Supplied Equations and Tables

Table 3.3. Faraday's Constant F and the Gas Constant R

Constant	Value
F	96,487 Coulombs/mole
R	8.314 Joules/degree K-mole
RT/F	$8.314 \times .300/96487 = 25.8$ mV at 27 °C

Fick's law of diffusion:

$$\bar{j}_d = -D\nabla C \tag{3.1}$$

Ohm's law of drift:

$$\bar{j}_e = -u_p \frac{Z_p}{|Z_p|} C_p \nabla \Phi \tag{3.2}$$

Einstein's equation:

$$D_p = \frac{u_p RT}{|Z_p|\, F} \tag{3.3}$$

Nernst-Planck equation (ion flux):

$$\bar{j}_p = -D_p \left(\nabla C_p + \frac{Z_p C_p F}{RT} \nabla \Phi \right) \tag{3.5}$$

Nernst-Planck equation (electric current density):

$$\begin{aligned} \bar{J}_p &= -D_p F Z_p \left(\nabla C_p + \frac{Z_p C_p F}{RT} \nabla \Phi \right) && (3.6) \\ &= -u_p \left(RT \frac{Z_p}{|Z_p|} \nabla C_p + |Z_p| C_p F \nabla \Phi \right) && (3.7) \end{aligned}$$

Nernst equation:

$$V_m^{eq} = \Phi_i - \Phi_e = \frac{-RT}{Z_p F} \ln\left(\frac{[C_p]_i}{[C_p]_e}\right) \qquad (3.21)$$

Passive membrane response to current step of I_0 from rest:

$$V_m(t) = I_0 R \left(1 - \mathrm{e}^{-t/\tau}\right) + V_{\text{rest}}$$

Passive membrane return to rest from initial membrane potential of $V_m(t=0)$:

$$V_m(t) = [V_m(0) - V_{\text{rest}}]\, \mathrm{e}^{-t/\tau} + V_{\text{rest}}$$

Passive membrane response to new steady state potential $V_m(t \to \infty)$ from initial value $V_m(t_0)$ at time $t \geq t_0$:

$$V_m(t) = (V_m(t \to \infty) - V_m(t_0))\left(1 - \mathrm{e}^{-(t-t_0)/\tau'}\right) + V_m(t_0)\,,$$

Boltzmann function for fraction of open channels:

$$\frac{[\text{open}]}{[\text{open+closed}]} = \frac{1}{1+\exp\left(\frac{w - z_g q_e V_m}{kT}\right)} \qquad (4.5)$$

Macroscopic channel kinetics:

$$N_c \underset{\beta}{\overset{\alpha}{\rightleftharpoons}} N_o \qquad (4.7)$$

$$\frac{\mathrm{d}N_c}{\mathrm{d}t} = \beta N_o - \alpha N_c \qquad (4.8)$$

$$\frac{\mathrm{d}N_o}{\mathrm{d}t} = \alpha N_c - \beta N_o \qquad (4.9)$$

Single channel probabilities:

$$p = \frac{\langle N_o \rangle}{N} \tag{4.15}$$

$$q = \frac{\langle N_c \rangle}{N} \tag{4.16}$$

Mean number of open channels:

$$\langle N_o \rangle = Np \tag{4.22}$$

Mean macroscopic conductance:

$$\langle G_\mathsf{K} \rangle = Np\,\gamma_\mathsf{K} \tag{4.29}$$

Gating particle kinetics for constant α_n and β_n:

$$\begin{aligned} n(t) &= n_\infty - (n_\infty - n_o)\,\mathrm{e}^{-t/\tau_n} \\ &= (n_\infty - n_o)\left[1 - \mathrm{e}^{-t/\tau_n}\right] + n_o \end{aligned} \tag{4.36}$$

$$\tau_n = \frac{1}{\alpha_n + \beta_n}, \quad n_\infty = \frac{\alpha_n}{\alpha_n + \beta_n} \tag{4.37}$$

