ELEC ENG 4CL4:

Control System Design

Notes for Lecture #14

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Tuning of PID Controllers

Because of their widespread use in practice, we present below several methods for tuning PID controllers. Actually these methods are quite old and date back to the 1950's. Nonetheless, they remain in widespread use today.

In particular, we will study.

- *Ziegler-Nichols Oscillation Method*
- *Ziegler-Nichols Reaction Curve Method*
- *Cohen-Coon Reaction Curve Method*

(1) Ziegler-Nichols (Z-N) Oscillation Method

This procedure is only valid for open loop stable plants and it is carried out through the following steps

- Set the true plant under proportional control, with a very small gain.
- Increase the gain until the loop starts oscillating. Note that linear oscillation is required and that it should be detected at the controller output.

- Record the controller critical gain $K_p = K_c$ and the oscillation period of the controller output, P_c.
- Adjust the controller parameters according to Table 6.1 (*next slide*); there is some controversy regarding the PID parameterization for which the Z-N method was developed, but the version described here is, to the best knowledge of the authors, applicable to the parameterization of standard form PID.

Table 6.1: *Ziegler-Nichols tuning using the oscillation method*

	$\mathbf{K_p}$	$\mathbf{T_r}$	$\mathbf{T_d}$
P	$0.50K_c$		
PI	$0.45K_c$	$\frac{P_c}{1.2}$	
PID	$0.60K_c$	$0.5P_c$	$\frac{P_c}{8}$

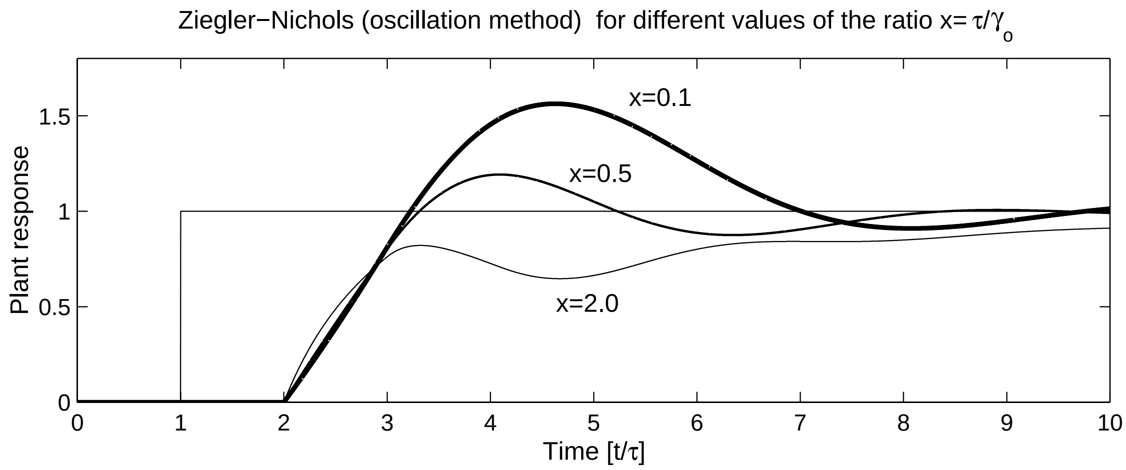
General System

If we consider a general plant of the form:

$$G_0(s) = \frac{K_0 e^{-s\tau}}{\gamma_0 s + 1}; \quad \gamma_0 > 0$$

then one can obtain the PID settings via Ziegler-Nichols tuning for different values of τ and γ_0. The next plot shows the resultant closed loop step responses as a function of the ratio $x \stackrel{\Delta}{=} \frac{\tau}{\gamma_0}$.

Figure 6.3: *PI Z-N tuned (oscillation method) control loop for different values of the ratio* $x \overset{\Delta}{=} \frac{\tau_0}{\gamma_0}$.



Numerical Example

Consider a plant with a model given by

$$G_o(s) = \frac{1}{(s+1)^3}$$

Find the parameters of a PID controller using the Z-N oscillation method. Obtain a graph of the response to a unit step input reference and to a unit step input disturbance.

