COMP ENG 4TL4:

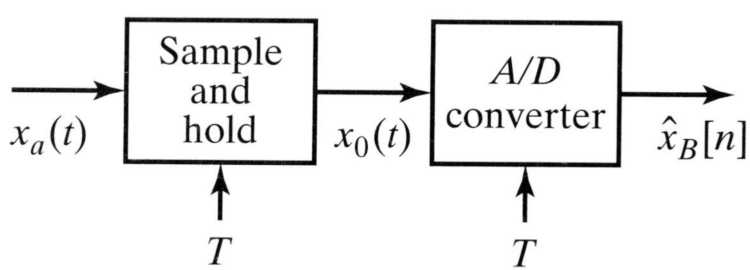
Digital Signal Processing

Notes for Lecture #4

Friday, September 12, 2003

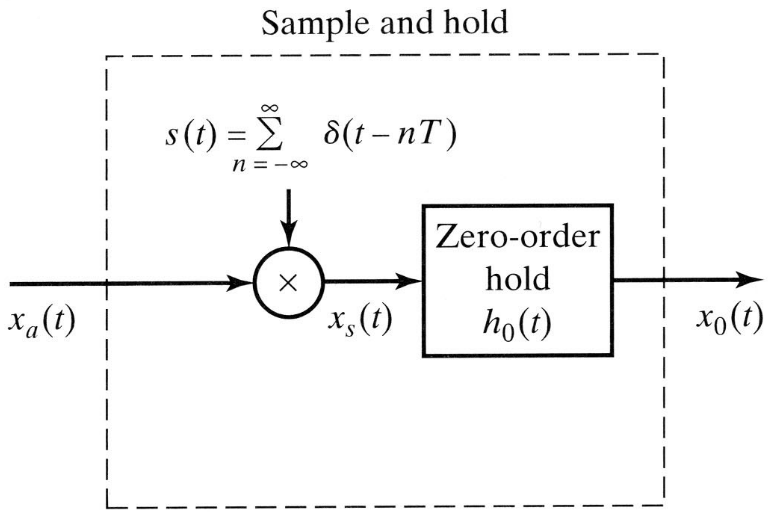
1.5 Physical aspects of sampling and reconstruction

A/D converters typically require a steady input voltage for some time period in order to complete the conversion. Ideal point sampling is consequently not possible, so an analog circuit referred to as a *sample-and-hold* is used. This effectively lowpass filters the analog signal, such that the required duration of the hold is the major factor in determining the upper limit to the A/D converter's bandwidth.

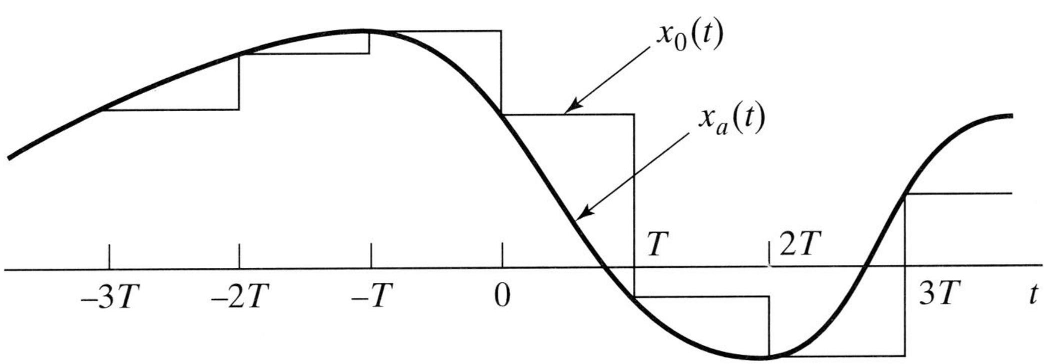


(Oppenheim and Schafer)

Figure 4.45 Physical configuration for analog-to-digital conversion.