GHK transmembrane potential equation:

$$V_m = \frac{RT}{F} \ln\left[\frac{P_\mathsf{K}[\mathsf{K}]_e + P_\mathsf{Na}[\mathsf{Na}]_e + P_\mathsf{Cl}[\mathsf{Cl}]_i}{P_\mathsf{K}[\mathsf{K}]_i + P_\mathsf{Na}[\mathsf{Na}]_i + P_\mathsf{Cl}[\mathsf{Cl}]_e}\right] \tag{5.1}$$

Hodgkin–Huxley potassium channel model:

$$g_{\mathsf{K}}(t, v_m) = \bar{g}_{\mathsf{K}} n^4(t, v_m) \tag{5.18}$$

$$\frac{\mathrm{d}n(t, v_m)}{\mathrm{d}t} = \alpha_n(v_m)\,(1-n) - \beta_n(v_m)\,n \tag{5.19}$$

or

$$\frac{\mathrm{d}n(t, v_m)}{\mathrm{d}t} = \frac{n_\infty - n}{\tau_n} \tag{5.22}$$

$$\alpha_n(v_m) = \frac{0.01\,(10 - v_m)}{\exp\left(\frac{10 - v_m}{10}\right) - 1} \tag{5.24}$$

$$\beta_n(v_m) = 0.125\,\exp\left(\frac{-v_m}{80}\right) \tag{5.25}$$

Hodgkin–Huxley sodium channel model:

$$g_{\mathrm{Na}}(t, v_m) = \bar{g}_{\mathrm{Na}} m^3(t, v_m)\, h(t, v_m) \tag{5.26}$$

$$\frac{\mathrm{d}m(t, v_m)}{\mathrm{d}t} = \alpha_m(v_m)\,(1-m) - \beta_m(v_m)\, m \tag{5.27}$$

$$\frac{\mathrm{d}h(t, v_m)}{\mathrm{d}t} = \alpha_h(v_m)\,(1-h) - \beta_h(v_m)\, h \tag{5.28}$$

$$\alpha_m(v_m) = \frac{0.1\,(25 - v_m)}{\exp\left(\frac{25-v_m}{10}\right) - 1};$$

$$\beta_m(v_m) = 4\exp\left(\frac{-v_m}{18}\right) \tag{5.36}$$

$$\alpha_h(v_m) = 0.07\exp\left(\frac{-v_m}{20}\right);$$

$$\beta_h(v_m) = \left\{\exp\left(\frac{30 - v_m}{10}\right) + 1\right\}^{-1} \tag{5.37}$$

Temperature scaling of HH ion channel gating rates:

$$Q = 3^P \tag{5.64}$$

$$P = \frac{T - 6.3}{10} \tag{5.65}$$

Temperature scaling of HH ion channel gating rates (cont.):

$$\frac{\mathrm{d}n}{\mathrm{d}t} = Q\alpha_n(1-n) - Q\beta_n n \tag{5.66}$$

$$\frac{\mathrm{d}m}{\mathrm{d}t} = Q\alpha_m(1-m) - Q\beta_m m \tag{5.67}$$

$$\frac{\mathrm{d}h}{\mathrm{d}t} = Q\alpha_h(1-h) - Q\beta_h h \tag{5.68}$$

Sodium pump efflux:

$$-\frac{\mathrm{d}\left[{}^{24}\mathrm{Na}^{+}\right]_i}{\mathrm{d}t} = k\left[{}^{24}\mathrm{Na}^{+}\right]_i \tag{5.70}$$

$$\Rightarrow \left[{}^{24}\mathrm{Na}^{+}\right]_i = A\exp(-kt) \tag{5.71}$$

Sodium pump stoichiometry:

$$\begin{gathered}\mathrm{ATP} + x\,[\mathrm{Na}]_i + y\,[\mathrm{K}]_o \longrightarrow \\ \mathrm{ADP} + \mathrm{P}_i + x\,[\mathrm{Na}]_o + y\,[\mathrm{K}]_i\,, \\ \text{where } x = 3 \text{ and } y = 2.\end{gathered} \tag{5.72}$$

Calcium channel model:

$$I_{\mathrm{Ca}} = 4\frac{P_{\mathrm{Ca}}V_m F^2}{RT}\left(\frac{[\mathrm{Ca}]_o - [\mathrm{Ca}]_i\,\mathrm{e}^{2V_m F/RT}}{1-\mathrm{e}^{2V_m F/RT}}\right) \tag{5.69}$$

