ELEC ENG 4CL4:

Control System Design

Notes for Lecture #19

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Goals in Feedback Control Design

Thus far we have learnt about:

- methods for analyzing the *stability* and *performance* of feedback control loops, and
- methods for synthesizing a controller.

What then should be our design goals, i.e., *what constitutes a "good" (or even "ideal") controller?*

Ensuring stability

The first requirement for any feedback control loop is *stability*. It is pointless to try to achieve any performance specification if the system is unstable.

Techniques that we have used to analyze the stability of a feedback control system are:

- *Routh's array*
- *root-locus diagrams*
- *Nyquist stability plots*
- *the robust stability theorem*

Performance requirements

Some performance requirements that we might consider are:

- *step response, including the rise time, maximum undershoot or overshoot, settling time*
- *steady-state error*
- *frequency response*
- *disturbance (noise) rejection*
- *performance robustness*

Achieving a specific performance criterion

Sometimes we might care about achieving just one performance criterion, and we are unconcerned about the other performance measures. In such cases, we can design our controller directly to meet this criterion while maintaining stability.

The design problem in Lab #2 is an example of this:- values for the system parameters K and a were found that ensured stability while achieving a satisfactorily-small steady-state error for a ramp reference signal.

Achieving “ideal” performance

Question: Given a set of performance criteria, is there a design methodology that we can use to obtain a good trade-off between all these requirements?

(We will refer to this as “ideal” performance rather than “optimal”, which would require a mathematical proof according to optimization theory.)

Answer: **Yes!**

In order to show this, we will return to the ideas developed in Lectures #1 and #2.

Returning to the goal of feedback

Recall that the goal of applying feedback was to create an approximate inverse of the plant such that the plant output $Y(s) \simeq R(s)$, the reference signal.

In a one-d.o.f., unity-feedback nominal control loop, this corresponds to:

$$Y(s) = T_o(s)\,R(s)$$

$$\Rightarrow T_o(s) \simeq 1.$$

Assessing $T_o(s) \simeq 1$

Ideal performance would therefore be achieved if we could produce a nominal complementary sensitivity $T_o(s)$ of approximately one for all frequencies ω within the bandwidth of the reference signal $R(s)$.

To evaluate how a unity nominal complementary sensitivity $T_o(s)$ can be obtained over a specific frequency range, we will make use of Bode diagrams.

Note that it will be helpful here to review the rules for creating approximate magnitude-frequency plots.

Approximate Bode magnitude-frequency plots

1. The DC gain (i.e., the gain for $\omega = 0$) is obtained by setting $s = 0$.
2. Each zero at the origin (i.e., $s = 0$) causes the magnitude-frequency response to *increase* by 20 dB/decade, starting at $\omega = 0$.
3. Each pole at the origin causes the magnitude-frequency response to *decrease* by 20 dB/decade, starting at $\omega = 0$.

Approximate Bode plots (cont.)

4. A <u>real-valued zero</u> at $s = c$ causes the magnitude-frequency response to *increase* by 20 dB/decade, starting at $\omega = |c|$.

5. A <u>real-valued pole</u> at $s = c$ causes the magnitude-frequency response to *decrease* by $20 \text{ dB}/\text{decade}$, starting at $\omega = |c|$.

Approximate Bode plots (cont.)

6. A <u>pair of complex-conjugate zeros</u> at $s = \mathrm{c}$ and c^* causes the magnitude-frequency response to *increase* by 40 dB/decade, starting at $\omega = |\mathrm{c}|$, and creates a *null* at $\omega = \mathfrak{Im}\{\mathrm{c}\}$.

