

BIOELECTRICITY
A QUANTITATIVE APPROACH

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and

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THIRD EDITION

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To our unseen co-authors, our wives:

VIVIAN PLONSEY

JEAN BARR

and our unnamed co-authors:

The students in BME 101

ABOUT THE AUTHORS

Robert Plonsey is Pfizer-Pratt Professor Emeritus of Biomedical Engineering at Duke University. He received the PhD in Electrical Engineering from the University of California in 1955. He received the Dr. of Technical Science from the Slovak Academy of Science in 1995 and was Chair, Department of Biomedical Engineering, Case Western Reserve, University, 1976-1980, Professor 1968-1983. Awards: Fellow of AAAS, William Morlock Award 1979, Centennial Medal 1984, Millennium Medal 2000, from IEEE Engineering in Medicine and Biology Society, Ragnar Granit Prize 2004, (First) Merit Award, 1997, International Union for Physiological & Engineering Science in Medicine, the Theo Pilkington Outstanding Educator Award, 2005, Distinguished Service award, Biomedical Engineering Science, 2004, ALZA distinguished lecturer, 1988. He was elected Member, National Academy of Engineering, 1986 (“For the application of electromagnetic field theory to biology, and for distinguished leadership in the emerging profession of biomedical engineering”).

Roger C. Barr is Professor of Biomedical Engineering and Associate Professor of Pediatrics at Duke University. In past years he served as the Chair of the Department of Biomedical Engineering at Duke, and then as Vice President and President of the IEEE Engineering in Medicine and Biology Society. He received the Duke University Scholar-Teacher Award in 1991. He is the author of more than 100 research papers about topics in bioelectricity and is a Fellow of the IEEE and American College of Cardiology. This text is a product of interactions with students, and in this regard he has taught the bioelectricity course sequence numerous times.

PREFACE

The study of electrophysiology has progressed rapidly because of the precise, delicate, and ingenious experimental studies of many investigators. The field has also made great strides by unifying these experimental observations through mathematical descriptions based on electromagnetic field theory, electrochemistry, etc., which underlie these experiments. In turn, these quantitative materials provide an understanding of many electrophysiological applications through a relatively small number of fundamental ideas.

This text is an introduction to electrophysiology, following a quantitative approach. The first chapter summarizes much of the mathematics required in the following chapters. The second chapter presents a very concise overview of the principles of electrical fields and the concomitant current flow in conducting media. It utilizes basic principles from the physical sciences and engineering but takes into account the biological applications. The following six chapters are the core material of this text. Chapter 3 includes a description of how voltages/currents exist across membranes and how these are evaluated using the Nernst–Planck equation. The membrane channels, which are the basis for cell excitability, are described in Chapter 4. An examination of the time course of changes in membrane voltages that produce action potentials are considered in Chapter 5. Propagation of action potentials down fibers is the subject of Chapter 6, and the response of fibers to artificial stimuli, such as those used in cardiac pacemakers, is treated in Chapter 7. The voltages and currents produced by these active processes in the surrounding extracellular space is described in Chapter 8. The subsequent chapters present more detailed material about the application of these principles to the study of the electrophysiology of cardiac and skeletal muscle with a modest inclusion of neural electrophysiology.

The material of this text was designed as an introduction to bioelectricity (electrophysiology), and one might think that fundamentals change very slowly. In fact the rapid growth of the field has reflected back changes in the underlying material. Since a quantitative approach to electrophysiology is a precursor to the various new applications; it is, in fact, a real challenge keeping things up-to-date. The second edition is the authors' effort to bring the text more into line with the current new applications found in recent texts.

In particular, we have introduced a few underlying factors in molecular biology as it interacts with electrophysiology. While the result is a very modest introduction it is hoped that the treatment will outline the importance of this topic in bioelectricity. In other applications we have also endeavored to bring matters up-to-date. This is done in both the chapters on applications as well as those devoted to fundamentals. We hope this conveys to the reader our excitement with this field.

In this third edition, we respond to the many requests from students and faculty colleagues that the book include more exercises with solutions. Thus the exercises have been reorganized, and many more exercises and solutions added. Additionally, Chapter 8 on extracellular potentials has been revised and extended, with many new figures, as we recognize that this chapter is key to understanding many clinical measurements. In addition a number of other chapters have been revised, with more information now included for the reader about the reasons why different topics are considered important and how they are related, information that allows one to better focus on those topics most important to particular instructors and students.

Each time we consider the material in the text we become aware, once again, of how many talented and energetic investigators and students of the field have made substantial contributions

to its progress. It is the nature of a textbook to reflect the integrated ideas of many individuals over more than century, so only a few of the many contributors are recognized by citation. Even so, a wealth of additional material is available to the reader, and that material provides a much more complete picture. We have included a few citations in the text on particular points and at the end of each chapter as additional material, so that the student has a entryway to the extensive library of published work that now is available.

The revisions also include many corrections and focused responses to suggestions received from colleagues, readers elsewhere, and especially from our students. We hope they will find the revisions to their liking. For the future we continue to invite comments and criticisms from students and faculty colleagues.

ROBERT PLONSEY
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