Solution

Applying the procedure we find:

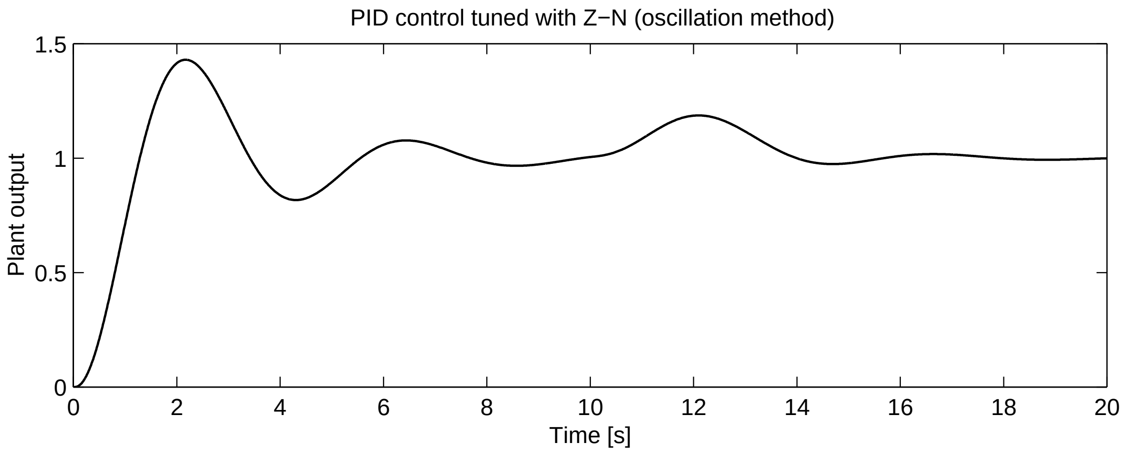
$$K_c = 8 \text{ and } \omega_c = \sqrt{3}.$$

Hence, from Table 6.1, we have

$$K_p = 0.6 * K_c = 4.8; \qquad T_r = 0.5 * P_c \approx 1.81; \qquad T_d = 0.125 * P_c \approx 0.45$$

The closed loop response to a unit step in the reference at $t = 0$ and a unit step disturbance at $t = 10$ are shown in the next figure.

Figure 6.4: *Response to step reference and step input disturbance*



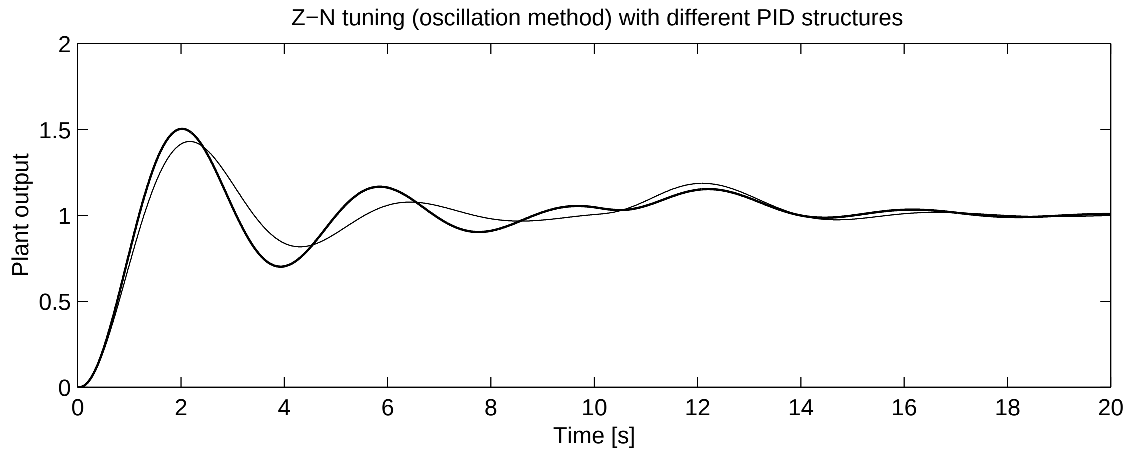
Different PID Structures?

A key issue when applying PID tuning rules (such as Ziegler-Nichols settings) is that of which PID structure these settings are applied to.

To obtain an appreciation of these differences we evaluate the PID control loop for the same plant in Example 6.1, but with the Z-N settings applied to the series structure, i.e. in the notation used in (6.2.5), we have

$$K_s = 4.8 \quad I_s = 1.81 \quad D_s = 0.45 \quad \gamma_s = 0.1$$

Figure 6.5: *PID Z-N settings applied to series structure (thick line) and conventional structure (thin line)*



Observation

In the above example, it has not made much difference, to which form of PID the tuning rules are applied. However, the reader is warned that this can make a difference in general.

(2) Reaction Curve Based Methods

A linearized quantitative version of a simple plant can be obtained with an open loop experiment, using the following procedure:

1. With the plant in open loop, take the plant manually to a normal operating point. Say that the plant output settles at $y(t) = y_0$ for a constant plant input $u(t) = u_0$.
2. At an initial time, t_0, apply a step change to the plant input, from u_0 to u_∞ (*this should be in the range of 10 to 20% of full scale*).

