OFDM Channel Estimation in the Presence of Interference Using the Estimated Generalized Least-squares Method

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Abstract – *A pilot-aided method for estimating a channel impulse response in orthogonal frequency division multiplex wireless systems is presented. In this scenario the received signal, in addition to undergoing multipath fading, may have significant correlation, hence the commonly used least-squares algorithms may be suboptimal. The covariance of the received signal is modeled using basis functions which results in a structured covariance matrix. The unknown parameters are estimated using an approximate maximum likelihood estimator. The nonlinear parameters (corresponding to the interferer) are computed first using the method of moments and then the maximum likelihood algorithm is applied to estimate the channel response. Since the number of unknown parameters is related to the number of pilot symbols, the algorithm outperforms the least-squares approach without significant loss in bandwidth efficiency. Numerical examples demonstrate the performance advantage of the proposed channel estimator.*

Keywords: OFDM, channel estimation, interference, maximum likelihood, generalized least squares, random growth curve.

This work was supported by the Air Force Office of Scientific Research under Grants F49620-99-1-0067 and F49620-00-1-0083, the National Science Foundation under Grant CCR-0105334, and the Office of Naval Research under Grant N00014-01-1-0681.

1 Introduction

Orthogonal frequency division multiplexing (OFDM) has received considerable interest in the last few years for its advantages in high-bit-rate transmissions over frequency selective fading channels. In OFDM systems, the input high-rate data stream is divided into many low-rate streams [1] that are transmitted in parallel, thereby increasing the symbol duration and reducing the intersymbol interference (ISI). These features have motivated the adoption of OFDM as a standard for digital audio broadcasting, digital video broadcasting, and broadband indoor wireless systems.

Coherent OFDM transmission invariably requires an estimation of the channel frequency response (i.e. the gains of the OFDM tones). Currently, there are two different types of channel parameter estimators: (i) blind estimators and (ii) pilot-aided estimators. Blind channel estimation techniques try to estimate the channel without any knowledge of the transmitted data. Blind estimation methods are attractive because of the possible savings in training overhead, however they are only effective when a large amount of data can be collected (so that stochastic estimation can be made reliably). This is clearly a disadvantage in the case of mobile wireless systems because of the time-varying nature of the channel. Pilot-aided channel estimation is the other type of approach in which training sequence consisting of known data symbols (pilots) is transmitted at the beginning of a session (or multiplexed into the user data stream at a later stage) and the initial estimation of channel parameters is performed using the received pilot signal.

In this paper we present a frequency domain channel estimation algorithm in the presence of interference using a pilot-aided channel estimation algorithm. In the presence of interference the received signal, in addition to multipath fading, may have frequency dependent covariance which means that commonly used least-squares algorithms may not be optimal. Sufficiently accurate estimation of the covariance matrices in such a model would impose significant training overhead since the estimation requires large number of pilot symbols. Therefore, we propose to model the frequency dependence of the covariance matrices using a priori known set of frequency dependent functions of joint (global) parameters which results in a structured covariance matrix. In this way the number of required pilot symbols will be dependent on the number of functions needed to achieve sufficiently accurate approximation. We also assume that the channel vector is deterministic and completely unknown. In this case the maximum likelihood estimator (MLE) is the best possible estimator as long as the channel impulse response is viewed as a deterministic unknown vector and the estimator is unbiased. Therefore, we propose estimating the unknown parameters using two estimators: (i) computationally intensive MLE and (ii) an approximate MLE in which the non-linear covariance parameters are estimated using a method of moments estimator.

In Section 2, we briefly describe the OFDM system with pilot-symbol-aided channel estimation. In Section 3, we derive the statistical model. In Section 4, we present the MLE and discuss numerical implementation of the proposed non-linear algorithms. Section 5 demonstrates the applicability of our results through numerical examples. In Section 6, we discuss possible extensions to multiple-input-multiple-output (MIMO) OFDM systems. Concluding remarks are given in Section 7.

2 The OFDM Model

In this section we briefly introduce the channel model for an OFDM system with pilot-symbol-aided channel estimator and describe the corresponding channel statistics. Our goal is to develop a model that will include unknown random effects due to a presence of unknown interference and enable statistically efficient channel estimation using sufficiently small number of pilots.

