COMP ENG 4TL4:

Digital Signal Processing

Notes for Lecture #5

Wednesday, September 17, 2003

2. TIME-DOMAIN ANALYSIS

2.1 Linear Time-Invariant (LTI) Systems

Definition of a system:

$$y[n] = \mathcal{T}\{x[n]\}$$

where $\mathcal{T}\{\cdot\}$ is an operator that maps an input sequence $x[n]$ into an output sequence $y[n]$.

Linear system:

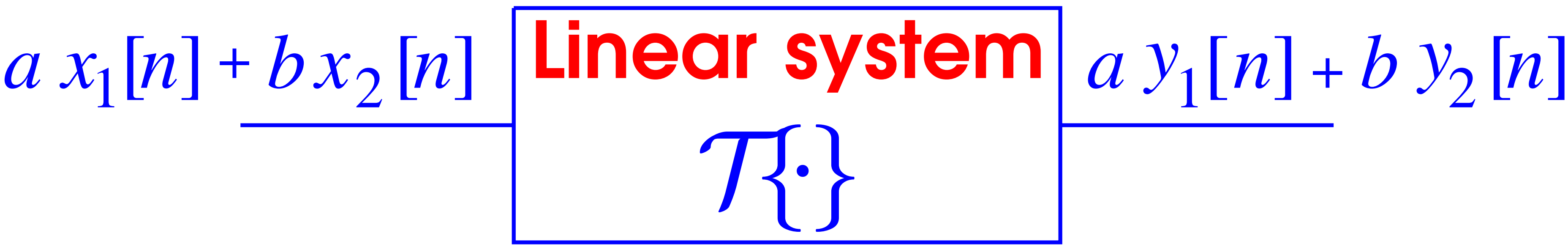
A system is linear if it obeys the *principle of superposition*.

Principle of superposition:

If the input of a system contains the *sum of multiple signals*, then the output of this system is the *sum of the system responses to each separate signal*.

A system is linear if and only if:

$$
\begin{aligned}
\mathcal{T}\left\{a x_1[n] + b x_2[n]\right\} &= a\mathcal{T}\left\{x_1[n]\right\} + bT\left\{x_2[n]\right\} \\
&= a y_1[n] + b y_2[n]
\end{aligned}
$$



<u>Example:</u> Let $y[n]$ = $x^2[n]$ (i.e., $\mathcal{T}\{\cdot\} = (\cdot)^2$). Then,

$$\begin{aligned}\mathcal{T}\{x_1[n] + x_2[n]\} &= x_1^2[n] + x_2^2[n] + 2x_1[n]\,x_2[n] \\ &\neq x_1^2[n] + x_2^2[n]\,.\end{aligned}$$

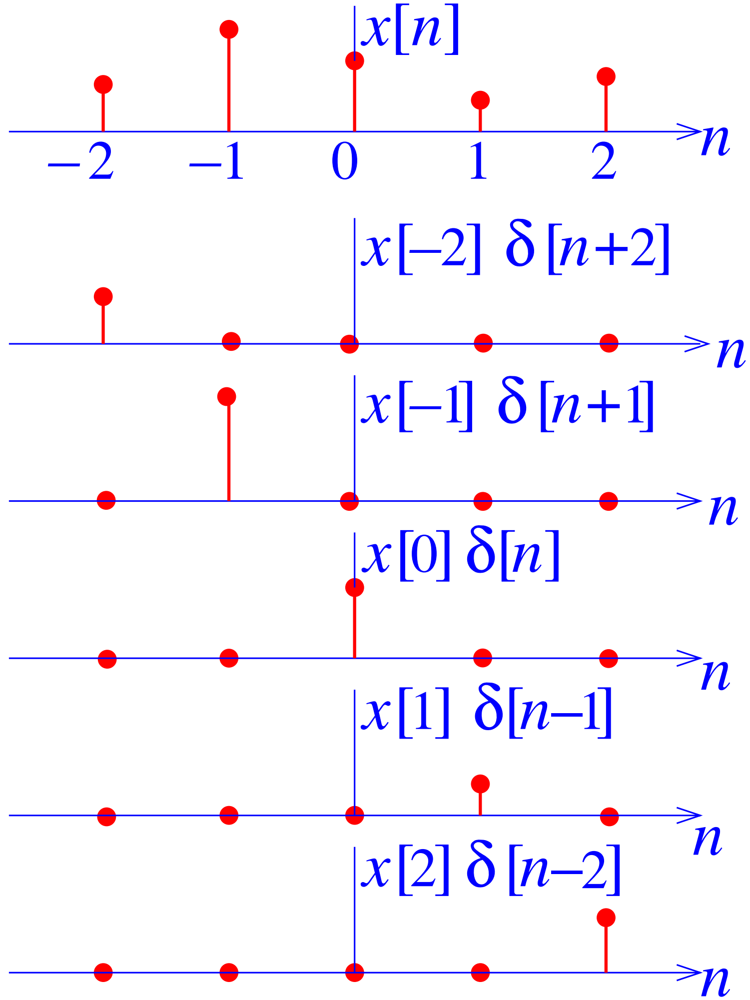
Hence, this system is nonlinear!

