ELEC ENG 4CL4:

Control System Design

Notes for Lecture #15

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(3) Cohen-Coon Reaction Curve Method

Cohen and Coon carried out further studies to find controller settings which, based on the same model, lead to a weaker dependence on the ratio of delay to time constant. Their suggested controller settings are shown in Table 6.3:

	K_p	T_r	T_d
P	$\frac{\nu_o}{K_o\tau_o}\left[1+\frac{\tau_o}{3\nu_o}\right]$		
PI	$\frac{\nu_o}{K_o\tau_o}\left[0.9+\frac{\tau_o}{12\nu_o}\right]$	$\frac{\tau_o[30\nu_o+3\tau_o]}{9\nu_o+20\tau_o}$	
PID	$\frac{\nu_o}{K_o\tau_o}\left[\frac{4}{3}+\frac{\tau_o}{4\nu_o}\right]$	$\frac{\tau_o[32\nu_o+6\tau_o]}{13\nu_o+8\tau_o}$	$\frac{4\tau_o\nu_o}{11\nu_o+2\tau_o}$

Table 6.3: *Cohen-Coon tuning using the reaction curve.*

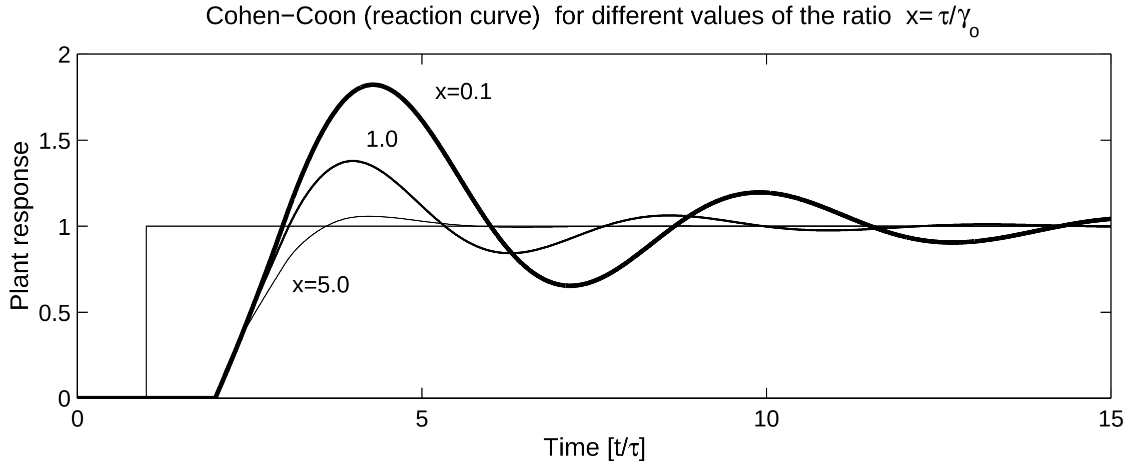
General System Revisited

Consider again the general plant:

$$G_0(s) = \frac{K_0 e^{-s\tau}}{\gamma_0 s + 1}$$

The next slide shows the closed loop responses resulting from Cohen-Coon Reaction Curve tuning for different values of $x = \frac{\tau}{\gamma_0}$.

Figure 6.8: *PI Cohen-Coon tuned (reaction curve method) control loop*



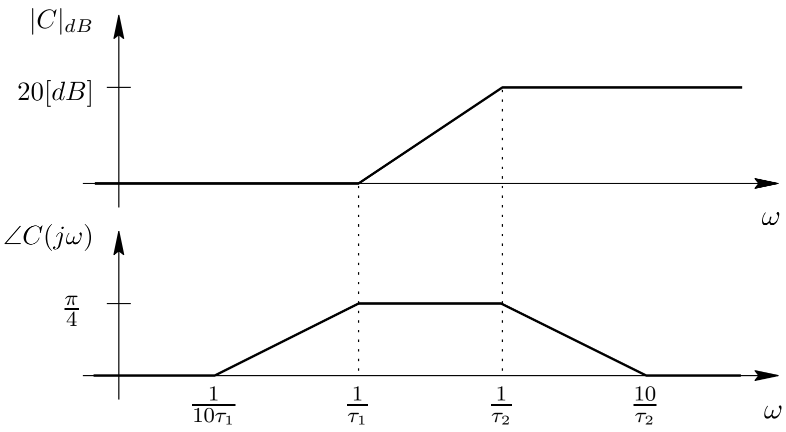
Lead-lag Compensators

Closely related to PID control is the idea of lead-lag compensation. The transfer function of these compensators is of the form:

$$C(s) = \frac{\tau_1 s + 1}{\tau_2 s + 1}$$

If $\tau_1 > \tau_2$, then this is a *lead network* and when $\tau_1 < \tau_2$, this is a *lag network*.