2.1 Channel Model

Consider an OFDM system that consists of n_{s} subcarriers of which $n_{\mathrm{u}}+1$ subcarriers at the central spectrum are used for transmission and the other subcarriers at both edges form the guard bands. Each transmission subcarrier is modulated by a data symbol x_{jk}, where j represents the subcarrier number, and k represents the time slot number (OFDM symbol number). The OFDM transmitters usually employ an inverse fast Fourier transform (IFFT) of size n_{s} for the modulation. In order to limit the transmit signal to a bandwidth smaller than $1/T$, where T is the sampling time interval of the OFDM signal, the subcarriers in the guard band are not used. A guard interval is also added for every OFDM symbol to avoid intersymbol interference caused by multipath fading channels. As a results, the output complex baseband representation of the transmitted signal is given by

$$s(t)=\sum_{k=-\infty}^{\infty}\sum_{j=-n_{\mathrm{u}}/2}^{n_{\mathrm{u}}/2} x_{jk}g(t-k(T+T_{\mathrm{g}}))e^{ij\Delta\omega t} \quad (1)$$

where x_{jk} is a subcarrier coefficient corresponding to an OFDM symbol, i is the imaginary unit number, $g(t)$ is a shaping pulse, usually rectangular, T is the useful symbol interval, $\Delta\omega$ is the OFDM subcarrier spacing, and T_{g} is the guard interval. Since the guard interval contains a repetition of a preceding part of the signal only, it is not of interest in this discussion and therefore we assume without loss of generality that $T_{\mathrm{g}}=0$ which also implies $n_{\mathrm{s}}=n_{\mathrm{u}}+1$. We also have $\Delta\omega=B/n_{\mathrm{s}}$ where B is the OFDM bandwidth .

The signal $s(t)$ is then transmitted through a multipath wireless channel characterized by

$$\boldsymbol{h}(t,\tau)=\sum_{i=1}^{n_{\mathrm{p}}}\boldsymbol{\gamma}_i(t)c(\tau-\tau_i), \quad (2)$$

where $\boldsymbol{h}(t,\tau)$ is an n_{r}-dimensional user channel response vector, n_{r} is the number of antennas on the receiver side, n_{p} the number of multipaths, $\boldsymbol{\gamma}_i(t)$ and τ_i are the delay and complex amplitude of the ith path, respectively, and $c(t)$ is the shaping pulse. The shaping pulse in OFDM systems can be modeled as a pulse with a raised cosine spectrum. In order to simplify the derivations we will assume $c(t)=\delta(t)$ where

$$\delta(t)=\begin{cases} 1 & t=0 \\ 0 & \text{elsewhere} \end{cases}. \quad (3)$$

We will show later that the only difference between $\boldsymbol{h}(t,\tau)$ for a square-root raised cosine and $\boldsymbol{h}(t,\tau)$ for $c(t)=\delta(t)$ is in a scaling factor which does not affect our derivation.

In the remainder of the paper, unless otherwise noted, we will assume that the channel is deterministic and time independent i.e., $\boldsymbol{\gamma}(t)\equiv\boldsymbol{\gamma}_i$ which yields $\boldsymbol{h}(t,\tau)\equiv\boldsymbol{h}(t)=\sum_{i=1}^{n_{\mathrm{p}}}\boldsymbol{\gamma}_i c(t-\tau_i)$. In Section 5 we will discuss possible extensions to time-varying channels using time interpolation based on appropriately selected basis functions.

For a time-independent channel the received signal in the time-domain is given by

$$\boldsymbol{y}(t) = s(t) * \boldsymbol{h}(t) + \boldsymbol{u}(t) + \boldsymbol{e}(t) \tag{4}$$

where $*$ denotes the convolution, $\boldsymbol{u}(t)$ is the interference, and $\boldsymbol{e}(t)$ is additive noise.

Using (4) and the Discrete Fourier Transform, the received signal in the frequency domain can be written as

$$\boldsymbol{y}_{jk} = \boldsymbol{h}_j x_{jk} + \boldsymbol{u}_{jk} + \boldsymbol{e}_{jk} \tag{5}$$

where $\boldsymbol{y}_{jk}$ is the n_{r}-dimensional received signal vector at the jth subcarrier and kth time slot, $\boldsymbol{h}_j$ is the n_{r}-dimensional user channel vector at the jth subcarrier, x_{jk} is a user data symbol at the jth subcarrier and the kth time slot, $\boldsymbol{u}_{jk}$ is the unknown interference vector at the jth subcarrier and the kth time slot, and $\boldsymbol{e}_{jk}$ is additive white Gaussian noise. Observe that we use the subscript j to denote frequency dependence and similarly, we use the subscript k to denote time dependence. In the frequency domain, the user channel vector is the Discrete Fourier Transform of the user channel impulse response $h(t)$. In particular,

$$\boldsymbol{h}_j = \sum_{l=1}^{n_{\mathrm{p}}} \boldsymbol{\alpha}_l e^{-j2\pi B(j-1-n_j/2)\tau_l/n_{\mathrm{s}}} \tag{6}$$

where B is the bandwidth of n_{s} subcarriers. An implication of (6) is that there will generally be no significant correlation between the channel at the first subcarrier ($j=1$) and the channel at the last subcarrier ($j=n_{\mathrm{s}}$) unless the length of the Fourier transform equals the number of frequency slots and all multipath delays correspond with the sampling times at the receiver.

