ELEC ENG 4CL4:
Control System Design

Notes for Lecture #29

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Chapter 13

Digital Control

Chapter 12 was concerned with building models for systems acting under digital control.

We next turn to the question of control itself.

Topics to be covered include:

- why one cannot simply treat digital control as if it were exactly the same as continuous control, and
- how to carry out designs for digital control systems so that the *at-sample* response is exactly treated.

Having the controller implemented in digital form introduces several constraints into the problem:

(a) the controller sees the output response only at the sample points,

(b) an anti-aliasing filter will usually be needed prior to the output sampling process to avoid folding of high frequency signals (such as noise) onto lower frequencies where they will be misinterpreted; and

(c) the continuous plant input bears a simple relationship to the (sampled) digital controller output, e.g. via a zero order hold device.

A key idea is that if one is only interested in the at-sample response, these samples can be described by discrete time models in either the shift or delta operator. For example, consider the sampled data control loop shown below

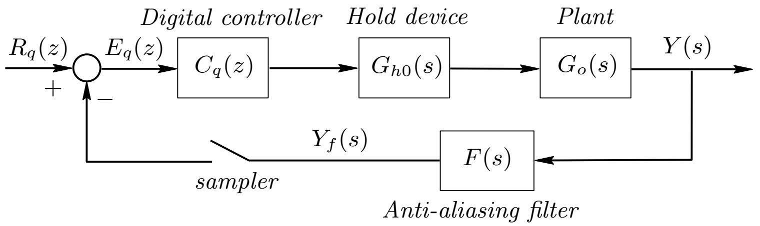


Figure 13.1: *Sampled data control loop*

If we focus only on the sampled response then it is straightforward to derive an equivalent discrete model for the at-sample response of the hold-plant-anti-aliasing filter combination. This was discussed in Chapter 12.

We use the transfer function form, and recall the following forms for the discrete time model:

(a) With anti-aliasing filter F

$$[FG_0G_{h0}]_q(z),\ Z\{\text{sampled impulse response of } F(s)G_0(s)G_{h0}(s)\}$$

(b) Without anti-aliasing filter

$$[G_0G_{h0}]_q(z),\ Z\{\text{sampled impulse response of } G_0(s)G_{h0}(s)\}$$

Control Ideas

Many of the continuous time control ideas studied in earlier chapters carry over directly to the discrete time case. Examples are given below.

The discrete sensitivity function is

$$S_{oq}(z) = \frac{E_q(z)}{R_q(z)} = \frac{1}{\left(1 + C_q(z) \left[F G_o G_{h0}\right]_q (z)\right)}$$

The discrete complementary sensitivity function is

$$T_{oq}(z) = \frac{Y_{fq}(z)}{R_q(z)} = \frac{C_q(z) \left[F G_o G_{h0}\right]_q (z)}{\left(1 + C_q(z) \left[F G_o G_{h0}\right]_q (z)\right)}$$

These can be used and understood in essentially the same way as they are used in the continuous time case.

Are there special features of digital control models?

Many ideas carry directly over to the discrete case. For example, one can easily do discrete pole assignment. Of course, one needs to remember that the discrete stability domain is different from the continuous stability domain. However, this simply means that the desirable region for closed loop poles is different in the discrete case.

We are led to ask if there are any real conceptual differences between continuous and discrete.

Zeros of Sampled Data Systems

We have seen earlier that open loop zeros of a system have a profound impact on achievable closed loop performance. The importance of an understanding of the zeros in discrete time models is therefore not surprising. It turns out that there exist some subtle issues here as we now investigate.

If we use shift operator models, then it is difficult to see the connection between continuous and discrete time models. However, if we use the equivalent delta domain description, then it is clear that discrete transfer

Functions converge to the underlying continuous time descriptions. In particular, the relationship between continuous and discrete (delta domain) poles is as follows (*See Chapter 12*):

$$p_i^\delta = \frac{e^{p_i \Delta} - 1}{\Delta}; \qquad i = 1, \ldots n$$

where p_i^δ, p_i denote the discrete (delta domain) poles and continuous time poles respectively.

The relationship between continuous and discrete zeros is more complex. Perhaps surprisingly, all discrete time systems turn out to have relative degree 1 irrespective of the relative degree of the original continuous system.

Hence, if the continuous system has n poles and $m(< n)$ zeros then the corresponding discrete system will have n poles and $(n\text{-}1)$ zeros. Thus, we have n-m+1 extra discrete zeros. We therefore (*somewhat artificially*) divide the discrete zeros into two sets.