Figure 6.9: *Approximate Bode diagrams for lead networks (τ_1=10τ_2)*



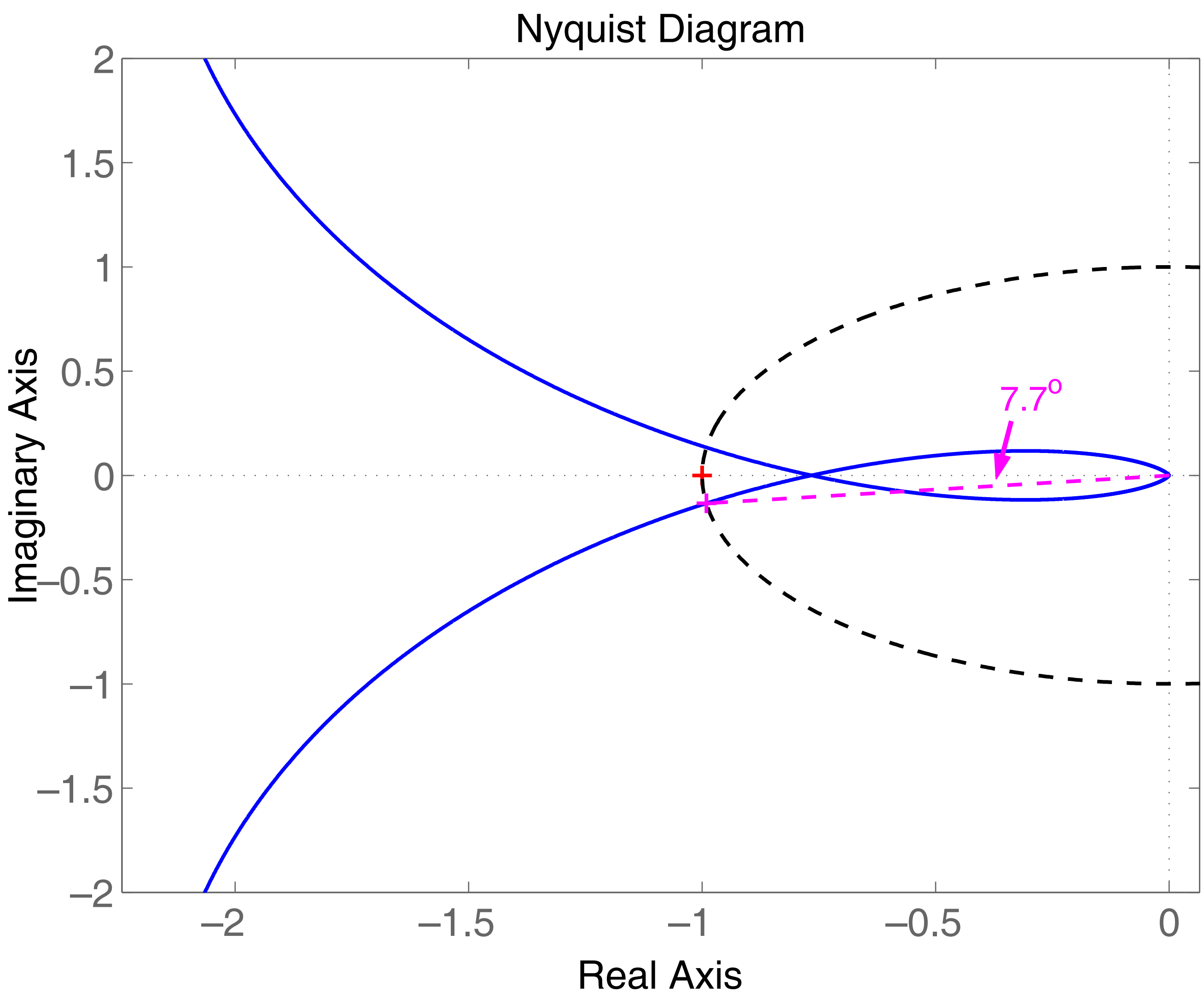
Observation

We see from the previous slide that the lead network gives phase advance at $\omega = 1/\tau_1$ without an increase in gain. Thus it plays a role similar to derivative action in PID.

