

## Supplied Equations and Tables

**Table 3.3.** Faraday's Constant  $F$  and the Gas Constant  $R$

Constant	Value
$F$	96,487 Coulombs/mole
$R$	8.314 Joules/degree K-mole
$RT/F$	$8.314 \times .300 / 96487 = 25.8$ mV at 27 °C

**Fick's law of diffusion:**

$$\bar{j}_d = -D\nabla C \quad (3.1)$$

**Ohm's law of drift:**

$$\bar{j}_e = -u_p \frac{Z_p}{|Z_p|} C_p \nabla \Phi \quad (3.2)$$

**Einstein's equation:**

$$D_p = \frac{u_p RT}{|Z_p| F} \quad (3.3)$$

**Nernst-Planck equation (ion flux):**

$$\bar{j}_p = -D_p \left( \nabla C_p + \frac{Z_p C_p F}{RT} \nabla \Phi \right) \quad (3.5)$$

**Nernst-Planck equation (electric current density):**

$$\bar{J}_p = -D_p F Z_p \left( \nabla C_p + \frac{Z_p C_p F}{RT} \nabla \Phi \right) \quad (3.6)$$

$$= -u_p \left( RT \frac{Z_p}{|Z_p|} \nabla C_p + |Z_p| C_p F \nabla \Phi \right) \quad (3.7)$$

**Nernst equation:**

$$V_m^{eq} = \Phi_i - \Phi_e = \frac{-RT}{Z_p F} \ln \left( \frac{[C_p]_i}{[C_p]_e} \right) \quad (3.21)$$

**Passive membrane response to current step of  $I_0$  from rest:**

$$V_m(t) = I_0 R \left( 1 - e^{-t/\tau} \right) + V_{rest}$$

**Passive membrane return to rest from initial membrane potential of  $V_m(t=0)$ :**

$$V_m(t) = [V_m(0) - V_{rest}] e^{-t/\tau} + V_{rest}$$

**Passive membrane response to new steady state potential  $V_m(t \rightarrow \infty)$  from initial value  $V_m(t_0)$  at time  $t \geq t_0$ :**

$$V_m(t) = (V_m(t \rightarrow \infty) - V_m(t_0)) \left( 1 - e^{-(t-t_0)/\tau'} \right) + V_m(t_0),$$

**Boltzmann function for fraction of open channels:**

$$\frac{[\text{open}]}{[\text{open} + \text{closed}]} = \frac{1}{1 + \exp\left(\frac{w - z g q_e V_m}{kT}\right)} \quad (4.5)$$

**Macroscopic channel kinetics:**

$$N_c \xrightleftharpoons[\beta]{\alpha} N_o \quad (4.7)$$

$$\frac{dN_c}{dt} = \beta N_o - \alpha N_c \quad (4.8)$$

$$\frac{dN_o}{dt} = \alpha N_c - \beta N_o \quad (4.9)$$

**Single channel probabilities:**

$$p = \frac{\langle N_o \rangle}{N} \quad (4.15)$$

$$q = \frac{\langle N_c \rangle}{N} \quad (4.16)$$

**Mean number of open channels:**

$$\langle N_o \rangle = Np \quad (4.22)$$

**Mean macroscopic conductance:**

$$\langle G_K \rangle = Np \gamma_K \quad (4.29)$$

**Gating particle kinetics for constant  $\alpha_n$  and  $\beta_n$ :**

$$\begin{aligned} n(t) &= n_\infty - (n_\infty - n_o) e^{-t/\tau_n} \\ &= (n_\infty - n_o) [1 - e^{-t/\tau_n}] + n_o \end{aligned} \quad (4.36)$$

$$\tau_n = \frac{1}{\alpha_n + \beta_n}, \quad n_\infty = \frac{\alpha_n}{\alpha_n + \beta_n} \quad (4.37)$$

