ELEC ENG 4CL4:

Control System Design

Notes for Lecture #21

Wednesday, March 3, 2004

Dr. Ian C. Bruce
Room: CRL-229
Phone ext.: 26984
Email: ibruce@mail.ece.mcmaster.ca

Structural Limitations

The above analysis of limitations has focussed on issues arising from the actuators, sensors and model accuracy. However, there is another source of errors arising from the nature of the plant. Specifically we have:

General Ideas: Performance *in the nominal linear control loop* is also subject to unavoidable constraints which derive from the particular structure of the nominal model itself. We discuss:

- *delays*
- *open loop zeros*
- *open loop poles*

Delays

Undoubtedly the most common source of structural limitation in process control applications is due to process delays. These delays are typically associated with the transportation of materials from one point to another. We have seen in Chapter 7, that the output sensitivity can, at best, be given by:

$$S_o^*(s) = 1 - e^{-s\tau}$$

Where τ is the delay.

To achieve this ideal result requires use of a Smith Predictor plus ideal controller.

If we were to achieve the idealized result, then the corresponding nominal complementary sensitivity would be

$$T_o^*(s) = e^{-s\tau}$$

This has gain 1 at all frequencies. Hence high frequency model errors will lead to instability unless the bandwidth is limited. Errors in the delay are particularly troublesome. We thus conclude:

(i) Delays limit disturbance rejection by requiring that a delay occur before the disturbance can be cancelled. This is reflected in the ideal sensitivity $S_0^*(s)$;

(ii) Delays further limit the achievable bandwidth due to the impact of model errors.

An interesting question which arises in this context is whether it is worthwhile using a Smith Predictor in practice.

The answer is probably *yes* if the system model (especially the delay) are accurately known. However, if the delay is poorly known, then robustness considerations limit the achievable bandwidth even if a Smith Predictor is used. Specifically, if the delay is known to say $\eta * 100\%$, then the bandwidth is limited to the order of ${}^{1}/_{\eta\tau}$. Say $\eta = 1/3$, then this gives a bandwidth of approximately $3/\tau$. On the other hand, a simple PID controller can probably achieve a bandwidth of $4/\tau$. Thus, one can see that accurate knowledge of the system model and delay is a precursor to gaining advantages from using a Smith Predictor.

Example 8.3: *Thickness control in rolling mills*

We recall the example of thickness control in rolling mills as mentioned in Chapter 1 (*see next slide for photo*). A schematic diagram for one stand of a rolling mill is given in Figure 8.3. In Figure 8.3 we have used the following symbols:

- F - Roll Force
- σ - unloaded roll gap
- H - input thickness
- V - input velocity
- h - exit thickness),
- v - exit velocity
- h_m - measured exit thickness,
- d - distance from mill to exit thickness measurement.

The distance from the mill to output thickness measurement introduces a (speed dependent) time delay of $(^d/_v)$. This introduces a fundamental limit to the controlled performances as described above.