There are two types of interference with respect to the cyclic prefix position: synchronous, when the interferer's cyclic prefix is aligned in time with the user's cyclic prefix, and asynchronous, when these prefixes are not aligned. In the case of synchronous interference $\boldsymbol{u}(t)$ can be represented as a convolution of the interferer's channel $g(t)$ and the interferer's data symbol sequence $\psi(t)$, i.e., in the frequency domain

$$\boldsymbol{u}_{jk} = \boldsymbol{g}_j \psi_{jk} \tag{7}$$

where ψ_{jk} is an interferer data symbol at the jth subcarrier and the kth time slot, and $\boldsymbol{g}_j$ is the n_{r}-dimensional interferer channel vector at the jth subcarrier

$$\boldsymbol{g}_j = \sum_{l=1}^{n_{\mathrm{p}}} \boldsymbol{\beta}_l e^{-j2\pi B(j-1-n_j/2)\tau_l/n_{\mathrm{s}}}. \tag{8}$$

When the interferer is asynchronous such a representation is not possible.

3 Statistical Model

In order to derive the MLE for the above OFDM model we first develop a statistical model of the received signal as a function of the channel and interference parameters. Our goal is to develop a model which will include the unknown random effect due to the presence of unknown interference and enable statistically efficient channel estimation using a small number of pilots.

We start by assuming that both the interference and ambient noise are zero-mean Gaussian wide-sense stationary (WSS) random processes. In addition, we assume that the ambient noise is uncorrelated in space and time according to:

$$\mathrm{E}[\boldsymbol{e}_{jk}\boldsymbol{e}_{j'k'}^H] = \sigma^2 \delta(j-j')\delta(k-k') I_{n_{\mathrm{r}}} \tag{9}$$

where superscript H denotes the Hermitian transpose and that the interference is uncorrelated in time according to:

$$\mathrm{E}[\boldsymbol{u}_{jk}\boldsymbol{u}_{j'k'}^H] = \Sigma_j \delta(j-j')\delta(k-k') I_{n_{\mathrm{r}}} \tag{10}$$

where I_{n} denotes the $n \times n$ identity matrix and Σ_j is the spatial correlation matrix of the interferer. In the remainder of the paper we will omit the dimension subscripts when the matrix dimension is obvious.

Then, the statistical properties of the received signal are completely determined by its mean

$$\boldsymbol{h}_j x_{jk} \tag{11}$$

and covariance

$$\boldsymbol{\epsilon}_{jk} = \boldsymbol{u}_{jk} + \boldsymbol{e}_{jk} \tag{12}$$

$$\mathrm{E}[\boldsymbol{\epsilon}_{jk}\boldsymbol{\epsilon}_{j'k'}{}^H] = \left(\Sigma_j + \sigma^2 I\right)\delta(j-j')\delta(k-k'). \tag{13}$$

In the synchronous case the covariance expression can be further simplified into

$$\mathrm{E}[\boldsymbol{\epsilon}_{jk}\boldsymbol{\epsilon}_{j'k'}{}^H] = \left(\boldsymbol{g}_j\boldsymbol{g}_{j'}^H \mathrm{E}[\psi_{jk}\psi_{j'k'}^*] + \sigma^2 I\right)\delta(j-j')\delta(k-k'), \tag{14}$$

where superscript * denotes complex conjugate. In the case of asynchronous interference we have $n_{\mathrm{s}}n_{\mathrm{r}}$ unknown parameters corresponding to the mean, and similarly $n_{\mathrm{s}}(n_{\mathrm{s}}-1)n_{\mathrm{r}}^2+1$ covariance parameters. In the case of synchronous interference, the number of the covariance parameters is smaller i.e. $n_{\mathrm{s}}(n_{\mathrm{s}}-1)n_{\mathrm{r}}+1$. Furthermore, the interference source ψ_{jk} can be modeled as a zero-mean Gaussian random process uncorrelated in space and time with unit variance. Therefore, the covariance in the synchronous case reduces to

$$\mathrm{E}[\boldsymbol{\epsilon}_{jk}\boldsymbol{\epsilon}_{j'k'}{}^H] = \left(\boldsymbol{g}_j\boldsymbol{g}_{j'}^H + \sigma^2 I\right)\delta(j-j')\delta(k-k').. \tag{15}$$

Observe that the number of unknown covariance parameters in this model is $n_\mathrm{s} n_\mathrm{r}$ which is significantly smaller compared with the models (13) and (14). However, the above model cannot be efficiently implemented since the number of the unknown parameters is large compared with the number of measurements $n_\mathrm{r} n_\mathrm{s} n_\mathrm{k}$ corresponding to n_k pilots.

In order to further decrease the number of unknown parameters we propose to exploit the structure of channel vectors $\boldsymbol{h}_j$ and $\boldsymbol{g}_j$. Following the approach of [2] we model the user channel using set of *a priori* known basis functions and the unknown corresponding coefficients. In [2] the user channel is approximated using DFT of L unknown sample-spaced time-domain tap gains, where L is chosen to encompass the maximum expected multipath delay spread and the size of DFT has to be much larger than the number of subcarriers.