(a)



(b)

(Oppenheim and Schafer)

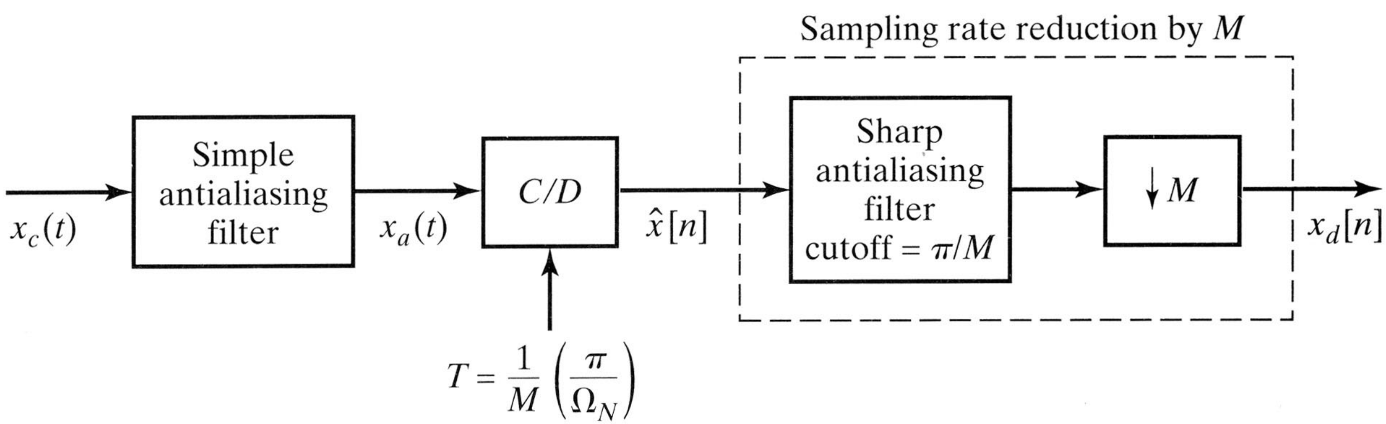
Figure 4.46 (a) Representation of an ideal sample-and-hold.
(b) Representative input and output signals for the sample-and-hold.

Common implementations of A/D include:

1. <u>Successive Approximation A/D</u>:
   - each bit is calculated in succession, starting from the MSB
   - simple but slow
2. <u>Flash A/D</u>:
   - all bits calculated in parallel
   - very fast but complicated: requires $2^{B+1}—1$ comparators
   - ➢ applications: e.g., image processing
3. <u>Half-Flash A/D</u>:
   - uses two flash A/D converters, each for half the number of bits
   - only half as fast as a full flash A/D, but requires only $2(2^{(B+1)/2}—1)$ comparators
4. <u>Sigma-delta A/D</u>:
   - high accuracy but slow and restricted to low-bandwidth signals
   - ➢ applications: e.g., audio processing, instrumentation

(*see Porat 3.5 for more details*)

There is a trade-off between the simplicity of an antialiasing filter to be placed in front of an A/D converter and how well it approximates an ideal lowpass filter, i.e., produces a truly bandlimited signal. One approach that avoids sharp-cutoff analog filters is to use a low-order analog antialiasing filter and a high sampling rate in the A/D, and then to apply a sharp lowpass digital filter followed by sampling rate reduction in the DSP. This also allows for flexibility in sampling rates used internally by the DSP.



(Oppenheim and Schafer)

Figure 4.43 Using oversampled A/D conversion to simplify a continuous-time antialiasing filter.

D/A converters typically use a zero-order hold circuit to approximate ideal conversion to an impulse train. Note that while the sample-and-hold circuit in the A/D described previous applies a zero-order hold to the unquantized signal, the zero-order hold in a D/A is applied to the quantized signal.

