EE 4CL4 – Control System Design

Solutions to Homework Assignment #1

1. **Explain how a high-gain feedback loop can be used to produce an implicit inverse of the plant model. (20 pts)**

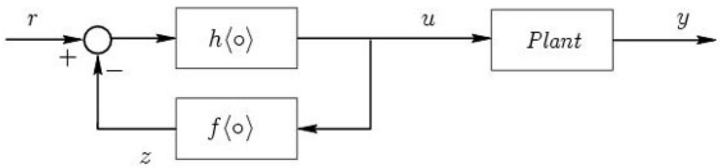

Figure 2.7: *Realisation of conceptual controller*

From Fig. 2.7 of Goodwin et al., $u = h\langle r - z\rangle = h\langle r - f\langle u\rangle\rangle$. Passing each side of this equation through the function $h^{-1}\langle\circ\rangle$ gives $h^{-1}\langle u\rangle = r - f\langle u\rangle$ or $f\langle u\rangle = r - h^{-1}\langle u\rangle$. Passing each side of this last equation through the function $f^{-1}\langle\circ\rangle$ then gives $u = f^{-1}\langle r - h^{-1}\langle u\rangle\rangle$. If $h^{-1}\langle u\rangle << r$, then $u \approx f^{-1}\langle r\rangle$. That is, the feedback loop produces an approximate inverse of $f\langle\circ\rangle$ under the condition that $h^{-1}\langle\circ\rangle$ is small, i.e., the gain of $h\langle\circ\rangle$ is high.

2. **List under what conditions might an open-loop controller be acceptable? Describe the advantages of using an open-loop controller as compared to a closed-loop controller such a case? (20 pts)**

An open-loop controller may be sufficient if:

a. a very accurate model of the plant is known,

b. the model and its inverse are stable, and

c. disturbances and initial conditions are negligible.

The advantages of an open-loop controller are:

d. no sensors required,

e. the controller may be reducible to a very simple system (e.g., an IIR filter), and

f. no transmission of sensor information required, and consequently a possible source of signal delay is removed.

3. **Explain why a closed-loop controller is preferable to an open-loop controller for most control problems? (20 pts)**

Closed-loop controllers are more forgiving in the presence of modelling errors, system instabilities, unknown initial conditions and disturbances to any signal in the system, because the *actual* output of the plant is being taken into account when producing the control signal.

4. **A nonlinear system has an input-output model given by:**

$$\frac{\mathrm{d}\,y(t)}{\mathrm{d}\,t}+\left(1+0.2\,y(t)\right)y(t)=u(t)+0.2u(t)^3$$

a. **Compute the operating points, i.e., values of *y* for a particular value of *u*, for u_Q = 2. (assume they are equilibrium points, i.e., $\frac{\mathrm{d}\,y(t)}{\mathrm{d}\,t}=0$)**

b. **Obtain a linearized model for each of the operating points above. (40 pts)**

a. If $\frac{\mathrm{d}\,y(t)}{\mathrm{d}\,t}=0$ and u_Q = 2, then $0.2{y_Q}^2+y_Q=u_Q+0.2{u_Q}^3 \Rightarrow 0.2{y_Q}^2+y_Q-3.6=0$ and consequently $y_Q = -7.4244$ or 2.4244.

b. The system can be linearized by using the first-order Taylor series to approximate the model, such that:

$$\frac{\mathrm{d}\,\Delta y(t)}{\mathrm{d}\,t}\approx \left.\frac{\partial f}{\partial y}\right|_{\substack{y=y_Q\\ u=u_Q}}\Delta y(t)+\left.\frac{\partial f}{\partial u}\right|_{\substack{y=y_Q\\ u=u_Q}}\Delta u(t),$$

where $\Delta y(t)=y(t)-y_Q$ and $\Delta u(t)=u(t)-u_Q$.

Solving this equation for the nonlinear model gives the linear equation:

$$\frac{\mathrm{d}\,\Delta y(t)}{\mathrm{d}\,t}+\left(1+0.4\,y_Q\right)\Delta y(t)=\left(1+0.6{u_Q}^2\right)\Delta u(t).$$

For the two equilibrium points (u_Q, y_Q) = (2, −7.4244) and (2, 2.4244) obtained in part a. above, the system can be modelled, respectively, as:

$$\frac{\mathrm{d}\,\Delta y(t)}{\mathrm{d}\,t}-1.9698\Delta y(t)=3.4\Delta u(t),\text{ and}$$

$$\frac{\mathrm{d}\,\Delta y(t)}{\mathrm{d}\,t}+1.9698\Delta y(t)=3.4\Delta u(t).$$