To choose an adequate basis functions model we observe that the main difficulty with estimating the unknown parameters in models (6) and (8) is the unknown, possibly large, number of multipaths and the corresponding multipath delays. Also, note that if several multipaths have the same delay they will be represented with only one term in the summation (6). Therefore, we propose to substitute the *real* wireless system with an *equivalent* system in which the number of multipaths and the corresponding delays are *a priori* known i.e.

$$\boldsymbol{h}_j \approx \sum_{l=1}^{n} \tilde{\boldsymbol{\alpha}}_l e^{-j2\pi B(j-1-n_j/2)\tilde{\tau}_l/n_\mathrm{s}}. \tag{16}$$

The above model will be exact whenever $\{\tau_1, \ldots, \tau_{n_\mathrm{p}}\} \subset \{\tilde{\tau}_1, \ldots, \tilde{\tau}_n\}$ and a very good approximation whenever $\max_i[\min_j(\tau_i - \tilde{\tau}_j)]$ is sufficiently small.

Using (16) we can model the user channel as:

$$\boldsymbol{h}_j = H_j \boldsymbol{\theta} \tag{17}$$

where $\boldsymbol{\theta}$ are the unknown coefficients vector, and

$$\begin{aligned} H_j &= \boldsymbol{f}_j^H \otimes I \\ \boldsymbol{f}_j &= [e^{i2\pi\tau_1 jBW/n_\mathrm{s}}, e^{i2\pi\tau_2 jBW/n_\mathrm{s}}, \ldots, e^{i2\pi\tau_l jBW/n_\mathrm{s}}]^H \end{aligned}$$

where I is $n_\mathrm{r} \times n_\mathrm{r}$ identity matrix, and $\otimes$ is the Kronecker product.

Similarly, in the case of a synchronous single interferer we propose to model the interferer's channel using a similar model i.e.,

$$\boldsymbol{u}_j = U_j \boldsymbol{\eta} \tag{18}$$

where U_j is the interferer channel interpolation matrix computed at the subcarrier j and $\boldsymbol{\eta}$ is the vector of interferer channel parameters. A natural choice for the matrix U_j is the same as for the user channel interpolation matrix H_j i.e., $U_j = \boldsymbol{f}_j^H \otimes I$. However, we are not limited to this choice and thus for generality, in the remainder of the paper we will assume that these two matrices are different.

4 Frequency Domain Channel Estimation

Recall that the received signal is given by

$$\boldsymbol{y}_{jk} = \boldsymbol{h}_j x_{jk} + \boldsymbol{u}_j \psi_{j,k} + \boldsymbol{e}_{jk} \tag{19}$$

with the following statistical description

$$\begin{aligned} \boldsymbol{y}_{jk} &\sim \mathcal{N}(x_{jk}\boldsymbol{h}_j, \Sigma_j) \\ \Sigma_j &= \sigma^2(I + \frac{1}{\sigma^2}\boldsymbol{u}_j \boldsymbol{u}_j^H). \end{aligned} \tag{20}$$

Our approach can be easily extended to a parametric case $E[\psi_{jk}\psi^*_{j'k'}] = G(j-j', \boldsymbol{\xi})\delta(k-k')$ where $G(\cdot)$ is a known (up to a parameter) matrix function and $\boldsymbol{\xi}$ are unknown parameters. We will further discuss this model further in Section 5.

The distribution of the received signal is thus given by

$$\boldsymbol{y}_{jk} \sim \frac{1}{\pi^{n_\mathrm{r}}|\Sigma_j|} \exp\left[-\frac{1}{\sigma^2}\boldsymbol{\epsilon}_{jk}{}^H \Sigma_j^{-1} \boldsymbol{\epsilon}_{jk}\right].$$

It is obvious that x_{jk} and $\boldsymbol{h}_j$ cannot be estimated simultaneously. In the pilot-aided channel estimation method we first send a training sequence of known symbols pilots x_{jk}. Using the received pilot symbols the log-likelihood function becomes

$$\begin{aligned} L(\boldsymbol{y}, \boldsymbol{h}, \boldsymbol{u}, \sigma) &= -n_k n n_\mathrm{r} \log\sigma - n_k \sum_{j=1}^{n} \log(1 + \frac{1}{\sigma^2}\boldsymbol{u}_j^H \boldsymbol{u}_j) - \\ &- \frac{1}{\sigma^2}\sum_{j=1}^{n}\sum_{k=1}^{n_k} \left(\boldsymbol{y}_{jk} - x_{jk}\boldsymbol{h}_j\right)^H \left(I + \frac{1}{\sigma^2}\boldsymbol{u}_j \boldsymbol{u}_j^H\right)^{-1} \left(\boldsymbol{y}_{jk} - x_{jk}\boldsymbol{h}_j\right), \end{aligned}$$

where

$$\begin{aligned} \boldsymbol{y} &= \left[\boldsymbol{y}_{11}^T, \ldots, \boldsymbol{y}_{n1}^T, \ldots, \boldsymbol{y}_{1k}^T, \ldots, \boldsymbol{y}_{nk}^T, \ldots, \boldsymbol{y}_{1n_k}^T, \ldots, \boldsymbol{y}_{nn_k}^T\right]^T, \\ \boldsymbol{h} &= [\boldsymbol{h}_j^T, \ldots, \boldsymbol{h}_n^T]^T, \\ \boldsymbol{u} &= [\boldsymbol{u}_j^T, \ldots, \boldsymbol{u}_n^T]^T. \end{aligned}$$