7. A <u>pair of complex-conjugate pole</u> at $s = \mathrm{c}$ and c^* causes the magnitude-frequency response to *decrease* by 40 $\mathrm{dB/decade}$, starting at $\omega = |\mathrm{c}|$, and creates a *resonance* at $\omega = \mathfrak{Im}\{\mathrm{c}\}$

Obtaining $T_o(s) \simeq 1$

Using these rules for the approximate Bode magnitude-frequency plot, we can devise the following method for obtaining $T_o(s) \simeq 1$ over a desired frequency range:

First, by including *integration in the controller*, i.e., a pole at $s = 0$, we can ensure that the DC gain is 1, i.e., $T_o(j0) = 1$.

We can show this by letting the denominator polynomial of the controller transfer function:

$$L(s) = s\bar{L}(s)\,.$$

Obtaining $T_o(j0) = 1$

We can now write the nominal complementary sensitivity as:

$$T_o(s) = \frac{B_o(s)\,P(s)}{A_o(s)\,s\bar{L}(s) + B_o(s)\,P(s)}.$$

Setting $s = j0$ gives:

$$T_o(j0) = \frac{\cancel{B_o(0)\,P(0)}}{\cancelto{0}{A_o(0)\cdot 0\,\bar{L}(0)} + \cancel{B_o(0)\,P(0)}}$$

$$= 1.$$

Perfect plant inversion

By using integration in the controller and obtaining $T_o(j0) = 1$, we say that we have obtained *perfect plant inversion* for $\omega = 0$.

Note that this is equivalent to obtaining zero steady-state error in response to a step reference change, which is a DC signal (i.e., has a frequency of 0) in the steady state, our original motivation for including integration in the controller.

Dominant poles and zeros

Second, if we obtain $T_o(j0) = 1$ via integration in the controller, then the rules for the approximate Bode plot show that the magnitude-frequency response of $T_o(s)$ will remain at approximately 1 from $\omega = 0$ up to $\omega = |d|$, where $|d|$ is magnitude of the pole or zero of $T_o(s)$ closest to the imaginary axis.

The pole(s) and zero(s) closest to the imaginary axis are therefore referred to as the *dominant* pole(s) and zero(s).

Bandwidth of $T_o(s)$

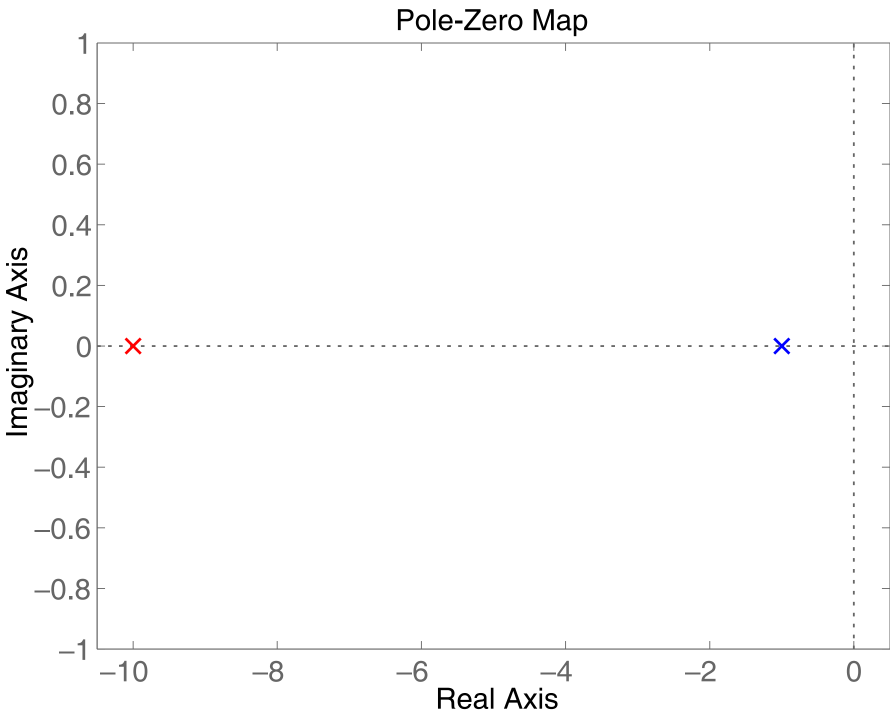
If the nominal plant model is *strictly proper* and the controller is *proper*, as is normally the case, then $T_o(s)$ is *strictly proper* and has a low-pass frequency response overall.