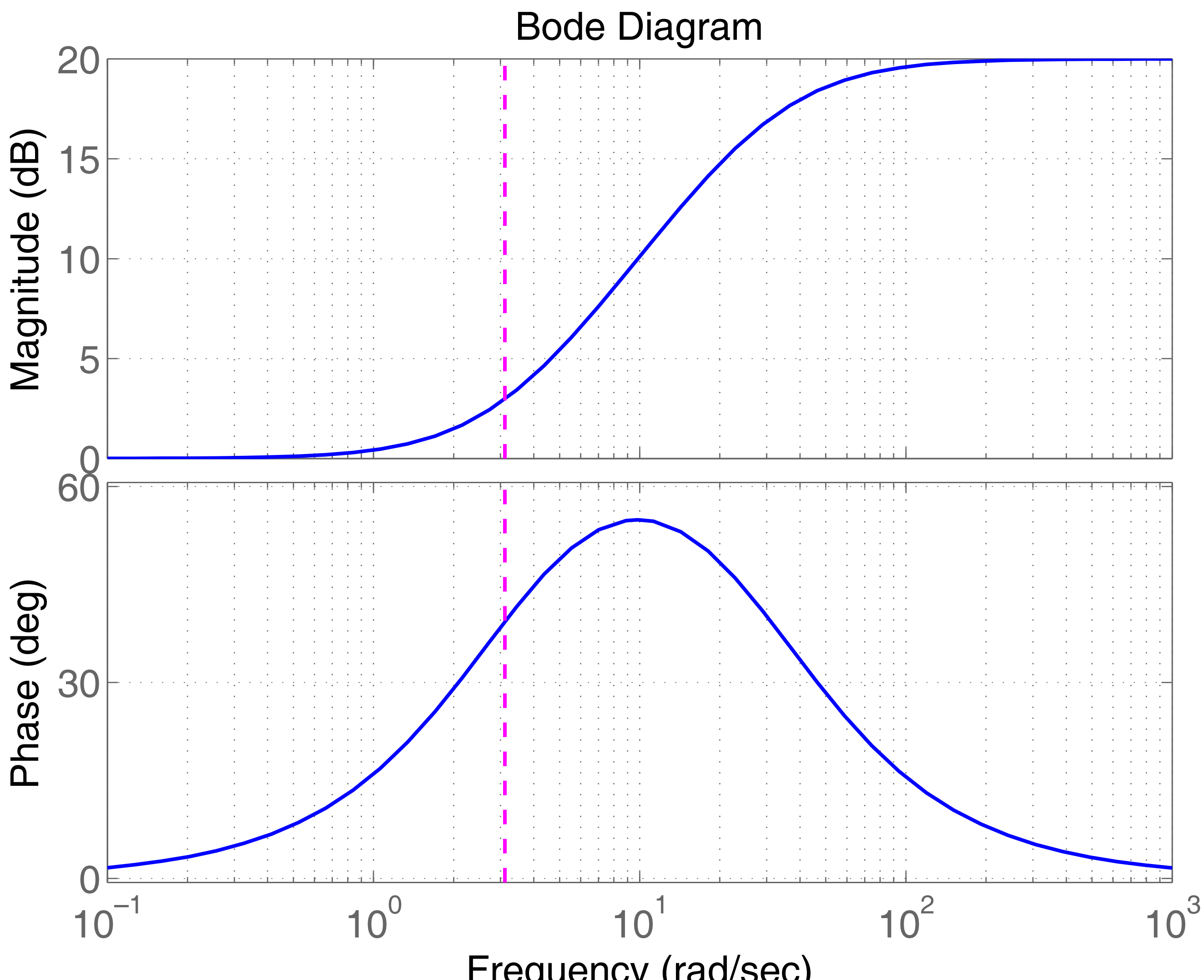
Application of lead compensator to increase phase stability margin

- Say we have a control loop with a very small phase margin at frequency ω_1.
- The lead compensator gives approximately 45° of phase lead at $\omega = 1/\tau_1$, without a significant change in gain.
- If we insert a lead compensator with $\tau_1 = 1/\omega_1$ into our control loop, we will increase the phase margin by close to 45°.

