

## ELEC ENG 4CL4 – CONTROL SYSTEM DESIGN

### Lab #2: MATLAB/Simulink analysis of control system stability

#### **Objectives:**

To gain experience in analysing the stability of a closed-loop feedback control system within MATLAB/Simulink.

#### **Assessment:**

Your grade for this lab will be based on your ability to a) apply various mathematical algorithms, b) write MATLAB code and c) create a Simulink model to investigate the stability of the control system described below and on your reporting of the results. The report should contain the mathematical derivations and calculations carried out, MATLAB plots of results, a schematic of your Simulink model, and answers to specific questions below.

A number of mathematical derivations and calculations in Sections 1 and 2 below should be done before the lab session, so that you can make maximal use of your time in the lab using MATLAB/Simulink.

The total grade will be out of 20 points. The written report will be worth 15 points, and the remaining 5 points will be based on your demonstration of your Simulink model and MATLAB code to the TA and/or a pre-lab quiz on the theory required for this lab.

Clearly label all plots and their axes (points for style will be deducted otherwise)

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Please attend the lab section to which you have been assigned.

You should complete this lab with one lab partner. If there are an odd number of students, then one group of three will be created by the TA, or you can choose to work on your own if a computer is available.

You may choose to complete the lab assignment partially or entirely in your own time (in groups preferably of two students but definitely no larger than three). However, if you choose to do this, you must show up at the start of your scheduled lab time to give the TA a brief demonstration of your MATLAB/Simulink code and model.

Each pair of students should complete one lab report together, which is to be submitted one week from the date of the lab.

## Description of tracked vehicle turning control:

The design of a turning control for a tracked vehicle involves the selection of two parameters. In Figure 1, the system shown in panel (a) has the model shown in panel (b). The parameters  $a$  and  $K$  affect the performance of the system, including its stability.

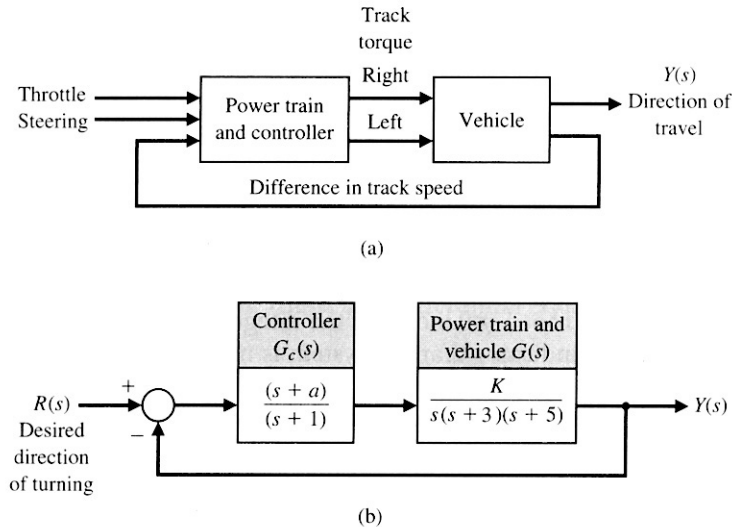


Figure 1 Tracked vehicle control system. (a) Turning control system for a two-track vehicle. (b) Linear system model.

### 1. Routh's criterion for internal stability

- Determine the closed-loop characteristic polynomial for the model shown in Fig. 1(b).
- Generate Routh's array for this characteristic polynomial.
- Determine under what conditions (i.e., values of  $a$  and  $K$ ) the closed-loop system is internally stable.
- In MATLAB, plot the curve of  $a$  versus  $K$  (for  $K > 0$ ) that divides the regions of stability and instability, and indicate on the plot which is the stable region.

### 2. Steady state error to a ramp reference

For a unity-gain closed-loop feedback system with open-loop transfer function  $G_c(s)G(s)$ , reference  $r(t)$ , output  $y(t)$  and error  $e(t) = r(t) - y(t)$ , the steady-state error is:

$$e_{ss} = \lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} \frac{sR(S)}{1 + G_c(S)G(S)}$$

- Derive the steady-state error  $e_{ss}$  for a ramp reference  $r(t) = At$ ,  $t > 0$ .
- Determine under what conditions (i.e., values of  $a$  and  $K$ ) the steady-state error is less than or equal to 22% of the ramp slope  $A$ .

### 3. Nyquist stability analysis

- a. Choose some value for  $a$  between 0 and 1. Now choose some value for  $K$  such that, given your choice for  $a$ , the system is stable and has a steady-state error less than or equal to 22% of the reference ramp slope  $A$ .
- b. In MATLAB, generate the Nyquist plot for  $G_c(s)G(s)$  with the values of  $a$  and  $K$  chosen above.

Hint #1: Use the MATLAB function `[bs, as] = zp2tf(...)` to find the transfer function (i.e., polynomial) representation of  $G_c(s)G(s)$  from the system's zeros, poles and gain.

Hint #2: Use the MATLAB Signal Processing Toolbox function `H = freqs(bs, as, w)` to find the frequency response of  $G_c(s)G(s)$ . Note that you need to provide an output variable (`H` in the example above) or otherwise this function will automatically generate a Bode plot, which is not what you want! A suitable vector for the frequency inputs to `freqs()` is

$$w = \text{logspace}(0, 4, 100)$$

- c. Does this Nyquist plot confirm that the system is stable for these values of  $a$  and  $K$ ?
- d. Leaving the value of  $a$  the same as above, increase the value of  $K$  such that the system should be unstable. Generate a Nyquist plot to confirm that the system is indeed unstable with these values of  $a$  and  $K$ .

### 4. Root-Locus Analysis

In MATLAB, generate the root-locus plot for this system, for values of  $K$  between 0 and some large number approximating  $\infty$ . Use the same value of  $a$  as above.

Hint #1: Use the MATLAB function `roots()` to find the roots of the characteristic polynomial.

On the root-locus plot, indicate the starting point and ending point (if it is finite) of each root locus. What do each of these correspond to in the open-loop transfer function?

### 5. Simulation of the model in Simulink

Create a Simulink model of the control system with a ramp reference as described in Section 2 above.

- a. Using the same values for  $a$  and  $K$  that you used in Section 3a above, plot the ramp reference  $r(t)$  and the model output  $y(t)$  versus  $t$  and confirm that the system is stable and that the steady-state error is less than or equal to 22% of the ramp slope  $A$ —try this for a couple of different values of  $A$ .
- b. Using the same values for  $a$  and  $K$  that you used in Section 3d above, plot the ramp reference  $r(t)$  and the model output  $y(t)$  versus  $t$  and confirm that the system is indeed unstable with these parameters.