**GHK transmembrane potential equation:**

$$V_m = \frac{RT}{F} \ln \left[ \frac{P_K [K]_e + P_{Na} [Na]_e + P_{Cl} [Cl]_i}{P_K [K]_i + P_{Na} [Na]_i + P_{Cl} [Cl]_e} \right] \quad (5.1)$$

**Hodgkin–Huxley potassium channel model:**

$$g_K(t, v_m) = \bar{g}_K n^4(t, v_m) \quad (5.18)$$

$$\frac{dn(t, v_m)}{dt} = \alpha_n(v_m) (1 - n) - \beta_n(v_m) n \quad (5.19)$$

or

$$\frac{dn(t, v_m)}{dt} = \frac{n_\infty - n}{\tau_n} \quad (5.22)$$

$$\alpha_n(v_m) = \frac{0.01 (10 - v_m)}{\exp\left(\frac{10 - v_m}{10}\right) - 1} \quad (5.24)$$

$$\beta_n(v_m) = 0.125 \exp\left(\frac{-v_m}{80}\right) \quad (5.25)$$

**Hodgkin–Huxley sodium channel model:**

$$g_{\text{Na}}(t, v_m) = \bar{g}_{\text{Na}} m^3(t, v_m) h(t, v_m) \quad (5.26)$$

$$\frac{dm(t, v_m)}{dt} = \alpha_m(v_m) (1 - m) - \beta_m(v_m) m \quad (5.27)$$

$$\frac{dh(t, v_m)}{dt} = \alpha_h(v_m) (1 - h) - \beta_h(v_m) h \quad (5.28)$$

$$\alpha_m(v_m) = \frac{0.1 (25 - v_m)}{\exp\left(\frac{25 - v_m}{10}\right) - 1};$$

$$\beta_m(v_m) = 4 \exp\left(\frac{-v_m}{18}\right) \quad (5.36)$$

$$\alpha_h(v_m) = 0.07 \exp\left(\frac{-v_m}{20}\right);$$

$$\beta_h(v_m) = \left\{ \exp\left(\frac{30 - v_m}{10}\right) + 1 \right\}^{-1} \quad (5.37)$$

**Temperature scaling of HH ion channel gating rates:**

$$Q = 3^P \quad (5.64)$$

$$P = \frac{T - 6.3}{10} \quad (5.65)$$

**Temperature scaling of HH ion channel gating rates (cont.):**

$$\frac{dn}{dt} = Q\alpha_n(1-n) - Q\beta_n n \quad (5.66)$$

$$\frac{dm}{dt} = Q\alpha_m(1-m) - Q\beta_m m \quad (5.67)$$

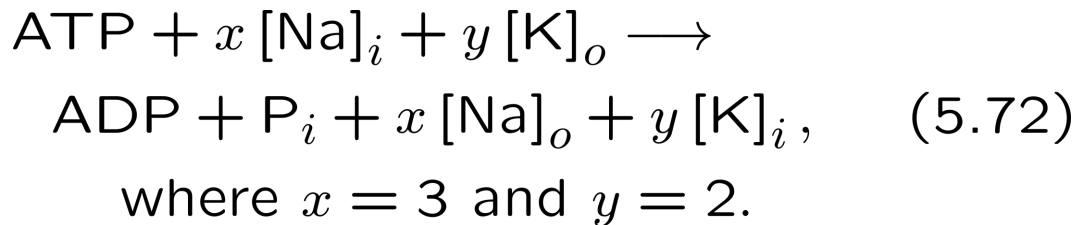
$$\frac{dh}{dt} = Q\alpha_h(1-h) - Q\beta_h h \quad (5.68)$$

**Sodium pump efflux:**

$$-\frac{d[{}^{24}\text{Na}^+]_i}{dt} = k[{}^{24}\text{Na}^+]_i \quad (5.70)$$

$$\Rightarrow [{}^{24}\text{Na}^+]_i = A \exp(-kt) \quad (5.71)$$

**Sodium pump stoichiometry:**



**Calcium channel model:**

$$I_{\text{Ca}} = 4 \frac{P_{\text{Ca}} V_m F^2}{RT} \left( \frac{[\text{Ca}]_o - [\text{Ca}]_i e^{2V_m F/RT}}{1 - e^{2V_m F/RT}} \right) \quad (5.69)$$