A modern rolling mill

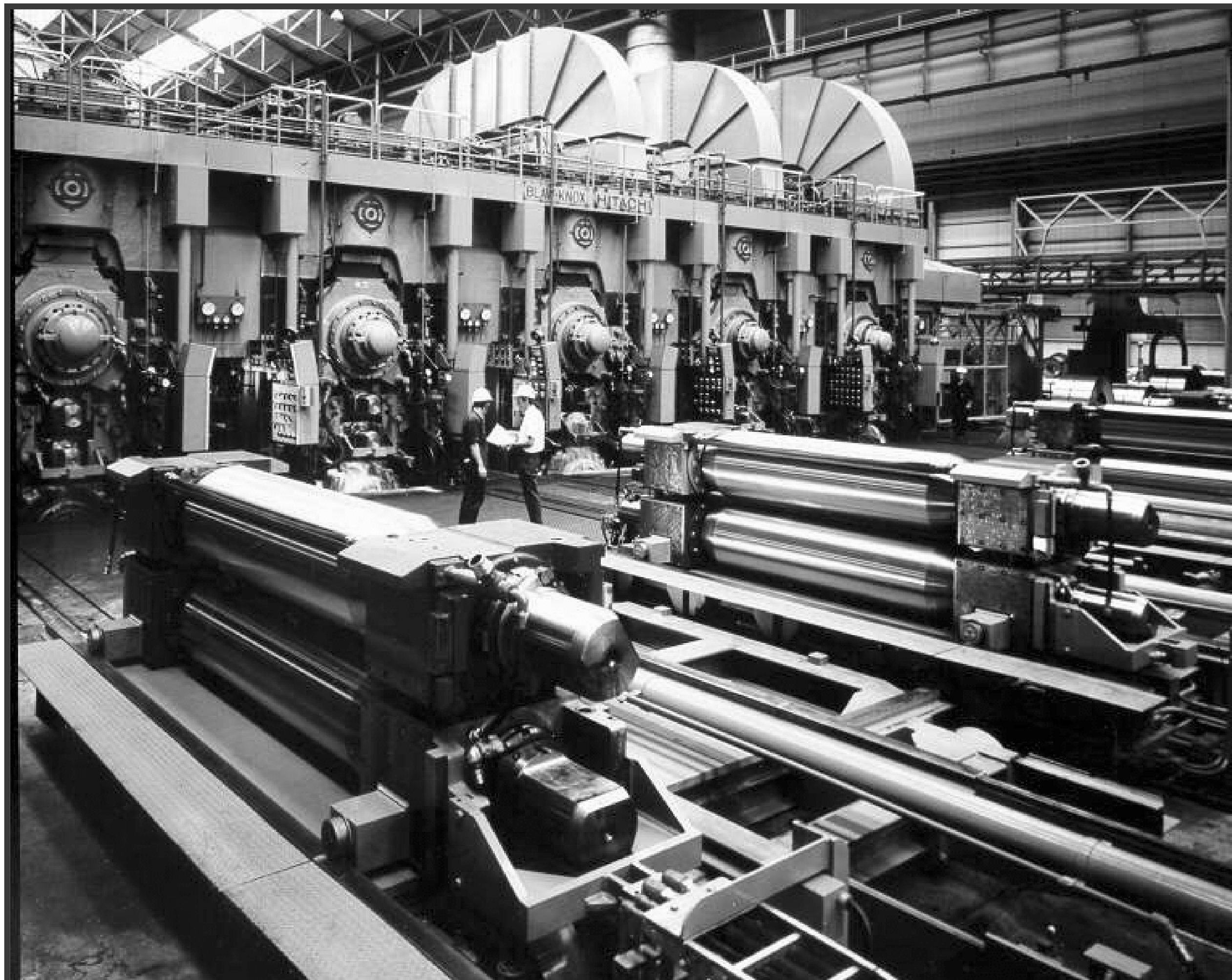
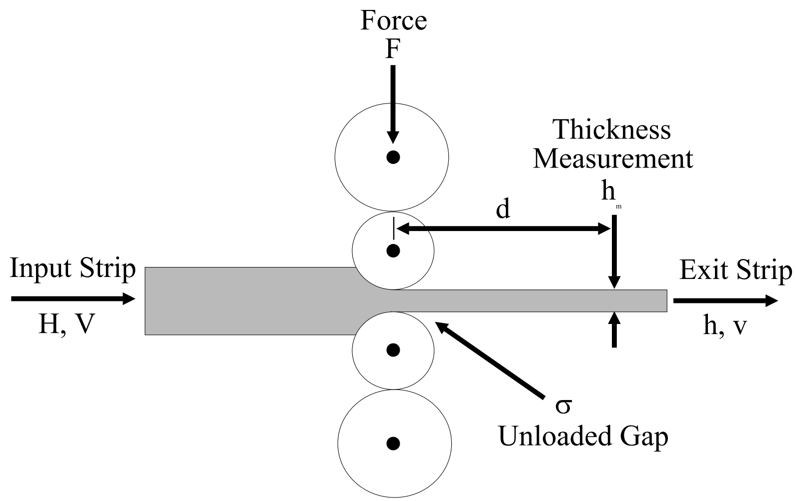


Figure 8.3: *Rolling mill thickness control*



Open Loop Poles and Zeros

We next study the effect of open loop poles and zeros on achievable performance. We shall see that open loop poles and zeros have a dramatic (*and predictable*) effect on closed loop performance.

We begin by examining the so-called interpolation constraints which show how open loop poles and zeros are reflected in the poles and zeros of the various closed loop sensitivity functions.

Interpolation Constraints

We recall that the relevant nominal sensitivity functions for a nominal plant $G_0(s) = {B_0(s)}/{A_0(s)}$ and a given unity feedback controller $C(s) = {P(s)}/{L(s)}$ are given below

$$T_o(s) = \frac{G_o(s)C(s)}{1 + G_o(s)C(s)} = \frac{B_o(s)P(s)}{A_o(s)L(s) + B_o(s)P(s)}$$

$$S_o(s) = \frac{1}{1 + G_o(s)C(s)} = \frac{A_o(s)L(s)}{A_o(s)L(s) + B_o(s)P(s)}$$

$$S_{io}(s) = \frac{G_o(s)}{1 + G_o(s)C(s)} = \frac{B_o(s)L(s)}{A_o(s)L(s) + B_o(s)P(s)}$$

$$S_{uo}(s) = \frac{C(s)}{1 + G_o(s)C(s)} = \frac{A_o(s)P(s)}{A_o(s)L(s) + B_o(s)P(s)}$$

Observations:

(i) The nominal complementary sensitivity $T_0(s)$ has a zero at all uncancelled zeros of $G_0(s)$.