Next, using the parametric model from Section III and the Woodbury's identity the likelihood functions becomes

$$\begin{aligned} L(\boldsymbol{y}, \boldsymbol{\theta}, \boldsymbol{\eta}, \sigma) &= -n_k n n_\mathrm{r} \log\sigma - n_k \sum_{j=1}^{n} \log(1 + \boldsymbol{\eta}^H U_j^H U_j \boldsymbol{\eta}) + \\ &+ \frac{1}{\sigma^2}\sum_{j=1}^{n} \frac{1}{(1 + \boldsymbol{\eta}^H U_j^H U_j \boldsymbol{\eta})} \sum_{k=1}^{n_k} \boldsymbol{e}_{jk}{}^H U_j \boldsymbol{\eta}\boldsymbol{\eta}^H U_j^H \boldsymbol{e}_{jk} - \\ &- \frac{1}{\sigma^2}\sum_{j=1}^{n}\sum_{k=1}^{n_k} \boldsymbol{e}_{jk}{}^H \boldsymbol{e}_{jk} \end{aligned} \tag{21}$$

where

$$e_{jk} = \left(\boldsymbol{y}_{jk} - x_{jk} H_j \boldsymbol{\theta}\right) \tag{22}$$

For a given ML estimate $\hat{\boldsymbol{\eta}}$ the covariance matrix $\Sigma_j = \sigma^2 D(\hat{\boldsymbol{\eta}})$ becomes unknown up to the parameter σ^2. Therefore, the ML estimates $\hat{\boldsymbol{\theta}}$ and $\hat{\sigma}^2$ are generalized least squares (GLS) estimates

$$\begin{aligned} \hat{\boldsymbol{\theta}} &= Q^{-1} \sum_{j=1}^{n} H_j^H D_j^{-1}(\hat{\boldsymbol{\eta}}) \boldsymbol{s}_j^{xy} \\ \hat{\sigma}^2 &= \frac{1}{n_k n n_{\mathrm{r}}} \sum_{j=1}^{n} \hat{\boldsymbol{e}}_{j-}^H D_j^{-1}(\hat{\boldsymbol{\eta}}) \hat{\boldsymbol{e}}_{j-}, \end{aligned} \tag{23}$$

where

$$\begin{aligned} \boldsymbol{s}_j^{xy} &= \sum_{k=1}^{n_k} x_{jk}^* \boldsymbol{y}_{jk} \\ \boldsymbol{e}_{j-} &= \sum_{k=1}^{n_k} \left(\boldsymbol{y}_{jk} - x_{jk} H_j \hat{\boldsymbol{\theta}}\right) \\ Q(\hat{\boldsymbol{\eta}}) &= \sum_{j=1}^{n} s_j^x H_j^H D_j^{-1}(\hat{\boldsymbol{\eta}}) H_j \\ D_j^{-1}(\hat{\boldsymbol{\eta}}) &= I - \frac{1}{1 + \hat{\boldsymbol{\eta}}^H U_j^H U_j \hat{\boldsymbol{\eta}}} U_J \hat{\boldsymbol{\eta}} \hat{\boldsymbol{\eta}}^H U_j^H. \end{aligned}$$

Using $\hat{\boldsymbol{\theta}}$ and $\hat{\sigma}^2$ we compute the MLE $\hat{\boldsymbol{\eta}}$ by maximizing the concentrated likelihood function $L(\boldsymbol{y}, \hat{\boldsymbol{\theta}}, \boldsymbol{\eta}, \hat{\sigma})$ i.e., we compute $\hat{\boldsymbol{\eta}}$ using the following equation

$$\frac{\partial L(\boldsymbol{y}, \hat{\boldsymbol{\theta}}, \boldsymbol{\eta}, \hat{\sigma})}{\partial \boldsymbol{\eta}} = 0.$$

Then,

$$\sum_{j=1}^{n} U_j^H G_j(\hat{\boldsymbol{\eta}}) U_j \hat{\boldsymbol{\eta}} = 0 \tag{24}$$

where

$$\begin{aligned} G_j(\hat{\boldsymbol{\eta}}) &= \frac{1}{\hat{\sigma}^2 {w_j}^2} \left\{ [\hat{\sigma}^2 w_j - \hat{\boldsymbol{\eta}}^H U_j^H E_j U_j \hat{\boldsymbol{\eta}}] I + w_j E_j \right\} \\ w_j &= 1 + \hat{\boldsymbol{\eta}}^H U_j^H U_j \hat{\boldsymbol{\eta}} && (25) \\ E_j &= \frac{1}{n_k} \sum_{k=1}^{n_k} \hat{\boldsymbol{e}}_{jk} \hat{\boldsymbol{e}}_{jk}^H && (26) \\ \hat{\boldsymbol{e}}_{jk} &= \boldsymbol{y}_{jk} - x_{jk} H_j \hat{\boldsymbol{\theta}}. && (27) \end{aligned}$$

Note that the solution to (4) is within the parameter space since $L(\boldsymbol{y}, \hat{\boldsymbol{\theta}}, \boldsymbol{\eta}, \hat{\sigma}) \to -\infty$ when $\boldsymbol{\eta} \to \infty$.