Cont/...

3. Record the plant output until it settles to the new operating point. Assume you obtain the curve shown on the next slide. This curve is known as the *process reaction curve*.

   In Figure 6.6, m.s.t. stands for *maximum slope tangent*.

4. Compute the parameter model as follows

$$K_o = \frac{y_\infty - y_o}{u_\infty - u_o}; \qquad \tau_o = t_1 - t_o; \qquad \nu_o = t_2 - t_1$$

Figure 6.6: *Plant step response*

The suggested parameters are shown in Table 6.2.

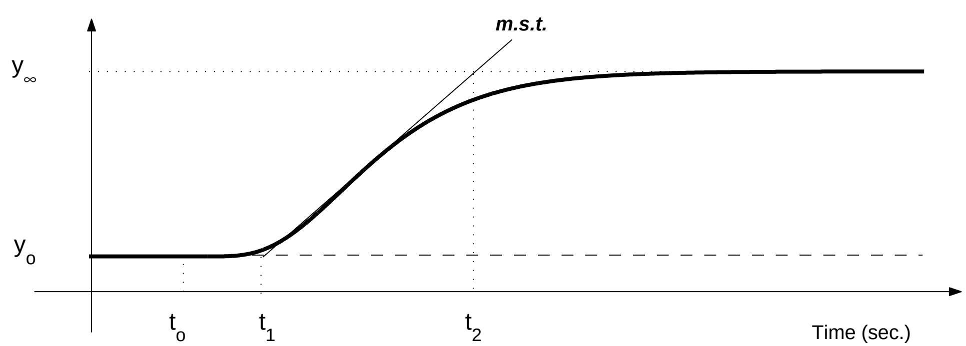


Table 6.2: *Ziegler-Nichols tuning using the reaction curve*

	$\mathbf{K_p}$	$\mathbf{T_r}$	$\mathbf{T_d}$
P	$\frac{\nu_o}{K_o\tau_o}$		
PI	$\frac{0.9\nu_o}{K_o\tau_o}$	$3\tau_o$	
PID	$\frac{1.2\nu_o}{K_o\tau_o}$	$2\tau_o$	$0.5\tau_o$

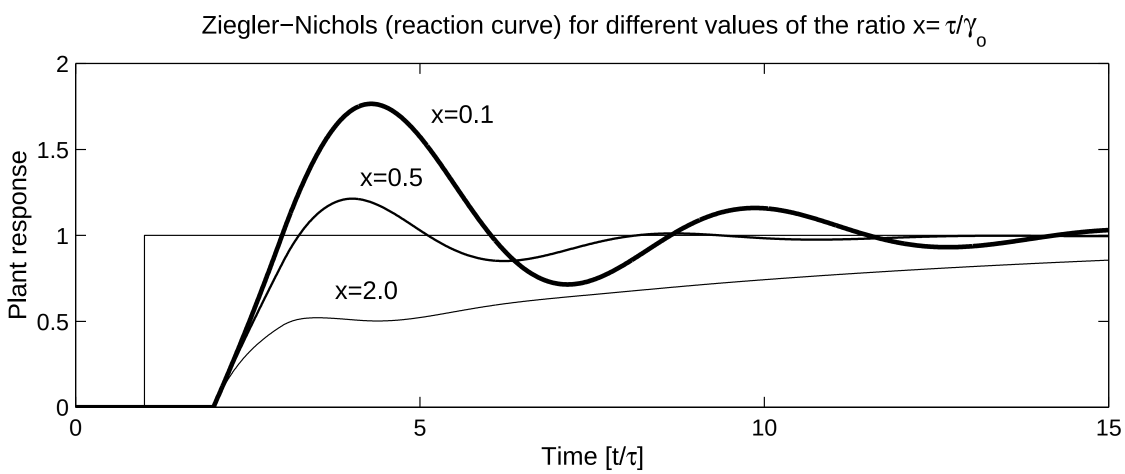
General System Revisited

Consider again the general plant:

$$G_0(s) = \frac{K_0 e^{-s\tau}}{\gamma_0 s + 1}$$

The next slide shows the closed loop responses resulting from Ziegler-Nichols Reaction Curve tuning for different values of $x \stackrel{\Delta}{=} \frac{\tau}{\gamma_0}$.

Figure 6.7: *PI Z-N tuned (reaction curve method) control loop*



Observation

We see from the previous slide that the Ziegler-Nichols reaction curve tuning method is very sensitive to the ratio of delay to time constant.