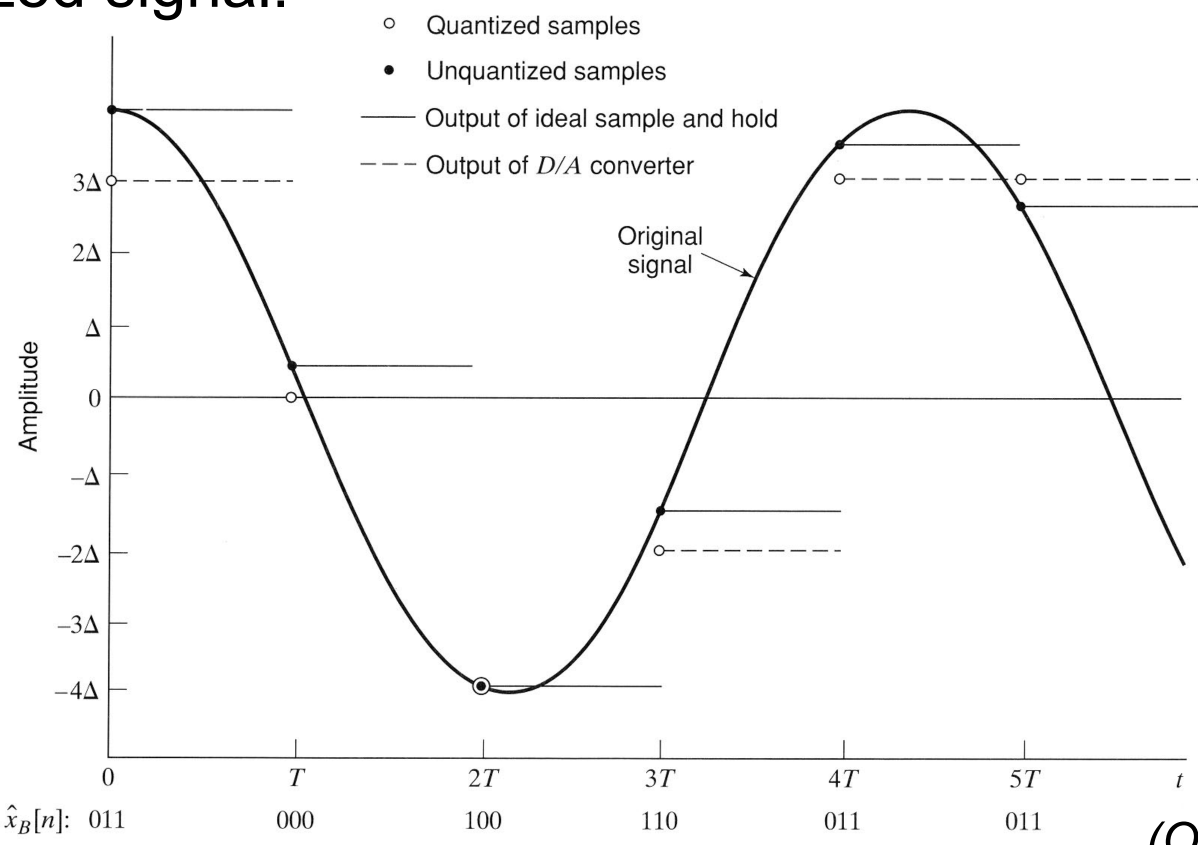
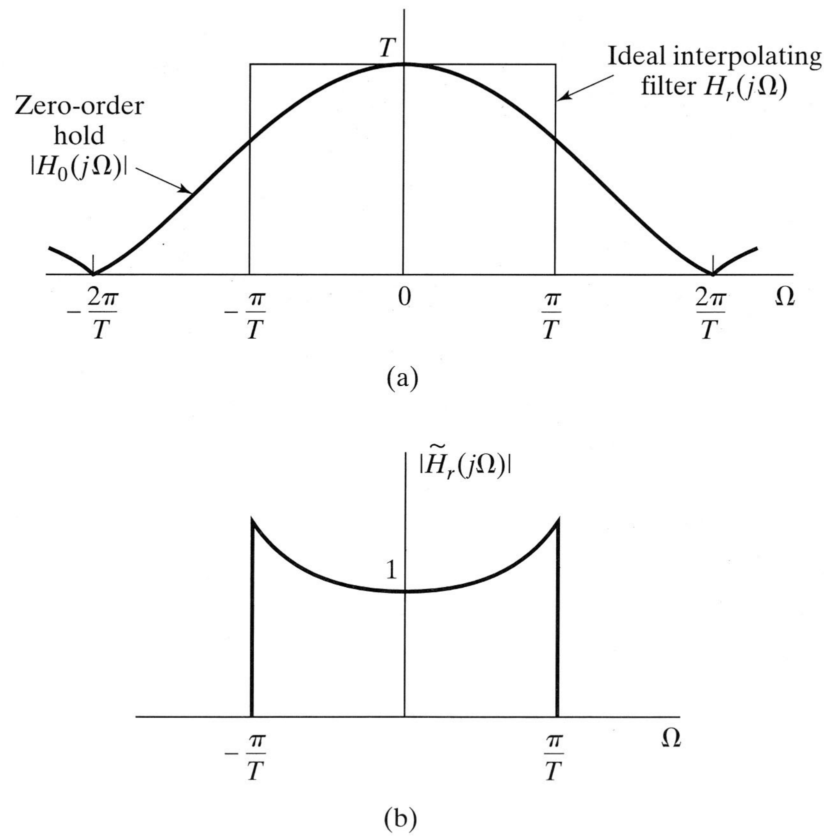