Nyquist stability plot for control loop without lead compensation



Bode plot of lead compensator



Nyquist stability plot for control loop with lead compensation

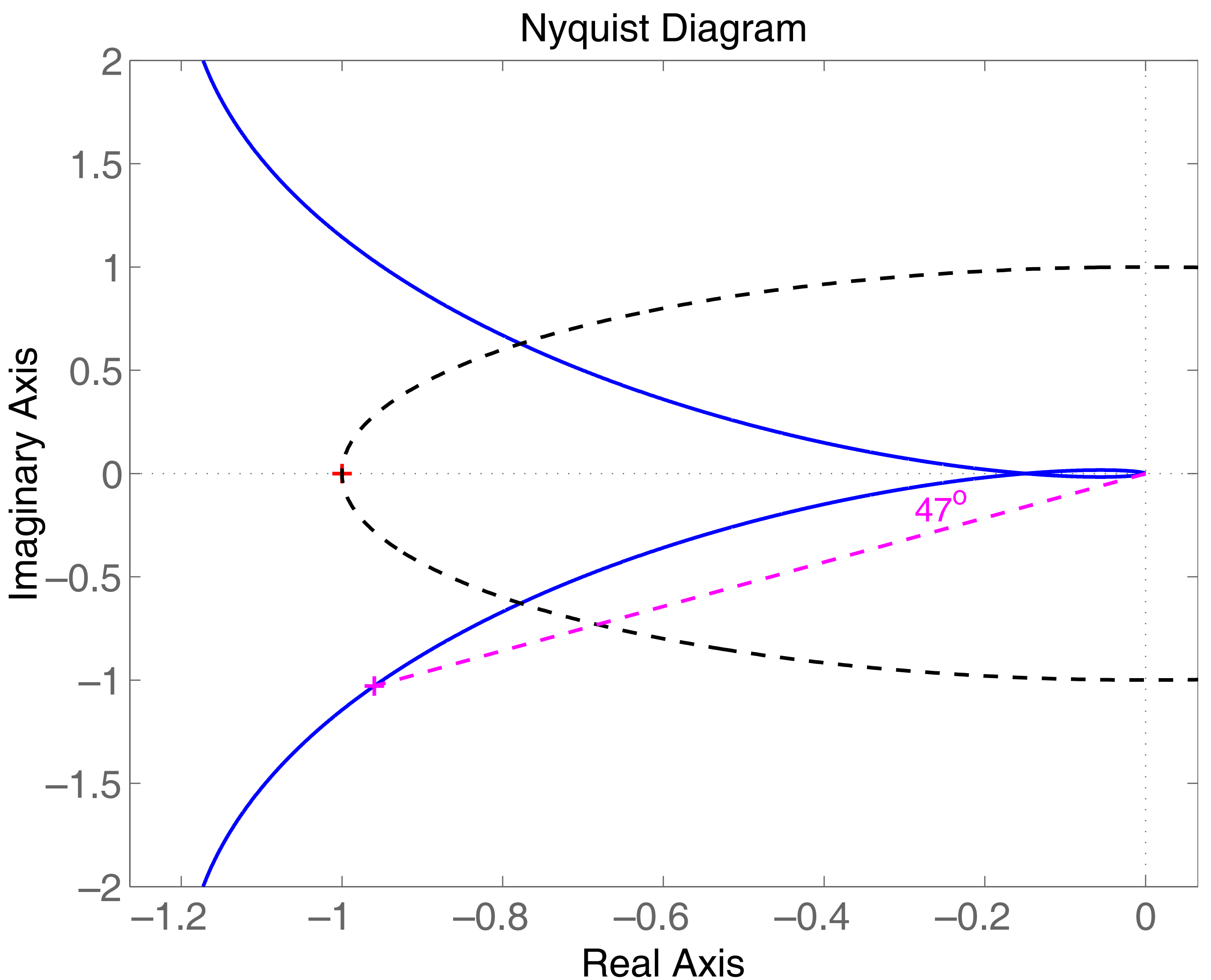
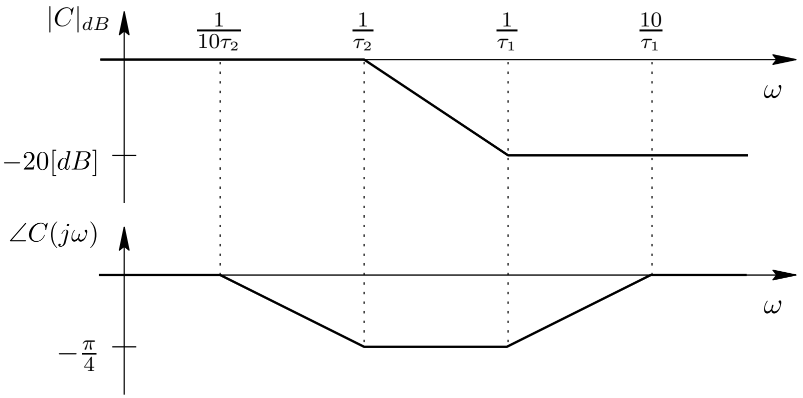