<u>A time-invariant system</u> has properties unvarying with time,i.e.:

$$\text{if} \quad y[n] = \mathcal{T}\{x[n]\} \quad \Rightarrow \quad y[n-k] = T\{x[n-k]\}.$$

A Linear Time-Invariant (LTI) system is *both linear and time-invariant* — sometimes referred to as a Linear Shift-Invariant (LSI) system.

2.2 Digital Signals via Impulse Functions



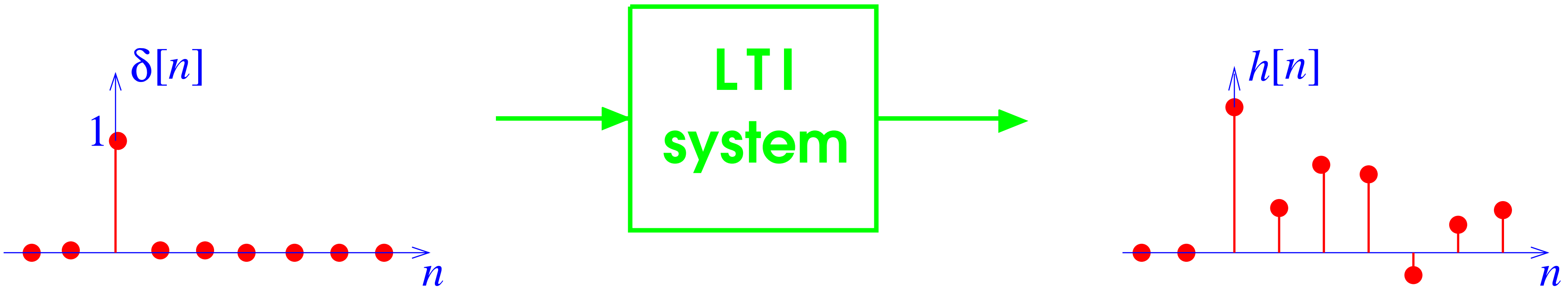
$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$

Let $h[n]$ be the response to $\delta[n]$ of the LTI system with transform $\mathcal{T}\{\cdot\}$.

Due to the time-invariance property, the response to $\delta[n-k]$ is simply $h[n-k] \Rightarrow$

$$\begin{aligned} y[n] &= \mathcal{T}\{x[n]\} \\ &= T\left\{\sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]\right\} \\ &= \sum_{k=-\infty}^{\infty} x[k]\,\mathcal{T}\{\delta[n-k]\} \\ &= \sum_{k=-\infty}^{\infty} x[k]\,h[n-k] \\ &= x[n] * h[n] \quad \text{convolution sum.} \end{aligned}$$

The sequence $h[n]$ is commonly referred to as impulse response of the LTI system.



