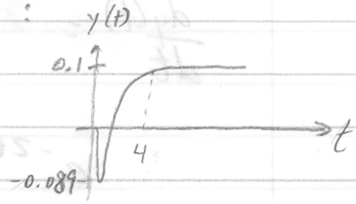
①

A system transfer function is given by:



$$H(s) = \frac{-s+1}{(s+2)(s+5)}$$

a) Will this system exhibit under or overshoot in response to a step?

zero is slow and unstable $\therefore$ NMP $\rightarrow$ undershoot

b) Compute the magnitude and time instant of the under or overshoot, and the steady-state output.

$$Y(s) = \frac{-s+1}{(s+2)(s+5)} \cdot \frac{1}{s} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+5}$$

$$= \frac{A(s+2)(s+5) + Bs(s+5) + Cs(s+2)}{s(s+2)(s+5)}$$

$$= \frac{As^2 + 7As + 10A + Bs^2 + 5Bs + Cs^2 + 2Cs}{s(s+2)(s+5)}$$

$A + B + C = 0$ $\qquad$ $7A + 5B + 2C = -1$ $\qquad$ $10A = 1$

$B + C = -\frac{1}{10}$ $\qquad$ $5B + 2C = -\frac{17}{10}$ $\qquad$ $A = \frac{1}{10}$

$B = -\frac{1}{10} - C$

$= -\frac{5}{10} = -\frac{1}{2}$ $\qquad$ $-\frac{5}{10} - 5C + 2C = -\frac{17}{10}$

$-3C = -\frac{12}{10}$

$C = \frac{12}{30} = \frac{4}{10} = \frac{2}{5}$

$$Y(s) = \frac{1/10}{s} - \frac{1/2}{s+2} + \frac{2/5}{s+5}$$

$$y(t) = 0.1 - 0.5e^{-2t} + 0.4e^{-5t}$$

$\therefore y_\infty = 0.1$

P.T.O. $\rightarrow$

$$\frac{dy(t)}{dt} = e^{-2t} - 2e^{-5t}$$

$$e^{-2t_u} = 2e^{-5t_u}$$

$$-2t_u = \ln 2 - 5t_u$$
$$3t_u = \ln 2$$
$$t_u = \frac{\ln 2}{3} = 0.2310$$

Assuming step occurs at $t=0$.

$$y(0.2310) = 0.1 - 0.5e^{-2(0.2310)} + 0.4e^{-5(0.2310)}$$
$$= -0.089$$

② Bode Plots → Nyquist Plot

Consider the following open-loop transfer function:

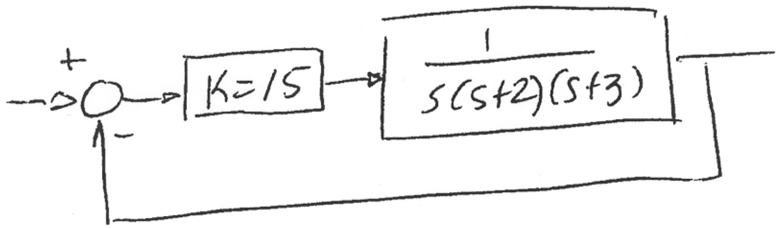
$$H(s) = \frac{15}{s(s+2)(s+3)}$$

To obtain the Bode plot, find the frequency response of $H(s)$: $s = j\omega$

$$H(j\omega) = \frac{15}{j\omega(j\omega+2)(j\omega+3)}$$

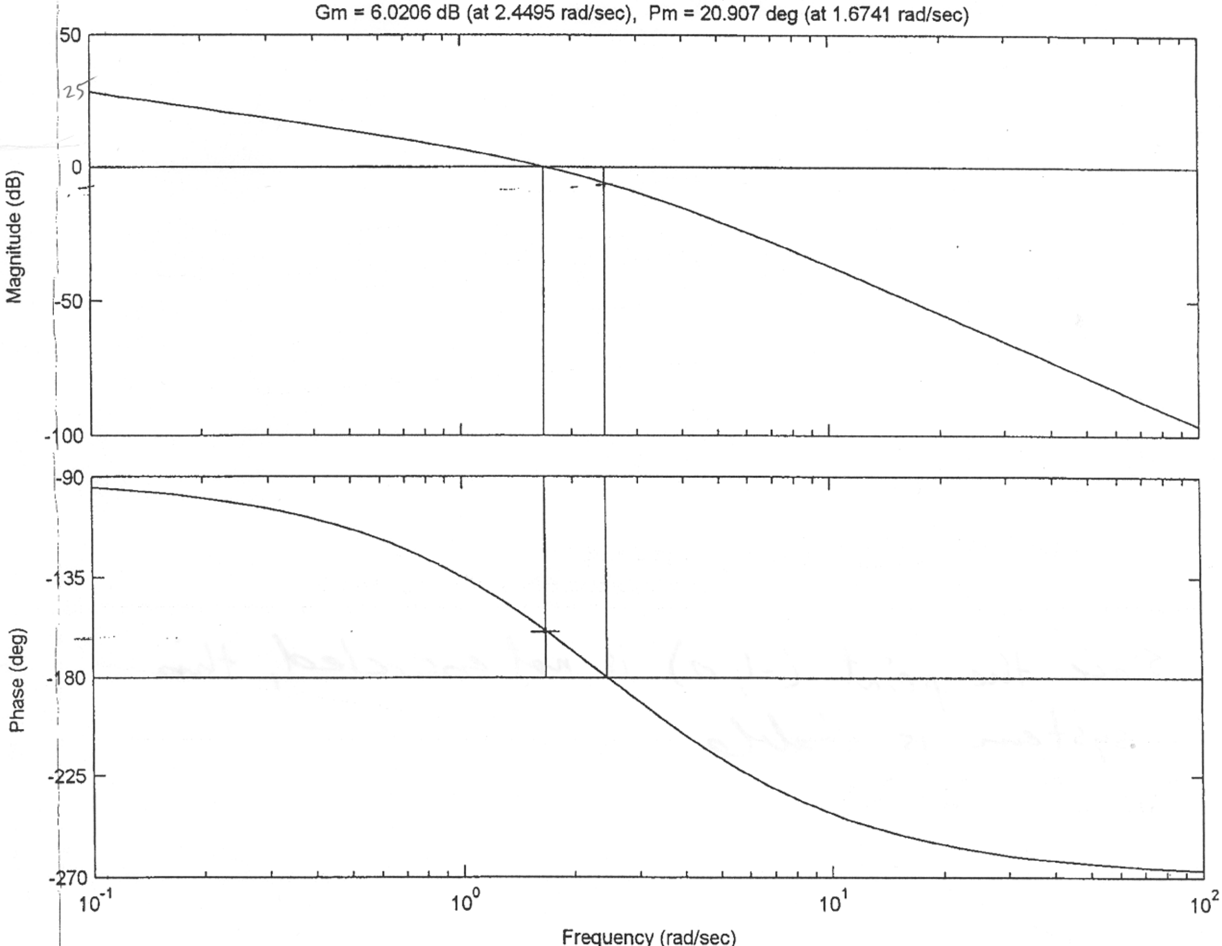
ω	MAG	PHASE	
0.001	2.5×10^3	$-90.05°$	
0.01	250	$-90.48°$	
0.1	25	$-94.7°$	
0.2	12.4	$-99.5°$	
0.5	4.8	$-113.5°$	
1.0	2.1	$-135°$	
→ 1.67	1.0 (0dB)	$-159°$	→ $180° - 159° = 21°$
2.0	0.74	$-169°$	
→ 2.45 ($=\sqrt{6}$)	0.5	$-180°$	
5.0	0.1	$-217°$	
10.0	0.01	$-242°$	
100	1.5×10^{-5}	$-267.1°$	
1000	1.5×10^{-8}	$-269.7°$	
$\to\infty$	$\to 0$	$\to -270°$	

Which gives the Bode plot: (next page)



Bode Diagram (using MARGIN command)

Gm = 6.0206 dB (at 2.4495 rad/sec), Pm = 20.907 deg (at 1.6741 rad/sec)