To solve the above equations we first obtain $\hat{\boldsymbol{\eta}}$ using ordinary least squares estimates of $\boldsymbol{\theta}$ and σ. Then, we can use the following iterative algorithm:

- At step i compute $\hat{\boldsymbol{\eta}}_{i+1}$ using $\hat{\sigma}_i$ and $\hat{\boldsymbol{\theta}}_i$.
- Using $\hat{\boldsymbol{\eta}}_{i+1}$ compute $\hat{\sigma}_{i+1}$ and $\hat{\boldsymbol{\theta}}_{i+1}$.
- Stop the algorithm if $\|\hat{\boldsymbol{\eta}}_i - \hat{\boldsymbol{\eta}}_{i+1}\|^2 + (\hat{\sigma}_i - \hat{\sigma}_{i+1})^2 \leq \epsilon$ otherwise increase i and repeat the process.

4.1 Symbol Detection

In the previous section we pointed out that the symbols x_{jk} and channel/interference parameters $(\boldsymbol{\theta}, \boldsymbol{\eta})$ cannot be estimated jointly. We have also shown that by sending *known* pilot symbols we can compute channel parameters $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\eta}}$. Then assuming that the channel and interferer properties do not change in time (or change sufficiently slowly) we can use these estimates to obtain ML estimates of the unknown symbols. Namely, we can rewrite likelihood function (21) as

$$\begin{aligned} L(\boldsymbol{x}) &= -\frac{1}{\sigma^2} \sum_{j=1}^{n} \sum_{k=1}^{n_k} {\boldsymbol{e}_{jk}}^H \boldsymbol{e}_{jk} + && (28) \\ &\quad + \frac{1}{\sigma^2} \sum_{j=1}^{n} \frac{1}{w_j} \sum_{k=1}^{n_k} {\boldsymbol{e}_{jk}}^H U_j \boldsymbol{\eta} \boldsymbol{\eta}^H U_j^H \boldsymbol{e}_{jk} && (29) \\ \boldsymbol{x}_i &= [x_{i1}, \ldots, x_{in_k}] && (30) \\ \boldsymbol{x} &= [\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n]. && (31) \end{aligned}$$

Then, maximizing $L(\boldsymbol{x})$ respect to $\boldsymbol{x}$ we estimate the unknown symbols using

$$\hat{x}_{jk}^* = \frac{\boldsymbol{y}_{jk}^H D_n^{-1}(\hat{\boldsymbol{\eta}}) H_j \hat{\boldsymbol{\theta}}}{\hat{\boldsymbol{\theta}}^H H_j^H D_n^{-1}(\hat{\boldsymbol{\eta}}) H_j \hat{\boldsymbol{\theta}}}. \tag{32}$$

4.2 Approximate Maximum Likelihood

Since our interest lies primarily in drawing inference about $\boldsymbol{\theta}$ (and subsequently x_{jk}) we propose an alternative estimation scheme based on the estimated generalized least squares (EGLS) method for $\boldsymbol{\theta}$ and method of moments estimator (MM) for $\boldsymbol{\eta}$ and σ^2, see [3]. These methods provide simple non-iterative estimates of the variance-covariance parameters without sacrificing any efficiency in the estimation of $\boldsymbol{\theta}$.

The EGLS objective function is the same as for the MLE except that $\Sigma_j(\boldsymbol{\eta})$ is replaced by $\Sigma_j(\hat{\boldsymbol{\eta}})$ where $\hat{\boldsymbol{\eta}}$ is any consistent estimator of $\boldsymbol{\eta}$. Thus the EGLS estimate of $\boldsymbol{\theta}$ is given by

$$\hat{\boldsymbol{\theta}}(\hat{\boldsymbol{\eta}}) = Q^{-1} \sum_{j=1}^{n} H_j^H D_j^{-1}(\hat{\boldsymbol{\eta}}) \boldsymbol{s}_j^{xy} \tag{33}$$

which is equivalent to the MLE whenever $\hat{\boldsymbol{\eta}}$ is the MLE. Furthermore, since $\hat{\boldsymbol{\eta}} \to \boldsymbol{\eta}$ as $n_k \to \infty$ the EGLS estimator can be shown to be asymptotically equivalent to the MLE.