Figure 6.10: *Approximate Bode diagrams for lag networks ($\tau_2=10\tau_1$)*



Observation

We see from the previous slide that the lag network gives low frequency gain increase. Thus it plays a role similar to integral action in PID.

Illustrative Case Study: *Distillation Column*

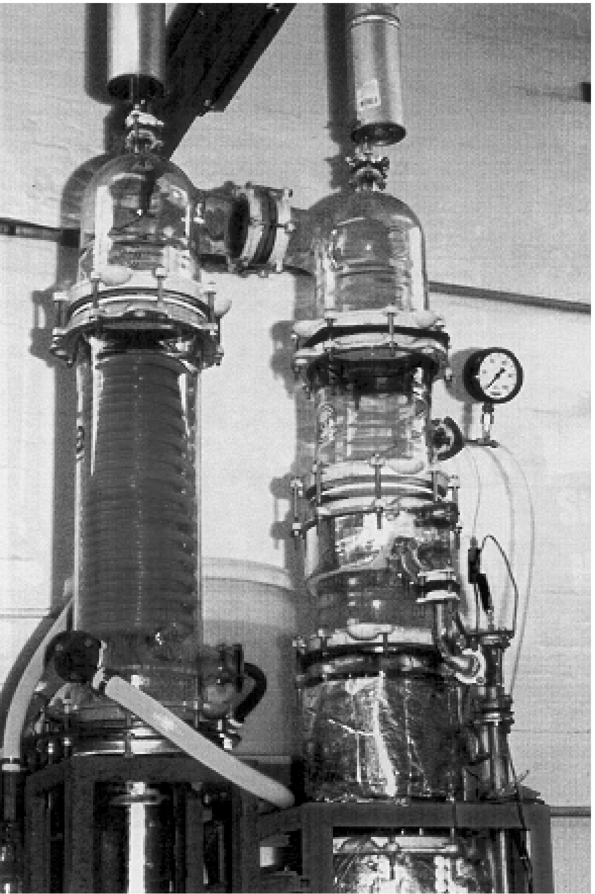
PID control is very widely used in industry. Indeed, one would we hard pressed to find loops that do not use some variant of this form of control.

Here we illustrate how PID controllers can be utilized in a practical setting by briefly examining the problem of controlling a distillation column.

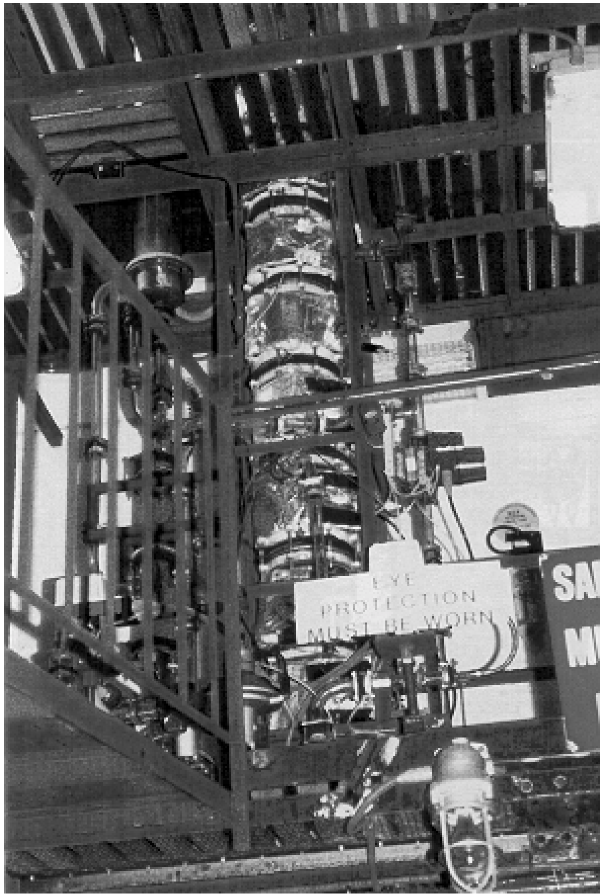
Example System

The specific system we study here is a pilot scale ethanol-water distillation column. Photos of the column (*which is in the Department of Chemical Engineering at the University of Sydney, Australia*) are shown on the next slide.