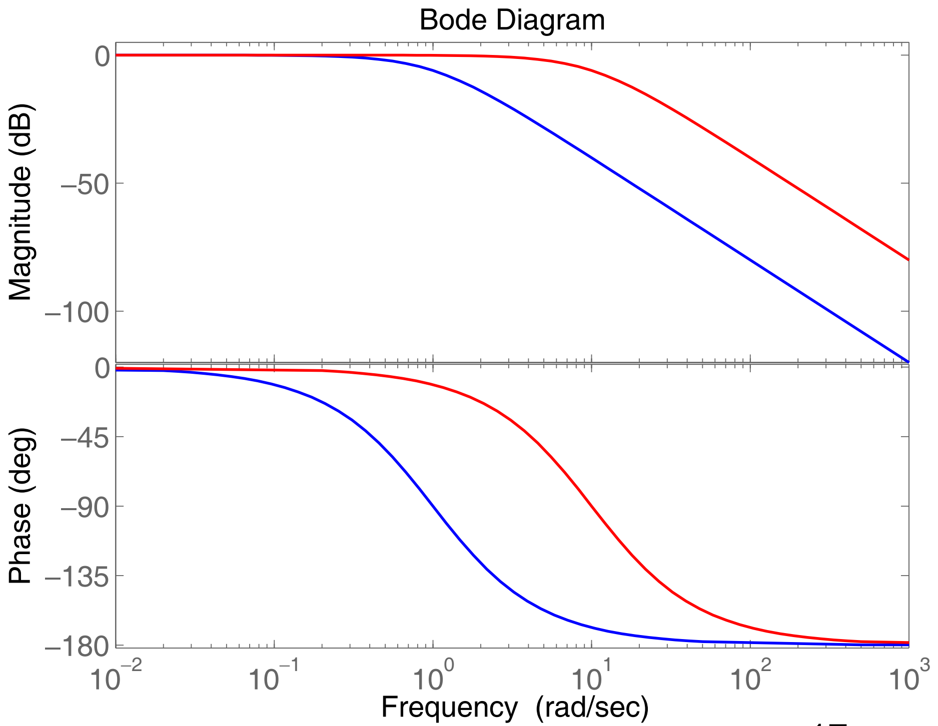
$\Rightarrow$ *The bandwidth of* $T_o(s)$ *is determined by the magnitudes of the closed-loop poles and zeros.*

Therefore, it is desirable, in general, to have the closed-loop poles and zeros be as far away from the imaginary axis as possible.

Example #1

$$T_o(s) = \frac{1}{(s+1)^2} \qquad T_o(s) = \frac{1}{(s+10)^2}$$





Poles of $T_o(s)$

Observing the nominal complementary sensitivity function:

$$T_o(s) = \frac{B_o(s)\,P(s)}{A_o(s)\,L(s) + B_o(s)\,P(s)},$$

we see that *root-locus analysis* or the *pole assignment method* can be used to obtain a desired characteristic polynomial:

$$A_{cl}(s) = A_o(s)\,L(s) + B_o(s)\,P(s)\,,$$

the roots of which are the *poles* of $T_o(s)$.

Zeros of $T_o(s)$

Observing the nominal complementary sensitivity function:

$$T_o(s) = \frac{B_o(s)\,P(s)}{A_o(s)\,L(s) + B_o(s)\,P(s)},$$

we see that the *zeros* of $T_o(s)$ consist of the *zeros of the controller*, the roots of $P(s)$, and the *zeros of the plant model*, the roots of $B_o(s)$, *unless they are cancelled by the denominator of the controller* $L(s)$.