(ii) The nominal sensitivity $S_0(s)$ is equal to one at all uncancelled zeros of $G_0(s)$. (This follows from (i) using the identity $S_0(s) + T_0(s) = 1$).

(iii) The nominal sensitivity $S_0(s)$ has a zero at all uncancelled poles of $G_0(s)$.

(iv) The nominal complementary sensitivity $T_0(s)$ is equal to one at all uncancelled poles of $G_0(s)$. (This follows from (iii) and the identity $S_0(s) + T_0(s) = 1$).

We next show how these interpolation constraints lead to performance limits.

Effect of Open Loop Integrators

Lemma 8.1: We assume that the plant is controlled in a one-degree-of-freedom configuration and that the open loop plant and controller satisfy:

$$A_o(s)L(s) = s^i (A_o(s)L(s))' \qquad i \geq 1$$

$$\lim_{s \to 0} (A_o(s)L(s))' = c_0 \neq 0$$

$$\lim_{s \to 0} (B_o(s)P(s)) = c_1 \neq 0$$

i.e., the plant-controller combination has *i* poles at the origin. Then, for a step output disturbance or step set point, the control error, *e*(*t*) satisfies

$$\lim_{t \to \infty} e(t) = 0 \qquad \forall i \geq 1$$

$$\int_0^\infty e(t)dt = 0 \qquad \forall i \geq 2$$

Also, for a negative unit ramp output disturbance or a positive unit ramp reference, the control error, *e*(*t*), satisfies

$$\lim_{t \to \infty} e(t) = \frac{c_0}{c_1} \qquad \text{for} \quad i = 1$$

$$\lim_{t \to \infty} e(t) = 0 \qquad \forall i \geq 2$$

$$\int_0^\infty e(t)dt = 0 \qquad \forall i \geq 3$$

Equal Area Result

CG contains double integrator

⬇

S has double zero at $s = 0$

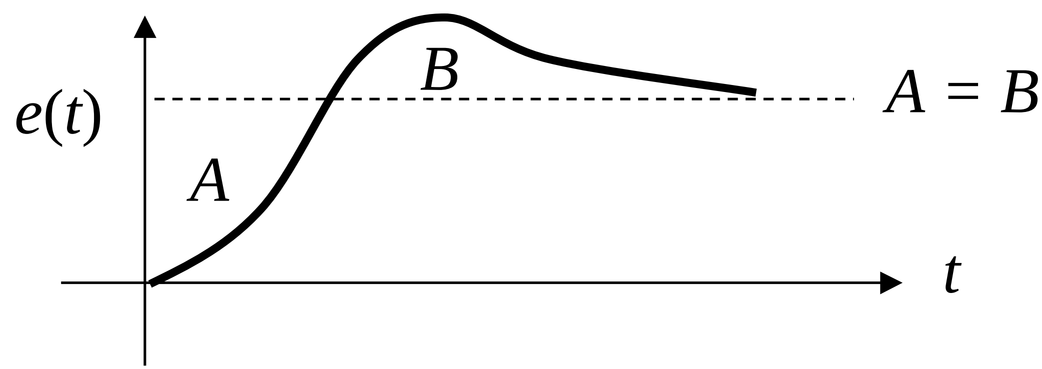
Hence

$$\int_0^\infty e(t)dt = \lim_{s\to 0}\int_0^\infty e(t)e^{-st}dt$$

$$= \lim_{s\to 0} E(s)$$

$$= S(s)\frac{1}{s} \; for\, unit\; step$$

$$= 0$$



The above conclusion holds for a *one-degree-of-freedom* feedback control system. Later in these slides we show that overshoot can actually be avoided if the architecture is changed to a *two-degree-of-freedom* control system.

Consequences

Say that we want to eliminate the effect of ramp input disturbances in steady state. This can be achieved by placing 2 integrators in the controller. However, we then see that the error to a step reference change must satisfy

$$\int_0^{\infty} e(t)dt = 0$$

This, in turn, implies that the error must change sign, i.e. *overshoot* must occur.

Thus it is impossible to have zero steady state error to ramp type input disturbances together with no overshoot to a step reference.