An important property of convolution:

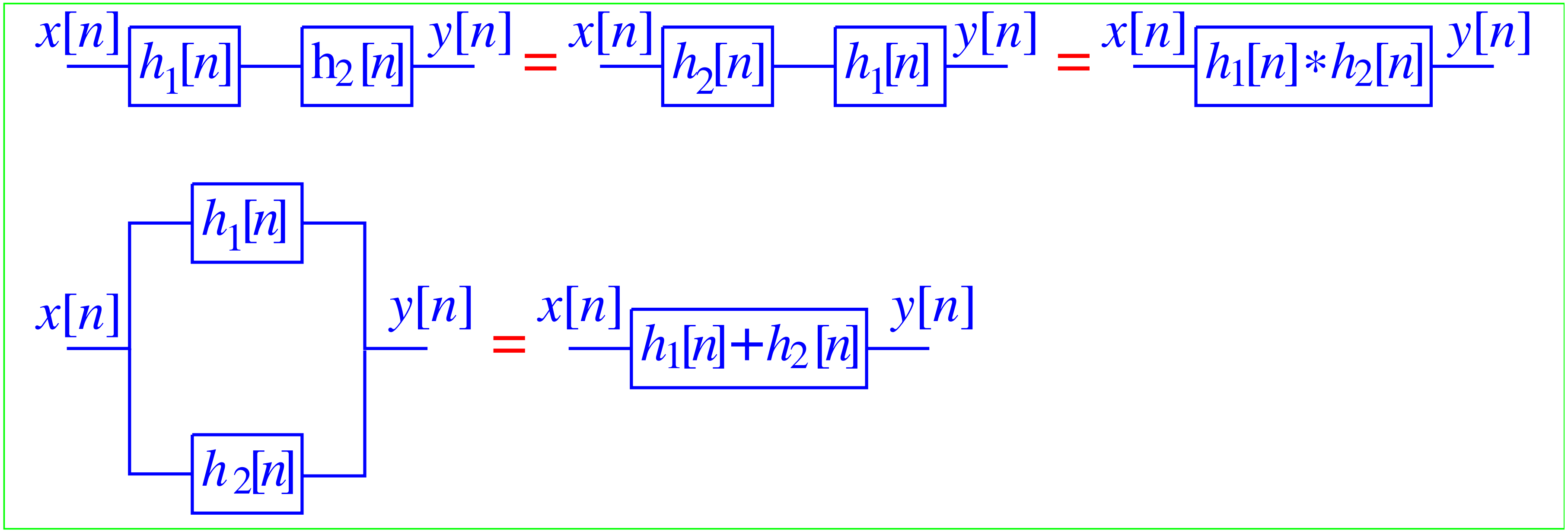
$$x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$
$$= h[n] * x[n]$$

$\Rightarrow$ the *order* in which two sequences are convolved is unimportant!

Other properties of convolution:

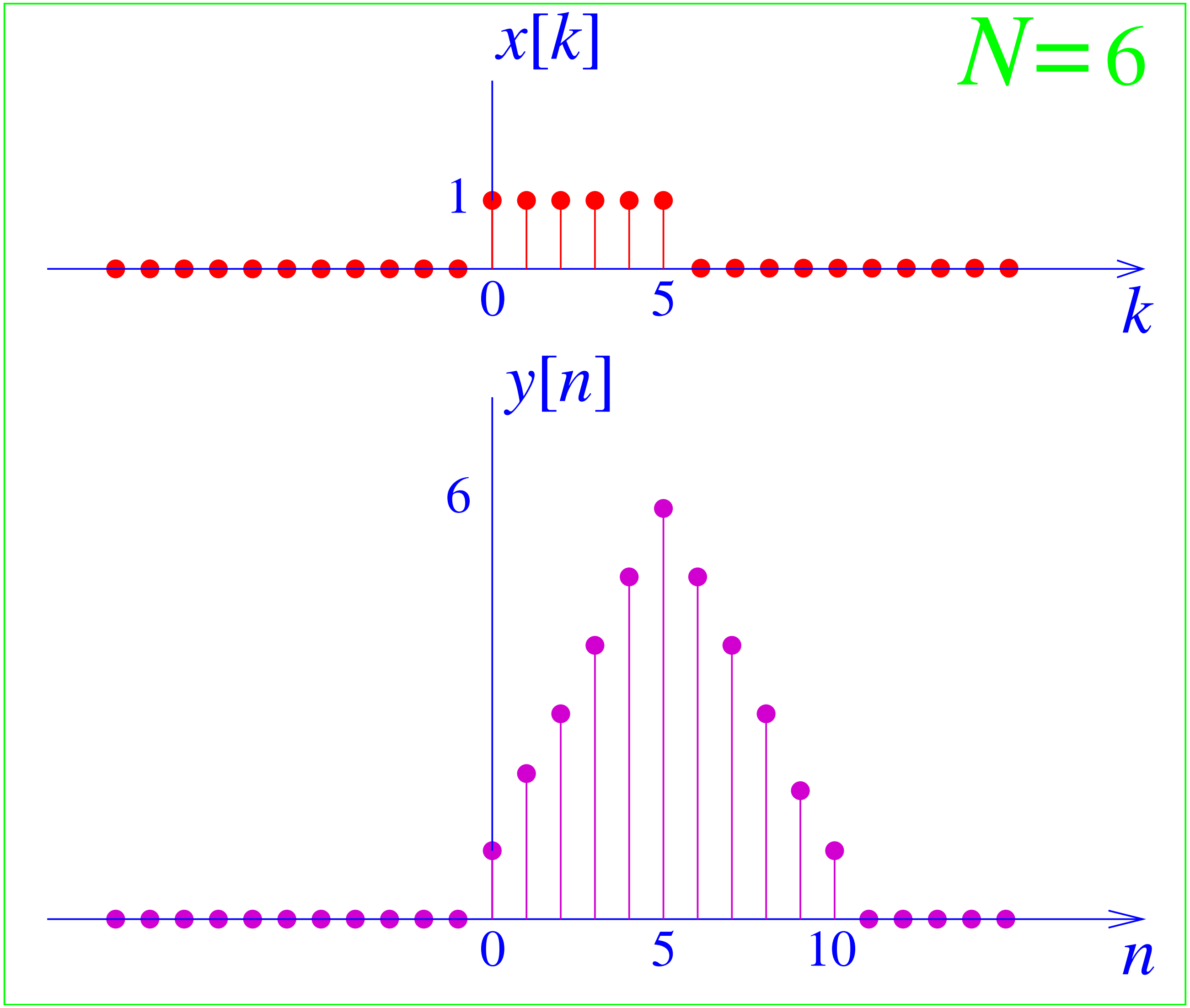
$$
\begin{aligned}
& x[n] * \{h_1[n] * h_2[n]\} \\
= \; & \{x[n] * h_1[n]\} * h_2[n] \qquad \text{associativity}
\end{aligned}
$$