Effect of zeros of $T_o(s)$

Any *zeros* of $T_o(s)$ that are closer to the imaginary axis than the dominant closed-loop pole will cause $|T_o(j\omega)|$ to deviate from 1 before the (low-pass) bandwidth of $T_o(s)$ is reached. This produces *overshoot* if the zero is *minimum-phase* and undershoot if it is *nonminimum-phase*.

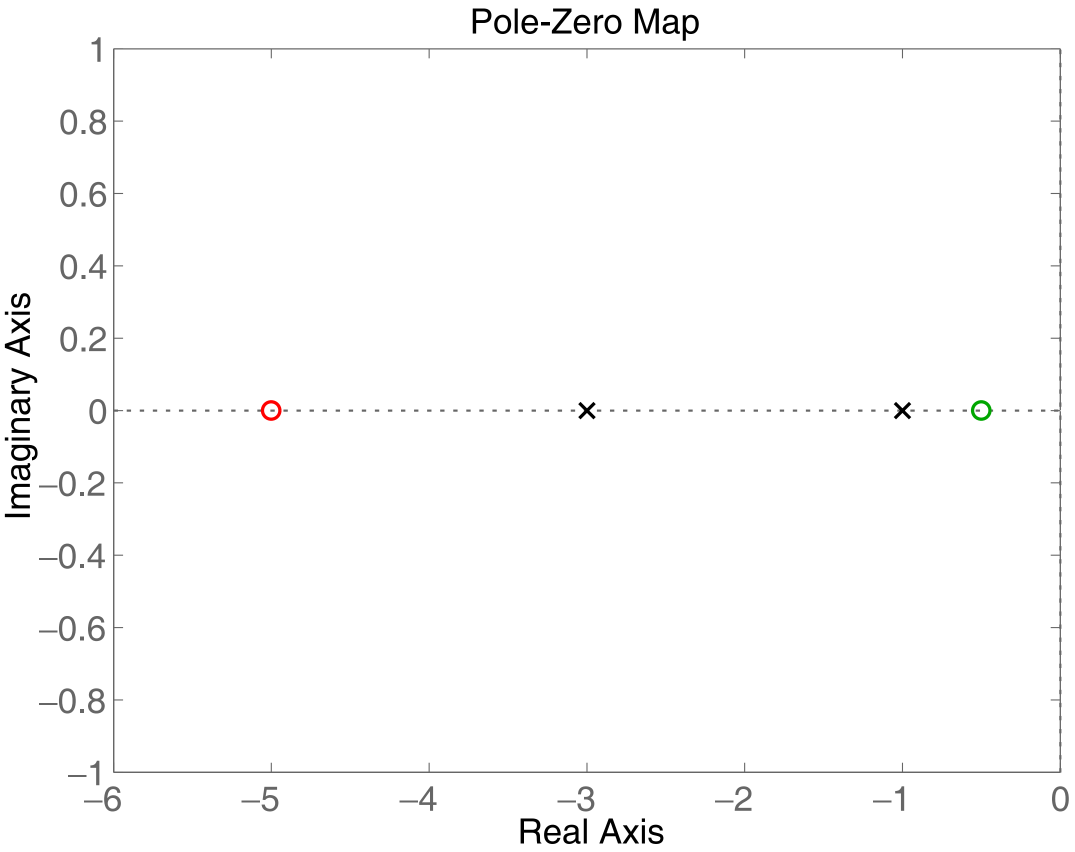
If all the *zeros* of $T_o(s)$ are further from the imaginary axis than all the closed-loop poles, then the zeros will have only a small effect on the response properties (e.g., frequency response, step response), and the closed-loop bandwidth will be determined by the dominant pole.