1. **System zeros:** $z_1^\delta, ..., z_m^\delta$ Having the property

$$\lim_{\Delta \to 0} z_i^\delta = z_i \qquad i = 1, \ldots, m$$

where z_i^δ are the discrete time zeros (expressed in the delta domain for convenience) and z_i are the zeros of the underlying continuous time system.

2. **Sampling zeros:** $z^{\delta}_{m+1}, \ldots, z^{\delta}_{n-1}$ Having the property

$$\lim_{\Delta\to 0} \left|z_i^{\delta}\right| = \infty \qquad\qquad i = m+1, \ldots, n-1$$

Of course, if $m = n - 1$ in the continuous time system, then there are no sampling zeros. Also, note that as the sampling zeros tend to infinity for $\Delta\to 0$, they then contribute to the continuous relative degree. This shows the consistency between the two types of model.

We illustrate by a simple example.

Example 13.1

Consider the continuous time servo system of Example 3.4, having continuous transfer function

$$G_o(s) = \frac{1}{s(s+1)}$$

where $n = 2$, $m = 0$. Then we anticipate that discretizing would result in one sampling zero, which we verify as follows.

With a sampling period of 0.1 seconds, the exact shift domain digital model is

$$G_{oq}(z) = K\frac{z - z_o^q}{(z-1)(z-\alpha_o)}$$

where $K = 0.0048$, $z_0^q = -0.967$ and $\alpha_0 = 0.905$.

The corresponding exact delta domain digital model is

$$G_\delta(\gamma) = \frac{K'(\gamma - z_o^\delta)}{\gamma(\gamma - \alpha_o')}$$

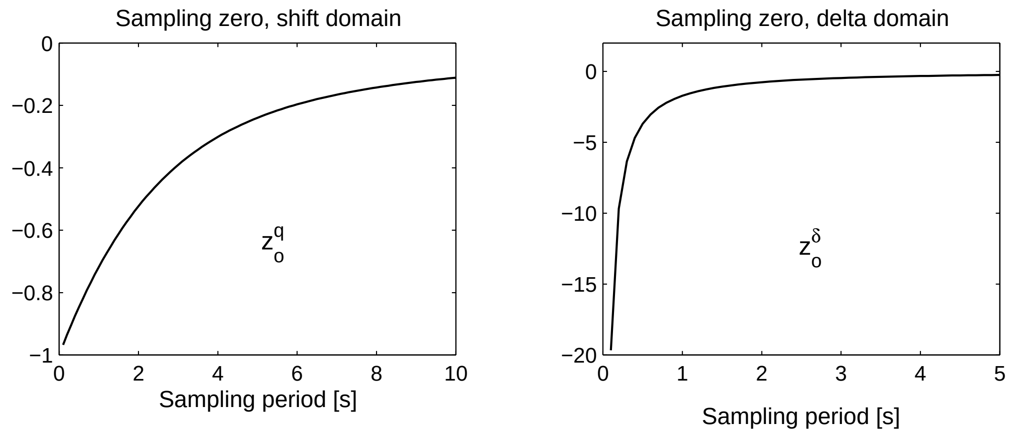
where $K' = 0.0048$, $z_0^q = -19.67$ and $\alpha_0 = -0.9516$.

We see that (*in the delta form*), the discrete system has a pole at $\gamma=0$ and a pole at $\gamma=-0.9516$. These are consistent with the continuous time poles at $s=0$ and $s=-1$.

Note, however, that the continuous system has relative degree 2, whereas the discrete system has relative degree 1 and a *sampling zero* at -19.67 (*in the delta formulation*).

The next slide shows a plot of the sampling zero as a function of sampling period.

Figure 13.2: *Location of sampling zero with different sampling periods. Example 13.1*



In the control of discrete time systems special care needs to be taken with the sampling zeros. For example, these zeros can be non-minimum phase even if the original continuous system is minimum phase. Consider, for instance, the minimum phase, continuous time system with transfer function given by

$$G_o(s) = \frac{s+4}{(s+1)^3}$$

For this system, the shift domain zeros of $[G_0 G_{h0}]_q(z)$ for two different sampling periods are

$$\Delta = 2[s] \quad \Rightarrow \text{zeros at -0.6082 and -0.0281}$$

$$\Delta = 0.5[s] \quad \Rightarrow \text{zeros at -1.0966 and 0.1286}$$

Note that $\Delta = 0.5[s]$, the pulse transfer function has a zero outside the stability region.

Thus, one needs to be particularly careful of sampling zeros when designing a digital control system.