Frankenhaeuser–Huxley potassium channel model:

$$I_{\mathsf{K}} = P_{\mathsf{K}} \frac{V_m^2 F^2}{RT} \left(\frac{[\mathsf{K}]_o - [\mathsf{K}]_i \, \mathrm{e}^{V_m F/RT}}{1 - \mathrm{e}^{V_m F/RT}} \right) \quad (12.27)$$

$$P_{\mathsf{K}} = \bar{P}_{\mathsf{K}} n^2 \quad (12.28)$$

$$\alpha_n = 0.02\,(v_m - 35) \left(1 - \mathrm{e}^{\frac{35 - v_m}{10}} \right)^{-1} \quad (12.30)$$

$$\beta_n = 0.05\,(10 - v_m) \left(1 - \mathrm{e}^{\frac{v_m - 10}{10}} \right)^{-1} \quad (12.31)$$

Cable equation intracellular axial resistance per unit length:

$$r_i = \frac{R_i}{\pi a^2} \quad \Omega/\mathrm{cm} \quad (2.55')$$

Cable equation membrane resistance times unit length:

$$r_m = \frac{R_m}{2\pi a} \quad \Omega\ \mathrm{cm} \quad (2.56')$$

Cable equation membrane capacitance per unit length:

$$c_m = C_m 2\pi a \quad \mu\mathrm{F/cm} \quad (2.57')$$

Cable equation extra- and intra-cellular axial electric potential gradients:

$$\frac{\partial \Phi_e}{\partial x} = -I_e r_e \quad (6.1)$$

$$\frac{\partial \Phi_i}{\partial x} = -I_i r_i \quad (6.2)$$

Cable equation intra- and extra-cellular axial current gradients:

$$\frac{\partial I_i}{\partial x} = -i_m \tag{6.3}$$

$$\frac{\partial I_e}{\partial x} = i_m + i_p \tag{6.4}$$

Cable equation total axial current:

$$I = I_i + I_e \tag{6.5}$$

Cable equation transmembrane current vs. membrane potential and applied current:

$$i_m = \frac{1}{(r_i + r_e)}\left(\frac{\partial^2 V_m}{\partial x^2} - r_e i_p\right) \tag{6.12}$$

Cable equation transmembrane current vs. intracellular potential:

$$i_m = \frac{1}{r_i}\frac{\partial^2 \phi_i}{\partial x^2} \tag{6.13}$$

Cable equation intra- and extra-cellular potentials in source-free region:

$$\phi_i(x,t) = \frac{r_i}{r_i + r_e} v_m(x,t) \tag{6.23}$$

$$\phi_e(x,t) = -\frac{r_e}{r_i + r_e} v_m(x,t) \tag{6.24}$$

Hodgkin-Huxley cable equation:

$$\frac{\partial V_m}{\partial t} = \frac{1}{C_m}\left[\frac{1}{2\pi a(r_i+r_e)}\left(\frac{\partial^2 V_m}{\partial x^2} - r_e i_p\right) - I_{\text{ion}}(x,t)\right]$$

$$\begin{aligned} I_{\text{ion}}(x,t) &= g_{\text{K}}(x,t)\,[V_m(x,t) - E_{\text{K}}] \\ &+ g_{\text{Na}}(x,t)\,[V_m(x,t) - E_{\text{Na}}] \\ &+ g_L\,[V_m(x,t) - E_L] \end{aligned} \tag{6.28}$$

Wave equation for temporal waveform:

$$V_m(x,t) = V_m\left(t - \frac{(x - x_0)}{\theta}\right) \tag{6.64$'$}$$

Wave equation for spatial waveform:

$$V_m(x,t) = V_m(x - \theta\,(t - t_0)) \tag{6.64$''$}$$

Hodgkin-Huxley propagating action potential equation:

$$\begin{aligned} \frac{a}{2R_i\theta^2}\frac{\mathrm{d}^2 V_m}{\mathrm{d}t^2} = C_m\frac{\mathrm{d}V_m}{\mathrm{d}t} &+ g_{\text{K}}(V_m - E_{\text{K}}) \\ &+ g_{\text{Na}}(V_m - E_{\text{Na}}) \\ &+ g_L(V_m - E_L) \end{aligned} \tag{6.68}$$