Condenser



Feed-point



Reboiler

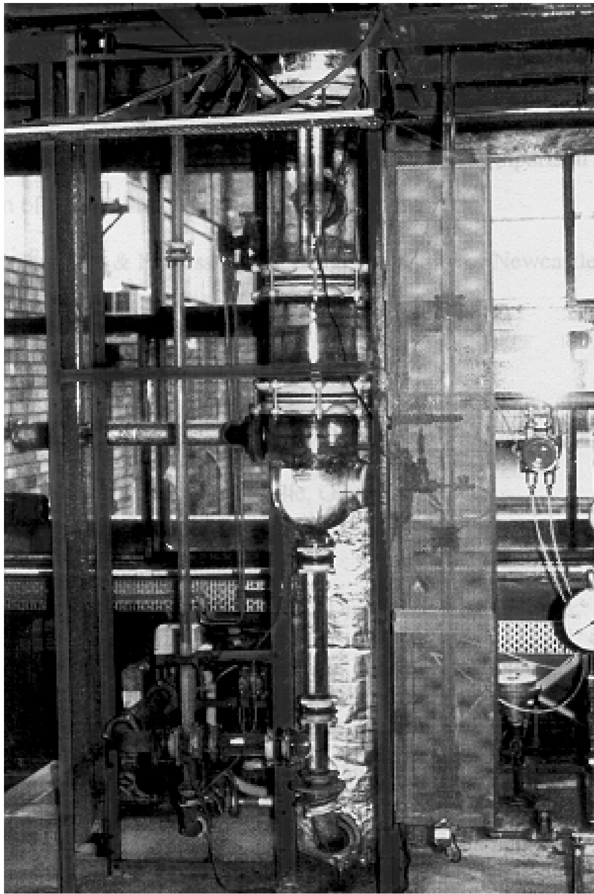
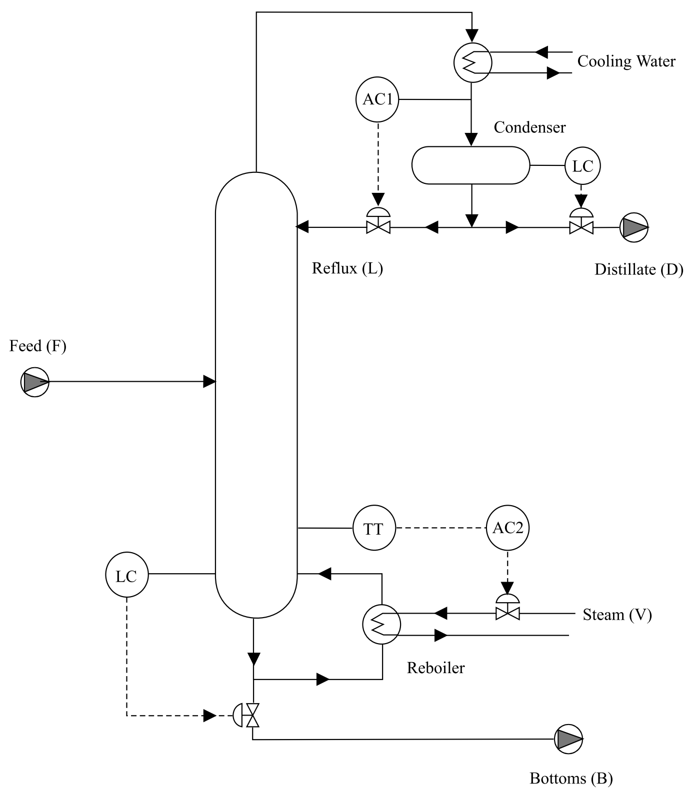


Figure 6.11: *Ethanol - water distillation column*

A schematic diagram of the column is given below:



Model

A locally linearized model for this system is as follows:

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$$

where

$$G_{11}(s) = \frac{0.66e^{-2.6s}}{6.7s+1}$$

$$G_{12}(s) = \frac{-0.0049e^{-s}}{9.06s+1}$$

$$G_{21}(s) = \frac{-34.7e^{-9.2s}}{8.15s+1}$$

$$G_{22}(s) = \frac{0.87(11.6s+1)e^{-s}}{(3.89s+1)(18.8s+1)}$$

Note that the units of time here are minutes.