Figure 4.49 Sampling, quantization, coding, and D/A conversion with a 3-bit quantizer.

(Oppenheim and Schafer)

While the zero-order hold has a lowpass frequency response, it does not have a sharp cutoff like the ideal reconstruction (interpolating) filter. Consequently, it is typically necessary to apply an additional sharp lowpass reconstruction filter that can also compensate for the frequency response of the zero-order hold.



(Oppenheim and Schafer)

Figure 4.54 (a) Frequency response of zero-order hold compared with ideal interpolating filter. (b) Ideal compensated reconstruction filter for use with a zero-order-hold output.

An alternative reconstruction circuit is a *first-order hold*, which works as follows:

At time $t = nT$ it computes the straight line connecting $(nT - T, x[nT - T])$ and $(nT, x[nT])$. During the interval $[nT, nT{+}T)$ it takes $x(t)$ as the ordinate of the point on that straight line whose abscissa it t, as shown below.

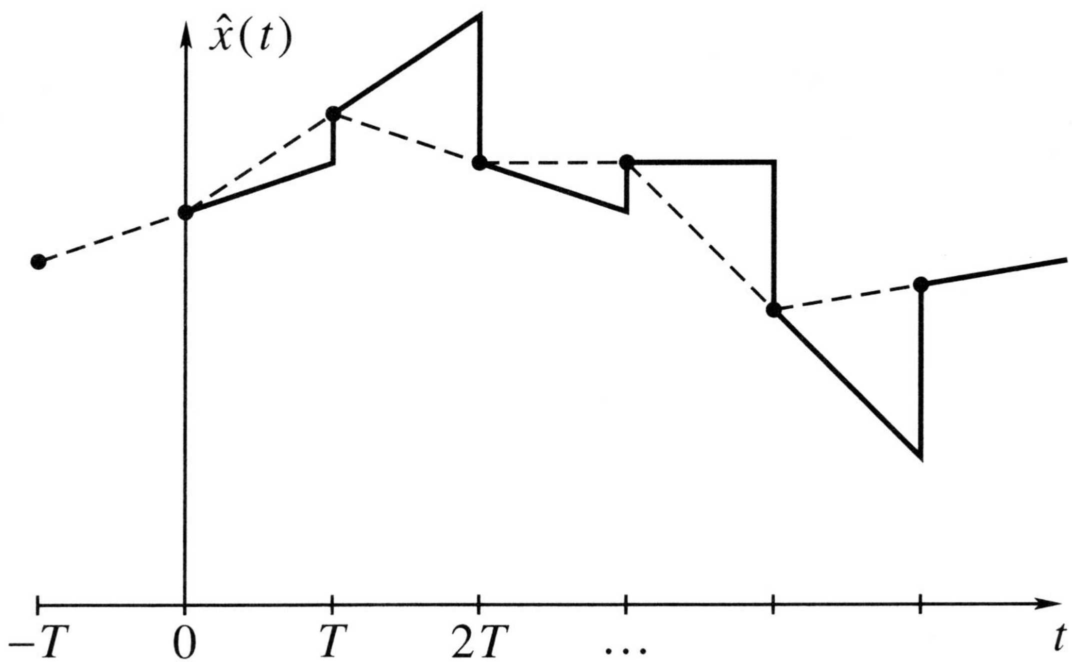


Figure 3.36 First-order hold. *(Modified from Porat)*

1.6 Basic types of digital signals

Unit-step function:

$$u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$

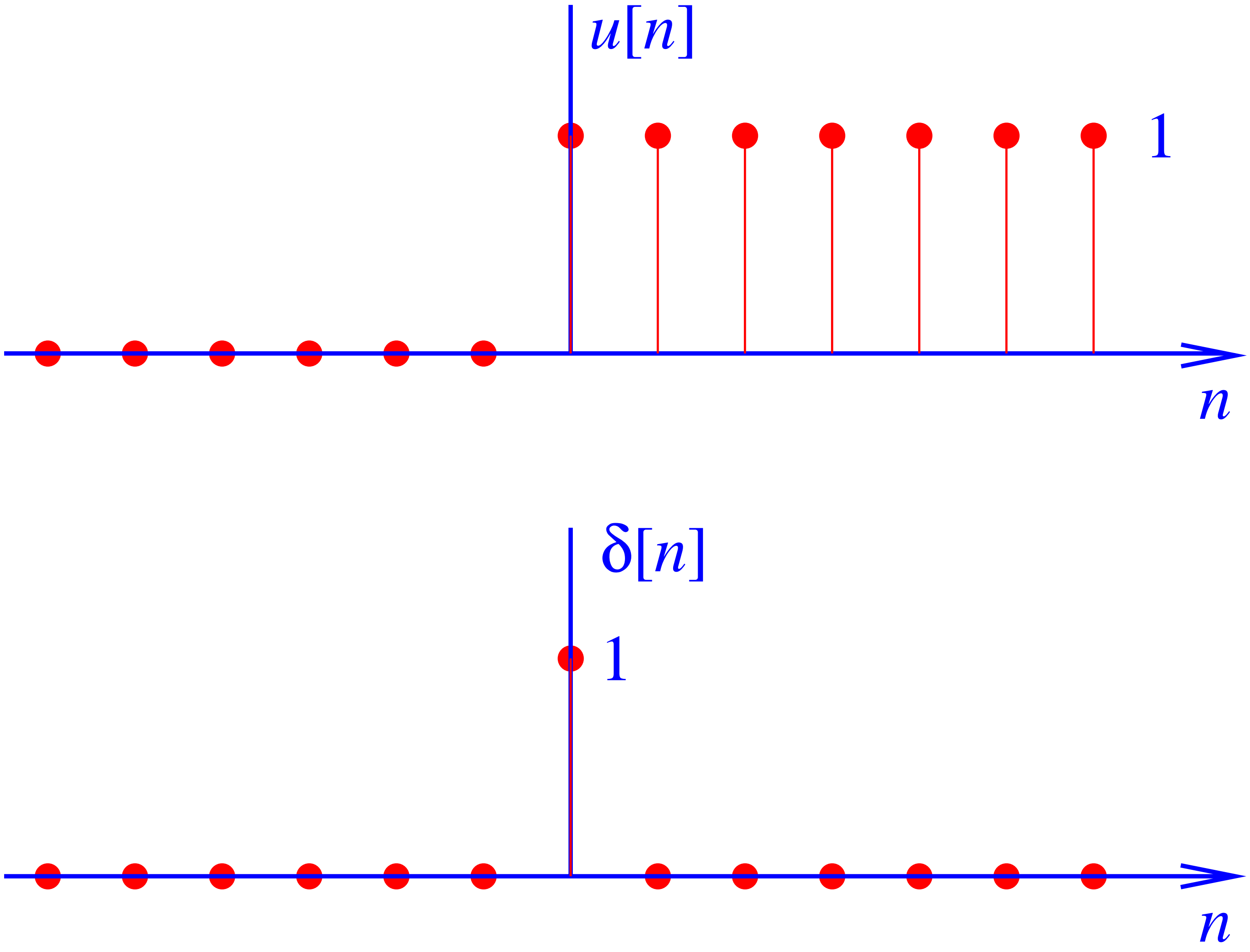
Unit-impulse function:

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

$$u[n] = \sum_{m=-\infty}^{n} \delta[m] \qquad \text{integration}$$

$$\delta[n] = u[n] - u[n-1] \qquad \text{differentiation}$$

The unit-step and unit-impulse functions



Periodic signals:

$$x(t) = x(t+T) \quad | \quad x[n] = x[n+N]$$

Finite-energy signals:

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, \mathrm{d}t < \infty \quad | \quad E = \sum_{n=-\infty}^{\infty} |x[n]|^2 < \infty$$

Finite-power signals:

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt \ < \ \infty \ |$$

$$P = \lim_{N \to \infty} \frac{1}{2N} \sum_{n=-N}^{N} |x[n]|^2 < \infty$$

Complex exponentials (cisoids):

$$x(t) = A\, e^{j(\omega t+\phi)} \quad | \quad x[n] = A\, e^{j(\omega n+\phi)}$$

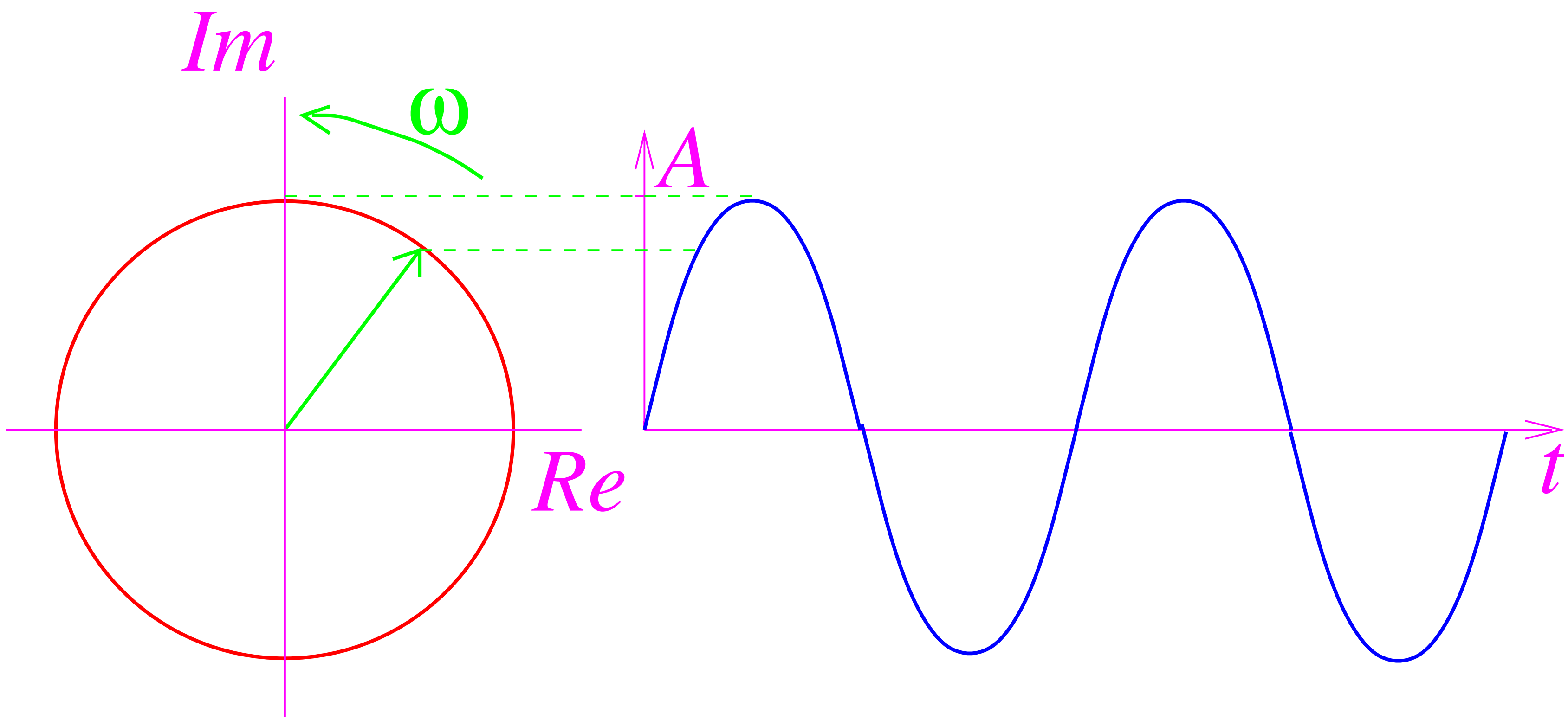
Sinusoids:

$$x(t) = A \sin(\omega t+\phi) \quad | \quad x[n] = A \sin(\omega n+\phi)$$

$$\omega \longrightarrow \omega T\,, \text{ i.e. } \quad \omega_{\text{discrete}} = \omega_{\text{analog}}\, T$$

$$\begin{aligned} e^{j(\omega n+\phi)} &= \cos(\omega n+\phi) + j \sin(\omega n+\phi) \\ \cos(\omega n+\phi) &= \{e^{j(\omega n+\phi)} + e^{-j(\omega n+\phi)}\}/2 \\ \sin(\omega n+\phi) &= \{e^{j(\omega n+\phi)} - e^{-j(\omega n+\phi)}\}/2j \end{aligned}$$

<u>A sine wave as the projection of a complex phasor onto the imaginary axis:</u>



Differences between sampled exponentials and their analog counterparts:

- analog exponentials and (co)sinusoids are periodic with $T = 2\pi/\omega$
- discrete sinusoids are not necessarily periodic (although their values lie on a periodic envelope).

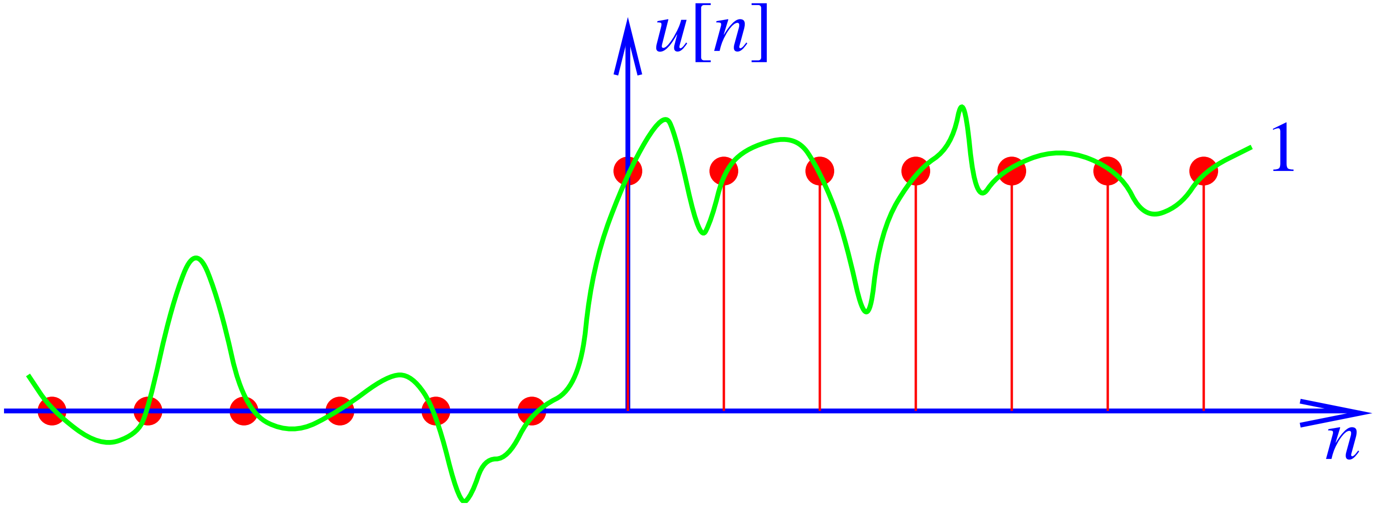
Periodicity condition:

$$
\begin{aligned}
x[n] &= x[n+N] \\
\Rightarrow \quad e^{j\omega n} &= e^{j\omega(n+N)} \qquad \Rightarrow \quad \exp\{j\omega N\} = 1 \\
\Rightarrow \omega &= \frac{2\pi m}{N} \quad \text{or} \quad f = \frac{m}{N} \qquad (\omega = 2\pi f)
\end{aligned}
$$

(this result applies to sines and cosines as well!)

- for sampled exponentials, the frequency ω should be measured in [radians] rather than [radians per second]
- digital signals have ambiguity – aliasing!

The unit-step function and one of many analog signals which can be drawn through its sample points



Ambiguity in digital sinusoids