Isopotential membrane patch response to an intracellular current step:

$$v_m = I_0 R\left(1 - \mathrm{e}^{-t/\tau}\right) \tag{7.3}$$

$$= S\left(1 - \mathrm{e}^{-t/\tau}\right) \tag{7.4}$$

Strength-duration relationship for intracellular stimulation:

$$I_{\text{th}} = I_R / \left(1 - \mathrm{e}^{-T/\tau}\right) \tag{7.8}$$

Chronaxie for intracellular stimulation:

$$T_c = \tau \ln 2 = 0.693\tau \tag{7.11}$$

Linear cable equation for extracellular current injection:

$$\lambda^2 \frac{\partial^2 v_m}{\partial x^2} - \tau \frac{\partial v_m}{\partial t} - v_m = r_e \lambda^2 i_p \tag{7.14}$$

Space constant and time constant:

$$\lambda = \sqrt{\frac{r_m}{r_i + r_e}} \quad \text{and} \quad \tau = r_m c_m \tag{7.15}$$

Steady-state solution to homogeneous infinite linear cable equation:

$$v_m = A\mathrm{e}^{-x/\lambda} + B\mathrm{e}^{x/\lambda} \tag{7.18}$$

Steady-state solution to infinite linear cable equation with extracellular current injection at the spatial origin:

$$v_m = -\frac{r_e \lambda I_0}{2}\, \mathrm{e}^{-|x|/\lambda} \tag{7.34}$$

General time-varying solution to infinite linear cable equation with intracellular current step at the spatial origin:

$$v_m(X,T) = \frac{r_i \lambda I_0}{4}\left\{ \mathrm{e}^{-|X|}\left[1-\mathrm{erf}\left(\frac{|X|}{2\sqrt{T}}-\sqrt{T}\right)\right] - \mathrm{e}^{|X|}\left[1-\mathrm{erf}\left(\frac{|X|}{2\sqrt{T}}+\sqrt{T}\right)\right]\right\} \qquad (7.48)$$

Extracellular field potential for applied monopole point current source:

$$\phi_a = \frac{I_0}{4\pi\sigma_e r} \qquad (7.55)$$

Initial transmembrane response to an applied extracellular field:

$$r_i \frac{\partial v_m}{\partial t} = \frac{1}{c_m}\frac{\partial^2 \phi_e}{\partial z^2} \qquad (7.59)$$

Extracellular field produced by a fiber:

$$\Phi_e(P) = \frac{1}{4\pi\sigma_e}\int_L \frac{i_m}{r}\mathrm{d}x \qquad (8.11)$$

Approximate extracellular field produced by a cylindrical fiber:

$$\Phi_e = \frac{a^2\sigma_i}{4\sigma_e}\int \frac{\partial^2 V_m/\partial x^2}{r}\mathrm{d}x \qquad (8.12)$$

Monopole source density for a cylindrical fiber:

$$I_\ell = \pi a^2 \sigma_i \frac{\partial^2 V_m}{\partial x^2} \qquad (8.19)$$

Lumped monopole strength for a cylindrical fiber:

$$
\begin{aligned}
M &= \pi a^2 \sigma_i \int_{x_a}^{x_b} \frac{\partial^2 V_m}{\partial x^2}\, \mathrm{d}x \\
&= \pi a^2 \sigma_i \left(\left.\frac{\partial V_m}{\partial x}\right|_{\substack{x= \\ x_b}} - \left.\frac{\partial V_m}{\partial x}\right|_{\substack{x= \\ x_a}} \right)
\end{aligned}
\tag{8.21}
$$

Extracellular field potential for idealized dipole source:

$$\Phi_d = \frac{1}{4\pi\sigma} \nabla \left(\frac{1}{r} \right) \cdot \bar{p} \tag{2.29}$$

Dipole source density for a cylindrical fiber:

$$\bar{\tau}_\ell \equiv -\pi a^2 \sigma_i \frac{\partial V_m}{\partial x} \bar{a}_x \approx I_i \bar{a}_x \tag{8.41}$$