We start by rewriting the received signal as

$$\boldsymbol{y}_{jk} = x_{jk} H_j \boldsymbol{\theta} + U_j \boldsymbol{\xi}_{jk} + \boldsymbol{e}_{jk} \tag{34}$$

where $\boldsymbol{\xi}_{jk}$ is a random vector uncorrelated distributed as

$$\begin{aligned}\boldsymbol{\xi}_{jk} &\sim \mathcal{N}\left(0, \boldsymbol{\eta}\boldsymbol{\eta}^H\right)\\ \mathrm{E}[\boldsymbol{\xi}_{jk}\boldsymbol{\xi}_{jk}^H] &= \boldsymbol{\eta}\boldsymbol{\eta}^H\delta(j-j')\delta(k-k'). \end{aligned} \quad (35)$$

Next, we note that the unobserved residuals can be written as

$$\boldsymbol{\epsilon}_{jk} = \boldsymbol{y}_{jk} - H_j\boldsymbol{\theta} = U_j\boldsymbol{\xi}_{jk} + \boldsymbol{e}_{jk}. \quad (36)$$

If we estimate $\boldsymbol{\xi}_{jk}$ via the usual least squares estimator

$$\hat{\psi}_{jk} = \left(U_j^H U_j\right)^{-1} U - j^H \boldsymbol{e}_{jk} \quad (37)$$

and σ^2 via the usual mean square for error,

$$\boldsymbol{s}_{jk} = \boldsymbol{e}_{jk}^H \left[I - U_j(U_j^H U_j)U_j^{-1}\right]\boldsymbol{e}_{jk}/(n_\mathrm{r}-1) \quad (38)$$

then the MM estimator can be obtained by taking the expectations of $\psi_{jk}\psi_{jk}^*$ and averaging them across the $n_\mathrm{s}n_\mathrm{k}$ measurements. Since a pooled MM estimator of σ^2 is

$$\hat{\sigma}^2 = \frac{1}{n_\mathrm{s}n_k(n_\mathrm{r}-1)}\sum_{j=1}^{n_\mathrm{s}}\sum_{k=1}^{n_k} s_{jk}^2 \quad (39)$$

and since

$$\mathrm{E}[\boldsymbol{\xi}_{jk}\boldsymbol{\xi}_{jk}^H] = \boldsymbol{\eta}\boldsymbol{\eta}^H + \hat{\sigma}^2\left(U_j^H U_j\right)^{-1} \quad (40)$$

for a fixed $\boldsymbol{\theta}$, it follows that a simple MM estimator of $\boldsymbol{\eta}\boldsymbol{\eta}^H$ is

$$(\widehat{\boldsymbol{\eta}\boldsymbol{\eta}^H}) = \frac{1}{n_\mathrm{s}n_\mathrm{k}}\sum_{j=1}^{n_\mathrm{s}}\sum_{k=1}^{n_k}\hat{\boldsymbol{\xi}}_{jk}\boldsymbol{\xi}_{jk}^H - \frac{\hat{\sigma}^2}{n_\mathrm{s}n_\mathrm{k}}\sum_{j=1}^{n_\mathrm{s}}\left(U_j^H U_j\right)^{-1}. \quad (41)$$

A nice feature of $\hat{\sigma}^2$ is that it is invariant to the value of $\boldsymbol{\theta}$ used in forming the residual vector $\boldsymbol{e}_{jk}$. Unfortunately, this invariance property does not hold for $\widehat{\boldsymbol{\eta}\boldsymbol{\eta}^H}$. By writing $\hat{\boldsymbol{\eta}}$ as $\hat{\boldsymbol{\eta}}(\boldsymbol{\theta})$ to reflect the dependence of MM estimator on $\boldsymbol{\theta}$ we can estimate $\boldsymbol{\eta}$ using the set of generalized estimating equations (GEE). However, this procedure would require an iterative algorithm such as iteratively reweighted least squares.

Rather than performing iteratively reweighted least squares, an alternative approach would be to use an MM estimator of $\boldsymbol{\eta}$ which, like $\hat{\sigma}^2$ is invariant with respect to $\boldsymbol{\theta}$. In general such an estimator may not be available. However, for a particular choice of interpolation matrices H_j and U_j we can derive a residual method of moments (RMM) estimate of $\boldsymbol{\eta}\boldsymbol{\eta}^H$, similar to the estimator proposed in [3]. Namely, for interpolation matrices such that

$$\begin{aligned} H_j &= U_j A_j \quad (42)\\ A_j &= I \otimes \boldsymbol{a}_j^H \quad (43)\end{aligned}$$

the RMM estimate (independent of $\boldsymbol{\theta}$) can be shown to be

$$(\widehat{\boldsymbol{\eta}\boldsymbol{\eta}^H})_\mathrm{RMM} = S - \frac{\hat{\sigma}^2}{n_\mathrm{s}n_\mathrm{k}}\sum_{j=1}^{n_\mathrm{s}}\left[1 - \boldsymbol{a}_j^H\left(A^H A\right)^{-1}\boldsymbol{a}_j\right]\left(U_j^H U_j\right)^{-1} \quad (44)$$