**Frankenhaeuser–Huxley potassium channel model:**

$$I_K = P_K \frac{V_m^2 F^2}{RT} \left( \frac{[K]_o - [K]_i e^{V_m F/RT}}{1 - e^{V_m F/RT}} \right) \quad (12.27)$$

$$P_K = \bar{P}_K n^2 \quad (12.28)$$

$$\alpha_n = 0.02 (v_m - 35) \left( 1 - e^{\frac{35 - v_m}{10}} \right)^{-1} \quad (12.30)$$

$$\beta_n = 0.05 (10 - v_m) \left( 1 - e^{\frac{v_m - 10}{10}} \right)^{-1} \quad (12.31)$$

**Cable equation intracellular axial resistance per unit length:**

$$r_i = \frac{R_i}{\pi a^2} \quad \Omega/\text{cm} \quad (2.55')$$

**Cable equation membrane resistance times unit length:**

$$r_m = \frac{R_m}{2\pi a} \quad \Omega \text{ cm} \quad (2.56')$$

**Cable equation membrane capacitance per unit length:**

$$c_m = C_m 2\pi a \quad \mu\text{F}/\text{cm} \quad (2.57')$$

**Cable equation extra- and intra-cellular axial electric potential gradients:**

$$\frac{\partial \Phi_e}{\partial x} = -I_e r_e \quad (6.1)$$

$$\frac{\partial \Phi_i}{\partial x} = -I_i r_i \quad (6.2)$$

**Cable equation intra- and extra-cellular axial current gradients:**

$$\frac{\partial I_i}{\partial x} = -i_m \quad (6.3)$$

$$\frac{\partial I_e}{\partial x} = i_m + i_p \quad (6.4)$$

**Cable equation total axial current:**

$$I = I_i + I_e \quad (6.5)$$

**Cable equation transmembrane current vs. membrane potential and applied current:**

$$i_m = \frac{1}{(r_i + r_e)} \left( \frac{\partial^2 V_m}{\partial x^2} - r_e i_p \right) \quad (6.12)$$

**Cable equation transmembrane current vs. intracellular potential:**

$$i_m = \frac{1}{r_i} \frac{\partial^2 \phi_i}{\partial x^2} \quad (6.13)$$

**Cable equation intra- and extra-cellular potentials in source-free region:**

$$\phi_i(x, t) = \frac{r_i}{r_i + r_e} v_m(x, t) \quad (6.23)$$

$$\phi_e(x, t) = -\frac{r_e}{r_i + r_e} v_m(x, t) \quad (6.24)$$



**Hodgkin-Huxley cable equation:**

$$\frac{\partial V_m}{\partial t} = \frac{1}{C_m} \left[ \frac{1}{2\pi a(r_i + r_e)} \left( \frac{\partial^2 V_m}{\partial x^2} - r_{ei} \right) - I_{ion}(x, t) \right]$$

$$\begin{aligned} I_{ion}(x, t) = & g_K(x, t) [V_m(x, t) - E_K] \\ & + g_{Na}(x, t) [V_m(x, t) - E_{Na}] \\ & + g_L [V_m(x, t) - E_L] \end{aligned} \quad (6.28)$$

**Wave equation for temporal waveform:**

$$V_m(x, t) = V_m \left( t - \frac{(x - x_0)}{\theta} \right) \quad (6.64')$$

**Wave equation for spatial waveform:**

$$V_m(x, t) = V_m(x - \theta(t - t_0)) \quad (6.64'')$$

**Hodgkin-Huxley propagating action potential equation:**

$$\begin{aligned} \frac{a}{2R_i\theta^2} \frac{d^2 V_m}{dt^2} = & C_m \frac{dV_m}{dt} + g_K(V_m - E_K) \\ & + g_{Na}(V_m - E_{Na}) \\ & + g_L(V_m - E_L) \end{aligned} \quad (6.68)$$