Lumped dipole strength for a cylindrical fiber:

$$
\begin{aligned}
D &= -\pi a^2 \sigma_i \int_{x_1}^{x_2} \frac{\partial V_m}{\partial x}\, \mathrm{d}x \\
&= \pi a^2 \sigma_i \left[V_m(x_1) - V_m(x_2) \right]
\end{aligned}
\tag{8.46}
$$

Heart vector/dipole:

$$\bar{H} = \int \bar{J}_i \, \mathrm{d}V \tag{9.83}$$

Lead voltage:

$$V_\ell = \bar{H} \cdot \bar{\ell} \tag{9.87}$$

Standard lead voltages:

$$V_{\mathrm{I}} = \Phi_{\mathsf{LA}} - \Phi_{\mathsf{RA}} \tag{9.74}$$

$$V_{\mathrm{II}} = \Phi_{\mathsf{LL}} - \Phi_{\mathsf{RA}} \tag{9.75}$$

$$V_{\mathrm{III}} = \Phi_{\mathsf{LL}} - \Phi_{\mathsf{LA}} \tag{9.76}$$

Binomial probability distribution:

$$f(x) = \frac{n!}{x!\,(n-x)!} p^x q^{n-x} \tag{10.1}$$

Poisson probability distribution:

$$f(x) = \frac{\mathrm{e}^{-m} m^x}{x!} \tag{10.6}$$

Postsynaptic current equation:

$$I_{\mathsf{syn}} = g_{\mathsf{syn}}(t)\,[V_m(t) - E_{\mathsf{syn}}] \qquad \text{(Koch 1.18)}$$

Postsynaptic conductance alpha equation:

$$g_{\mathsf{syn}}(t) = \mathrm{const} \cdot t\, \mathrm{e}^{-t/t_{\mathsf{peak}}} \qquad \text{(Koch 1.21)}$$

Ligand-gated fast chemical synapse kinetic model with two binding sites and slow unbinding:

$$\underset{\text{closed}}{\mathrm{R}} \; \overset{\text{binding-1}}{\underset{k_{-1}}{\overset{\mathrm{A}\;\; k_1}{\rightleftharpoons}}} \; \underset{\text{closed}}{\mathrm{AR}} \; \overset{\text{binding-2}}{\underset{k_{-2}}{\overset{\mathrm{A}\;\; k_2}{\rightleftharpoons}}} \; \underset{\text{closed}}{\mathrm{A_2R}} \; \overset{\text{opening}}{\underset{\underset{\text{closing}}{\alpha(\mathrm{E})}}{\overset{\beta}{\rightleftharpoons}}} \; \underset{\text{open}}{\mathrm{A_2R^*}} \qquad \text{(H6.3)}$$

Anodic and cathodic Faradaic reactions for stainless-steel electrodes:

$$\mathrm{Fe} \longrightarrow \mathrm{Fe}^{++} + 2e^{-} \tag{12.2}$$

$$2\mathrm{H_2O} + 2e^{-} \longrightarrow \mathrm{H_2} \uparrow \mathrm{OH}^{-} \tag{12.3}$$

Anodic and cathodic Faradaic reactions for platinum electrodes:

$$\mathrm{Pt} + \mathrm{H_2O} \rightarrow \mathrm{PtO} + 2\mathrm{H}^{+} + 2e^{-} \tag{12.4}$$

$$\mathrm{Pt} + \mathrm{H}^{+} + e^{-} \longrightarrow \mathrm{Pt\text{-}H} \tag{12.5}$$

Strength-duration relationship for extracellular stimulation:

$$I_{\mathrm{th}} = \frac{I_R}{\left(1 - \mathrm{e}^{-Kt}\right)} \tag{12.7}$$

Chronaxie for extracellular stimulation:

$$t_c = \frac{\ln 2}{K} \tag{12.13}$$

Activating function for myelinated axon:

$$\frac{\Phi_{e,n-1} - 2\Phi_{e,n} + \Phi_{e,n+1}}{\Delta z^2}$$

END OF SUPPLIED EQUATIONS AND TABLES

THE END