Decentralized PID Design

We will use two PID controllers:

One connecting Y_1 to U_1

The other, connecting Y_2 to U_2 .

In designing the two PID controllers we will initially ignore the two transfer functions G_{12} and G_{21}. This leads to two separate (and non-interacting) SISO systems. The resultant controllers are:

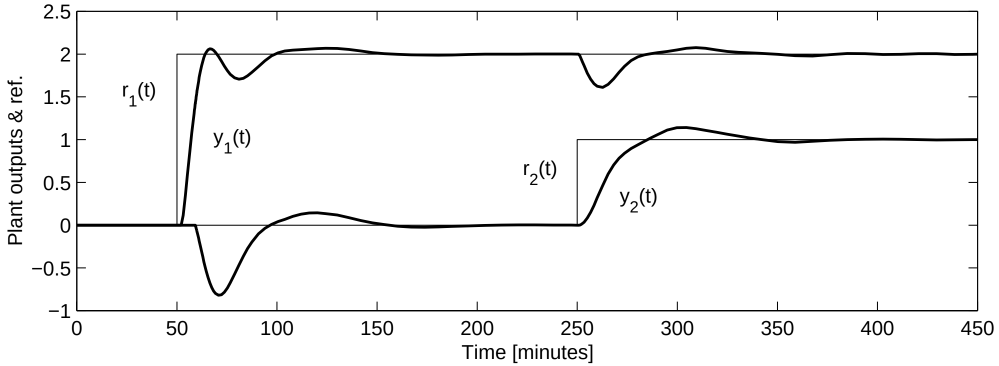
$$C_1(s) = 1 + \frac{0.25}{s}$$
$$C_s(s) = 1 + \frac{0.15}{s}$$

We see that these are of PI type.

Simulations

We simulate the performance of the system with the two decentralized PID controllers. A two unit step in reference 1 is applied at time $t = 50$ and a one unit step is applied in reference 2 at time $t = 250$. The system was simulated with the true coupling (i.e. including G_{12} and G_{21}). The results are shown on the next slide.

Figure 6.12: Simulation results for PI control of distillation column



It can be seen from the figure that the PID controllers give quite acceptable performance on this problem. However, the figure also shows something that is very common in practical applications - namely the two loops interact i.e. a change in reference r_1 not only causes a change in y_1 (as required) but also induces a transient in y_2. Similarly a change in the reference r_2 causes a change in y_2 (as required) and also induces a change in y_1. In this particular example, these interactions are probably sufficiently small to be acceptable. Thus, in common with the majority of industrial problems, we have found that two simple PID (actually PI in this case) controllers give quite acceptable performance for this problem. Later we will see how to design a full multivariable controller for this problem that accounts for the interaction.

Summary

- PI and PID controllers are widely used in industrial control.
- From a modern perspective, a PID controller is simply a controller of (up to second order) containing an integrator. Historically, however, PID controllers were tuned in terms of their **P**, **I** and **D** terms.
- It has been empirically found that the PID structure often has sufficient flexibility to yield excellent results in many applications.

- The basic term is the proportional term, **P**, which causes a corrective control actuation proportional to the error.

- The integral term, **I** gives a correction proportional to the integral of the error. This has the positive feature of ultimately ensuring that sufficient control effort is applied to reduce the tracking error to zero. However, integral action tends to have a destabilizing effect due to the increased phase shift.

- The derivative term, **D**, gives a predictive capability yielding a control action proportional to the rate of change of the error. This tends to have a stabilizing effect but often leads to large control movements.

- Various empirical tuning methods can be used to determine the PID parameters for a given application. They should be considered as a first guess in a search procedure.

- Attention should also be paid to the PID structure.
- Systematic model-based procedures for PID controllers will be covered in later chapters.
- A controller structure that is closely related to PID is a lead-lag network. The lead component acts like **D** and the lag acts like **I**.