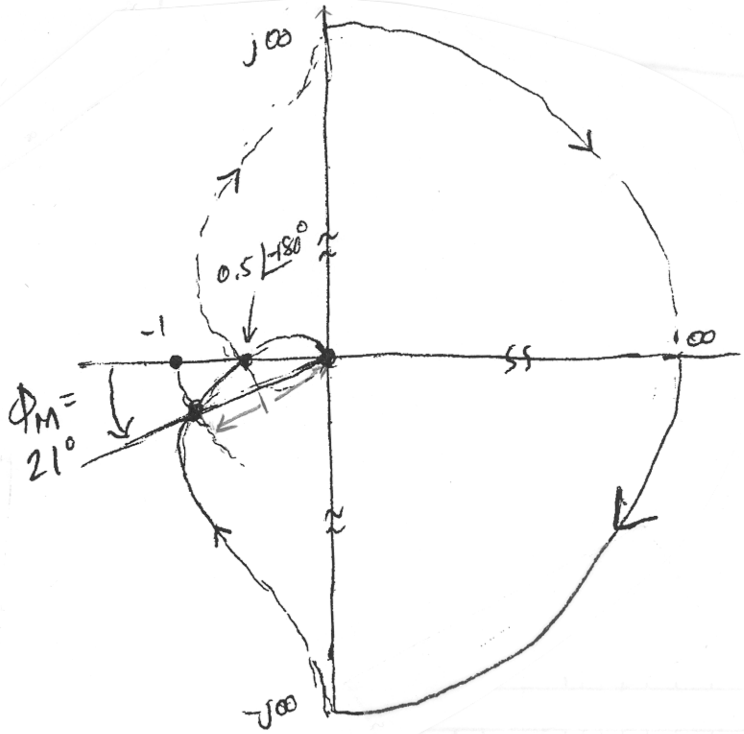


n = [0 0 0 15]

d = [1 5 6 15]

margin(n,d)

From the Bode plot, the Nyquist Plot can be drawn by combining the magnitudes and phases:



Since the point (-1, 0) is not encircled, this system is stable.

③ Example 5.2, p 128

Is this system stable?

$$G_o(s) = \frac{3}{(s+4)(-s+2)} \qquad C(s) = \frac{-s+2}{s}$$

where $G_o(s) = \frac{B_o(s)}{A_o(s)}$ $\qquad C(s) = \frac{P(s)}{L(s)}$

for stability $A_o(s)L(s) + B_o(s)P(s) = 0$ has to have all roots in the left-half of the plane.

$G_o(s)$ has a positive root → pole in positive plane

However the controller has a zero at the same location!

Nominal complementary sensitivity

$$T_o(s) = \frac{G_o(s)C(s)}{1+G_o(s)C(s)}$$

$$T_o(s) \left(= \frac{3(-s+2)}{1+s(s+4)(-s+2)} \right) = \frac{B_o(s)P(s)}{A_o(s)L(s)+B_o(s)P(s)}$$

?

Input-disturbance sensitivity:

$$S_{io}(s) = \frac{G_o(s)}{1+G_o(s)C(s)} = \frac{B_o(s)L(s)}{A_o(s)L(s)+B_o(s)P(s)}$$

$$T_o(s) = \frac{3(-s+2)}{s(s+4)(-s+2)+3(-s+2)} = \frac{3\cancel{(-s+2)}}{\cancel{(-s+2)}(s^2+4s+3)}$$

$$= \frac{3}{s^2+4s+3} = \frac{3}{(s+1)(s+3)} \quad \text{stable}$$

$$S_{io}(s) = \frac{3s}{s(s+4)(-s+2)+3(-s+2)} = \frac{3s}{(-s+2)(s^2+4s+3)}$$

so this is not stable!!

So Lemma 5.1 holds

$A_o(s)L(s) + B_o(s)P(s)$ should have roots in the negative half of the plane only.

(4) Robust stability

Given:

$$G(s) = e^{-\tau s} F(s) \qquad G_o(s) = \left(\frac{-\tau s + 2k}{\tau s + 2k}\right)^k F(s)$$

Let

$$C(s) = 1, \quad F(s) = \frac{1}{s(s+1)^2}, \quad \tau = \frac{1}{2}, \quad k = 1$$

Will this controller make the system robustly stable?

Need to satisfy $|T_o(j\omega)||G_\Delta(j\omega)| < 1$

$$G_\Delta(s) = \frac{G(s) - G_o(s)}{G_o(s)} = e^{-\tau s}\left(\frac{-\tau s + 2k}{\tau s + 2k}\right)^{-k} - 1$$

$$|G_\Delta(j\omega)| = |e^{-j\tau\omega} - e^{-j2k\phi}|, \text{ where } \phi = \arctan\left(\frac{\omega\tau}{2k}\right)$$

$$T_o(s) = \frac{G_o(s)C(s)}{1 + G_o(s)C(s)} = \frac{B_o(s)P(s)}{A_o(s)L(s) + B_o(s)P(s)}$$

$$= \frac{-\frac{1}{2}s + 2}{(\frac{1}{2}s+2)s(s+1)^2 + (\frac{1}{2}s+2)} = \frac{-\frac{1}{2}s + 2}{\frac{1}{2}s^4 + 3s^3 + \frac{9}{2}s^2 + \frac{3}{2}s + 2}$$

Tutorial 4, Problem 4 (continued)