Example #2

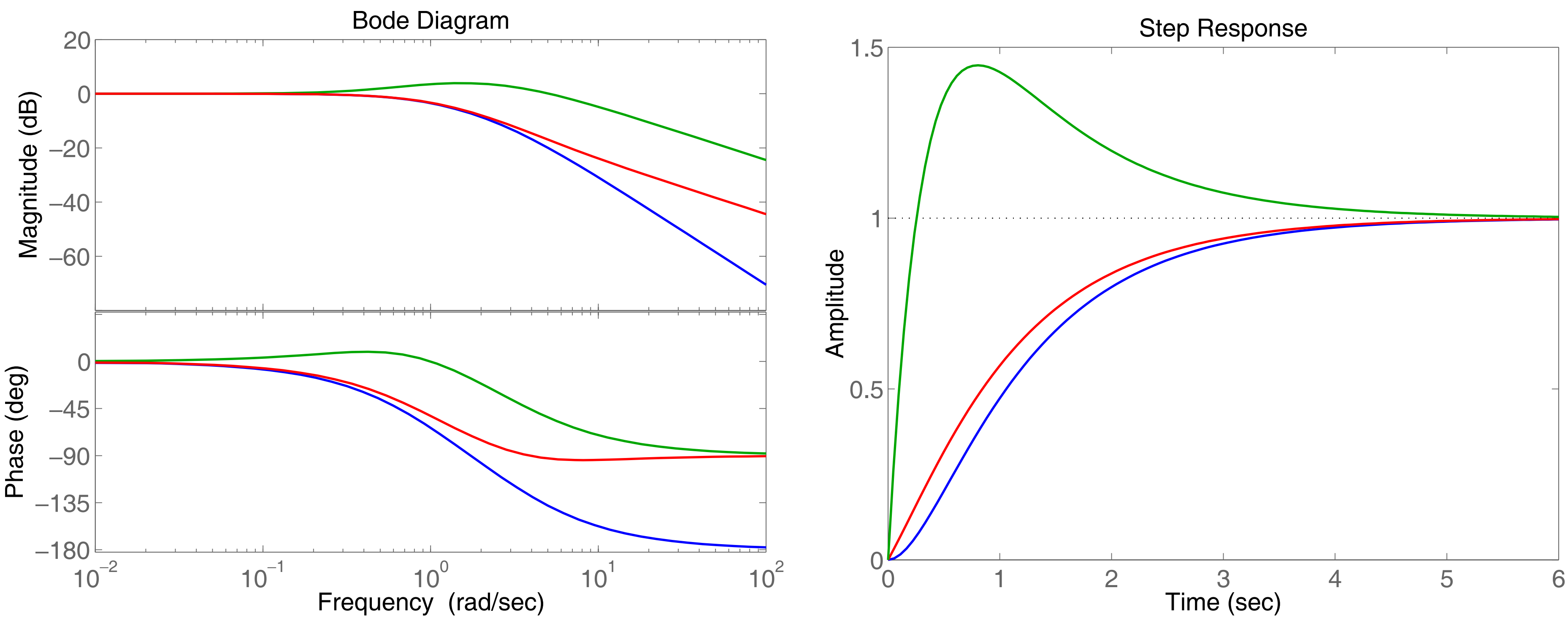
$$T_o(s) = \frac{3}{(s+1)(s+3)}$$

$$T_o(s) = \frac{6(s+0.5)}{(s+1)(s+3)}$$

$$T_o(s) = \frac{0.6(s+5)}{(s+1)(s+3)}$$



Example #2 (cont.)



The minimum-phase zero at $s = -0.5$ produces an overshoot, while the minimum-phase zero at $s = -5$ has little effect on the step response.

What is a good bandwidth?

From Example #2 we see that we might want to limit the bandwidth of $T_o(s)$ to avoid the overshoot produced by the zero at $s = -0.5$, i.e., move the closed-loop poles closer to the imaginary axis than the zero.

In the next set of lectures we will:

1. look at the fundamental limitations of a one-d.o.f., unity-feedback control loop, and
2. determine what is a good closed-loop bandwidth given these limitations.