**Isopotential membrane patch response to an intracellular current step:**

$$v_m = I_0 R (1 - e^{-t/\tau}) \quad (7.3)$$

$$= S (1 - e^{-t/\tau}) \quad (7.4)$$

**Strength-duration relationship for intracellular stimulation:**

$$I_{th} = I_R / (1 - e^{-T/\tau}) \quad (7.8)$$

**Chronaxie for intracellular stimulation:**

$$T_c = \tau \ln 2 = 0.693\tau \quad (7.11)$$

**Linear cable equation for extracellular current injection:**

$$\lambda^2 \frac{\partial^2 v_m}{\partial x^2} - \tau \frac{\partial v_m}{\partial t} - v_m = r_e \lambda^2 i_p \quad (7.14)$$

**Space constant and time constant:**

$$\lambda = \sqrt{\frac{r_m}{r_i + r_e}} \quad \text{and} \quad \tau = r_m c_m \quad (7.15)$$

**Steady-state solution to homogeneous infinite linear cable equation:**

$$v_m = Ae^{-x/\lambda} + Be^{x/\lambda} \quad (7.18)$$

**Steady-state solution to infinite linear cable equation with extracellular current injection at the spatial origin:**

$$v_m = -\frac{r_e \lambda I_0}{2} e^{-|x|/\lambda} \quad (7.34)$$

**General time-varying solution to infinite linear cable equation with intracellular current step at the spatial origin:**

$$v_m(X, T) = \frac{r_i \lambda I_0}{4} \left\{ e^{-|X|} \left[ 1 - \operatorname{erf} \left( \frac{|X|}{2\sqrt{T}} - \sqrt{T} \right) \right] - e^{|X|} \left[ 1 - \operatorname{erf} \left( \frac{|X|}{2\sqrt{T}} + \sqrt{T} \right) \right] \right\} \quad (7.48)$$

**Extracellular field potential for applied monopole point current source:**

$$\phi_a = \frac{I_0}{4\pi\sigma_e r} \quad (7.55)$$

**Initial transmembrane response to an applied extracellular field:**

$$r_i \frac{\partial v_m}{\partial t} = \frac{1}{c_m} \frac{\partial^2 \phi_e}{\partial z^2} \quad (7.59)$$

**Extracellular field produced by a fiber:**

$$\Phi_e(P) = \frac{1}{4\pi\sigma_e} \int_L \frac{i_m}{r} dx \quad (8.11)$$

**Approximate extracellular field produced by a cylindrical fiber:**

$$\Phi_e = \frac{a^2 \sigma_i}{4\sigma_e} \int \frac{\partial^2 V_m / \partial x^2}{r} dx \quad (8.12)$$

**Monopole source density for a cylindrical fiber:**

$$I_\ell = \pi a^2 \sigma_i \frac{\partial^2 V_m}{\partial x^2} \quad (8.19)$$

**Lumped monopole strength for a cylindrical fiber:**

$$\begin{aligned} M &= \pi a^2 \sigma_i \int_{x_a}^{x_b} \frac{\partial^2 V_m}{\partial x^2} dx \\ &= \pi a^2 \sigma_i \left( \left. \frac{\partial V_m}{\partial x} \right|_{x_b} - \left. \frac{\partial V_m}{\partial x} \right|_{x_a} \right) \end{aligned} \quad (8.21)$$

**Extracellular field potential for idealized dipole source:**

$$\Phi_d = \frac{1}{4\pi\sigma} \nabla \left( \frac{1}{r} \right) \cdot \bar{p} \quad (2.29)$$

**Dipole source density for a cylindrical fiber:**

$$\bar{\tau}_\ell \equiv -\pi a^2 \sigma_i \frac{\partial V_m}{\partial x} \bar{a}_x \approx I_i \bar{a}_x \quad (8.41)$$