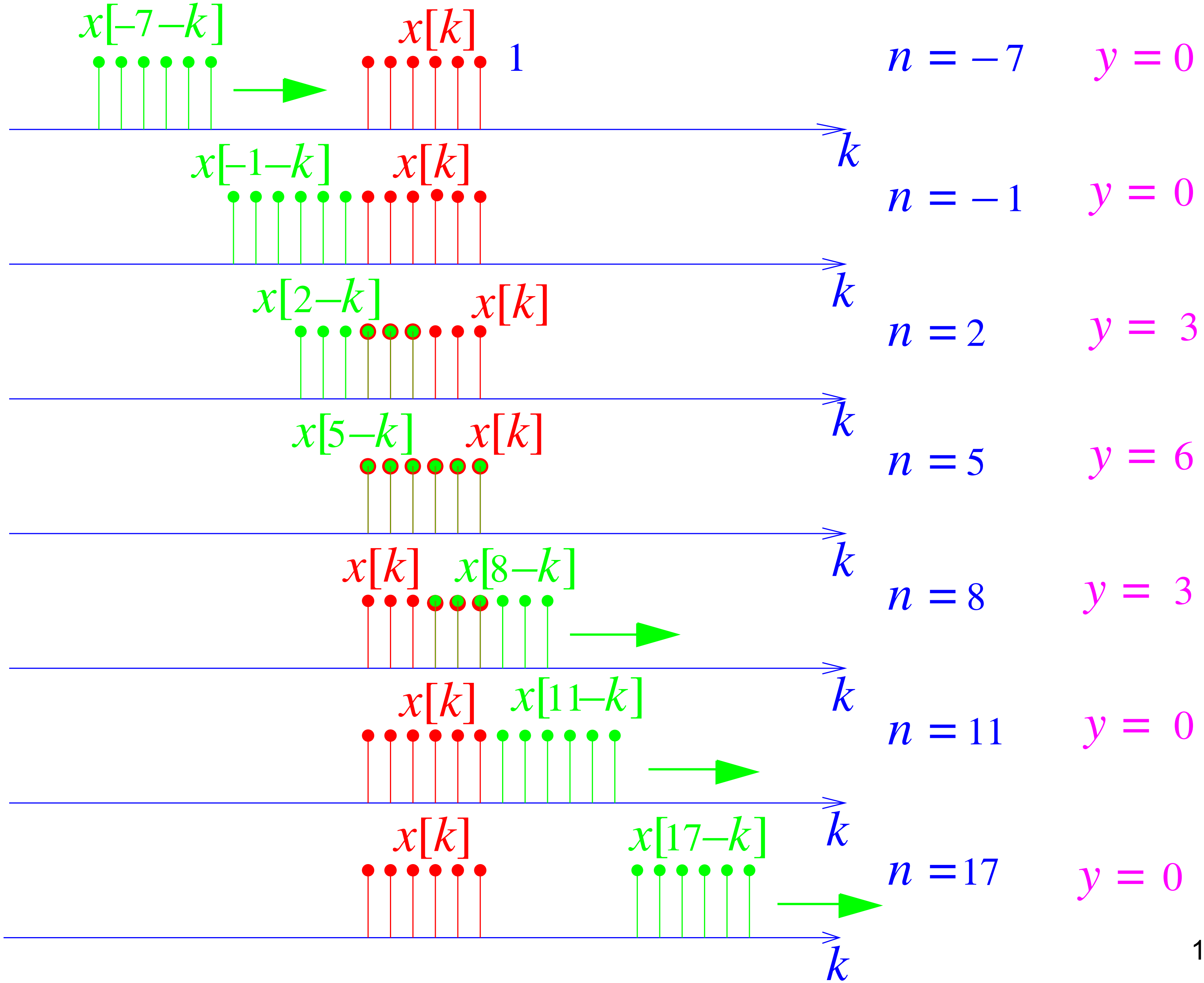
$$
\begin{aligned}
& x[n] * \{h_1[n] + h_2[n]\} \\
= \; & x[n] * h_1[n] + x[n] * h_2[n] \qquad \text{distributivity}
\end{aligned}
$$



Example: Convolution of two rectangles $x[n]$.

$$y[n] = x[n] * x[n]$$





$x[-7-k]$
$x[k]$
1
$n = -7$
$y = 0$
k
$x[-1-k]$
$x[k]$
$n = -1$
$y = 0$
k
$x[2-k]$
$x[k]$
$n = 2$
$y = 3$
k
$x[5-k]$
$x[k]$
$n = 5$
$y = 6$
k
$x[k]$
$x[8-k]$
$n = 8$
$y = 3$
k
$x[k]$
$x[11-k]$
$n = 11$
$y = 0$
k
$x[k]$
$x[17-k]$
$n = 17$
$y = 0$
k