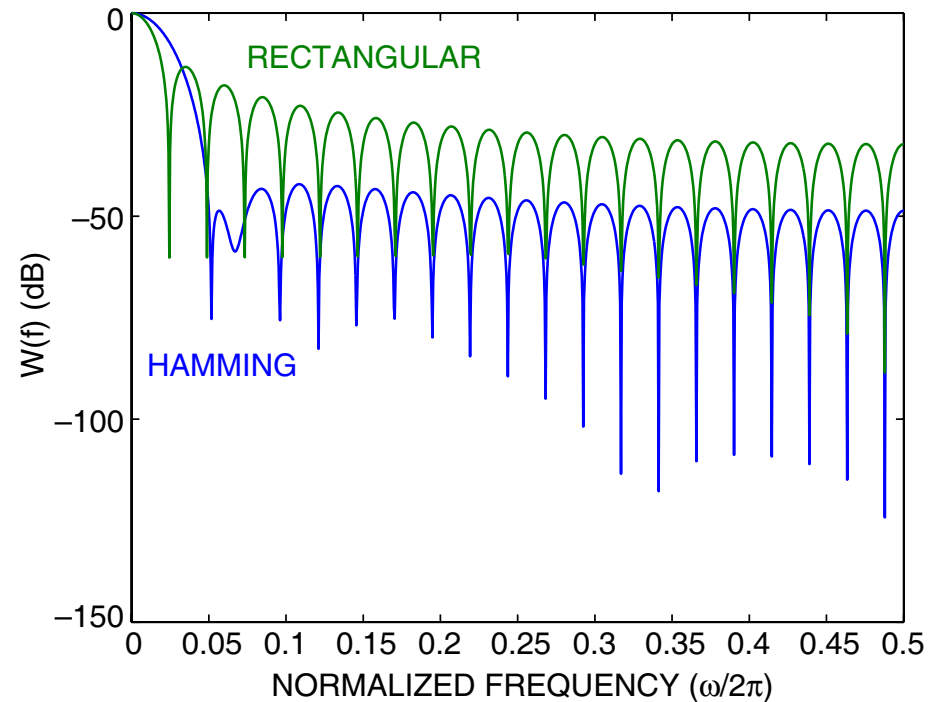
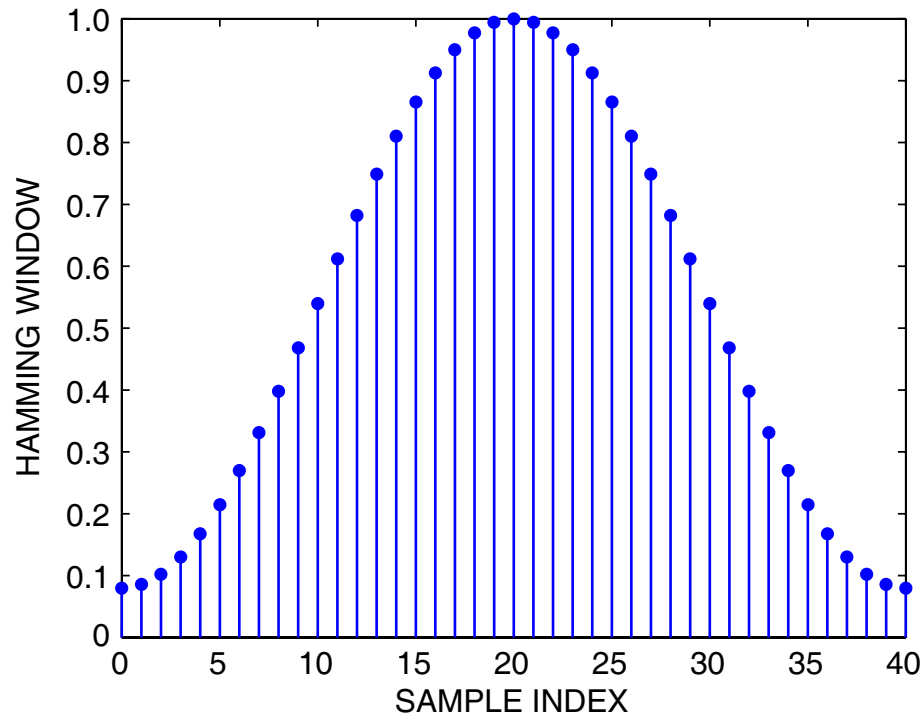
$$w[n] = \begin{cases} 0.42 - 0.5 \cos(2\pi n/M) \\ \quad + 0.08 \cos(4\pi n/M), & 0 \leq n \leq M, \\ 0, & \text{otherwise.} \end{cases}$$



# Hanning window compared to the rectangular window in the frequency domain: ( $M = 40$ )



# Hamming window compared to the rectangular window in the frequency domain: ( $M = 40$ )



# Blackman window compared to the rectangular window in the frequency domain: ( $M = 40$ )