**Lumped dipole strength for a cylindrical fiber:**

$$\begin{aligned} D &= -\pi a^2 \sigma_i \int_{x_1}^{x_2} \frac{\partial V_m}{\partial x} dx \\ &= \pi a^2 \sigma_i [V_m(x_1) - V_m(x_2)] \end{aligned} \quad (8.46)$$

**Heart vector/dipole:**

$$\bar{H} = \int \bar{J}_i dV \quad (9.83)$$

**Lead voltage:**

$$V_\ell = \bar{H} \cdot \bar{\ell} \quad (9.87)$$

**Standard lead voltages:**

$$V_I = \Phi_{LA} - \Phi_{RA} \quad (9.74)$$

$$V_{II} = \Phi_{LL} - \Phi_{RA} \quad (9.75)$$

$$V_{III} = \Phi_{LL} - \Phi_{LA} \quad (9.76)$$

**Binomial probability distribution:**

$$f(x) = \frac{n!}{x!(n-x)!} p^x q^{n-x} \quad (10.1)$$

**Poisson probability distribution:**

$$f(x) = \frac{e^{-m} m^x}{x!} \quad (10.6)$$

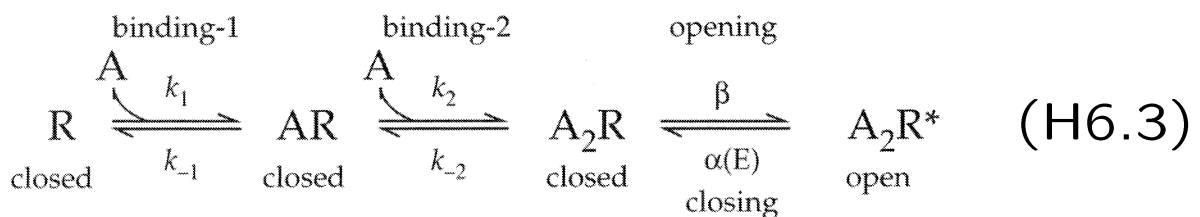
**Postsynaptic current equation:**

$$I_{\text{syn}} = g_{\text{syn}}(t) [V_m(t) - E_{\text{syn}}] \quad (\text{Koch 1.18})$$

**Postsynaptic conductance alpha equation:**

$$g_{\text{syn}}(t) = \text{const} \cdot t e^{-t/t_{\text{peak}}} \quad (\text{Koch 1.21})$$

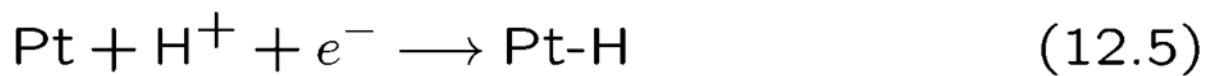
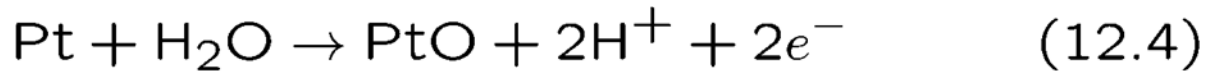
**Ligand-gated fast chemical synapse kinetic model with two binding sites and slow unbinding:**



**Anodic and cathodic Faradaic reactions for stainless-steel electrodes:**



**Anodic and cathodic Faradaic reactions for platinum electrodes:**



**Strength-duration relationship for extracellular stimulation:**

$$I_{\text{th}} = \frac{I_R}{(1 - e^{-Kt})} \quad (12.7)$$

**Chronaxie for extracellular stimulation:**

$$t_c = \frac{\ln 2}{K} \quad (12.13)$$

**Activating function for myelinated axon:**

$$\frac{\Phi_{e,n-1} - 2\Phi_{e,n} + \Phi_{e,n+1}}{\Delta z^2}$$

**END OF SUPPLIED EQUATIONS AND TABLES**

**THE END**