where

$$\begin{aligned} A &= \left[\boldsymbol{a}_1 \ldots \boldsymbol{a}_{n_\mathrm{s}}\right]^H, \quad (45)\\ S &= \tilde{B}\left(I - A(A^H A)A^H)\right)\tilde{B}, \quad (46)\\ \tilde{B} &= \left[\tilde{b}_{11}, \ldots, \tilde{b}_{1n_k}\tilde{b}_{21}, \ldots\ldots, \tilde{b}_{n_\mathrm{s}n_k}\right]^H, \quad (47)\\ \tilde{b}_{jk} &= \left(U_j^H U_J\right)^{-1} U_j^H \boldsymbol{y}_{jk}. \quad (48)\end{aligned}$$

The RMM estimate is similar to the residual or restricted maximum likelihood (REML) estimation in that it corrects for small number of pilots bias.

The above algorithm estimates the covariance $\Lambda = \boldsymbol{\eta}\boldsymbol{\eta}^H$ without exploiting the structural information i.e. it estimates $n(n-1)$ unknown parameters. To account for the additional information an iterative algorithm with respect to $\boldsymbol{\eta}$ is required. It is possible, then, that it will provide little improvement in computing time over the fully iterated MLE. To avoid iterations, we propose to employ a suboptimal approach and fit $\hat{\Lambda}$ to $\boldsymbol{\eta}\boldsymbol{\eta}^H$ using the generalized least squares (GLS). First, let $\hat{\lambda}_{ij}$ be the i-th row and j-th column element of matrix $\hat{\Lambda}$. Then the objective function for the GLS can be written as

$$\sum_{i,j} w_{ij}(\boldsymbol{\eta})\|\hat{\lambda_{ij}} - \eta_i\eta_j^*\|^2 \quad (49)$$

where

$$\boldsymbol{\eta} = [\eta_1, \ldots, \eta_l]^T \quad (50)$$

and $w_{ij}(\boldsymbol{\eta})$ covariances of $\hat{\lambda}_{jk}$. Observe that $\hat{\Lambda}$ has the non-central Wishart distribution with the non-centrality factor dependent on $\boldsymbol{\eta}$ and thus $w_{ij}(\boldsymbol{\eta})$ is also dependent on $\boldsymbol{\eta}$. The GEE (50) are highly non-linear and thus may not significantly reduce the computational complexity. Instead, we propose to use the empirical covariances $\hat{\Lambda}^{-1}$ which yields the following set of l non-linear equations

$$\sum_{j=1}^{l} w_{ij}\eta_j^*\left(\hat{\lambda}_{ij} - \eta_i^*\eta_j\right) = 0 \qquad \text{for } i = 1, \ldots, l \quad (51)$$

which can be solved easily using any standard method for solving non-linear equations. It is Important to note that the choice of weighting factors w_{ij} will have a little impact in terms of inference about $\boldsymbol{\theta}$ but it may impact any inference related to σ^2 and $\boldsymbol{\eta}$.

A common drawback with MM type estimators is that they may occasionally produce negative-definite estimates. To ensure having a non-negative definite estimate of Λ we can apply the correction procedure described in [3]. However this procedure requires an iterative algorithm in which case MLE may be better a choice.

4.3 MANOVA Model Estimator

The proposed approximate MLE exploits covariance structure $\Sigma_j(\boldsymbol{\theta})$ which results in the computationally expensive implementation but with smaller training overhead requirements. For comparison purposes, in this section we derive the MLE algorithm for unstructured covariance model. Using these results, we will compare performance of the approximate MLE and MANOVA estimator for various interference scenarios in Section 6.

First, we collect the received waveforms into matrix

$$Y_k = \begin{bmatrix} \boldsymbol{y}_{1k} \\ \vdots \\ \boldsymbol{y}_{n_s k} \end{bmatrix}. \tag{52}$$

Next, we rewrite the received signal model as

$$\boldsymbol{y}_k = H_k \boldsymbol{\theta} + E_k \tag{53}$$

where

$$H_k = \begin{bmatrix} x_1 H_1 \\ \vdots \\ x_{n_s} H_{n_s} \end{bmatrix}. \tag{54}$$

In the unstructured covariance model (MANOVA) the columns of E_k are independent and identically distributed (i.i.d.) according to a multivariate normal distribution with zero mean and covariance matrix Σ

$$\text{Vec}(E_k) \sim \mathcal{N}(0, \Sigma \otimes I) \tag{55}$$

where Vec is the vector operator defined in the following way

$$E = [\boldsymbol{e}_1 \cdots \boldsymbol{e}_n] \tag{56}$$

$$\text{Vec}(E) = \begin{bmatrix} \boldsymbol{e}_1 \\ \vdots \\ \boldsymbol{e}_{n_s} \end{bmatrix} \tag{57}$$

The log-likelihood function for the above model is

$$L(\boldsymbol{y}, \boldsymbol{\theta}, \Sigma) = -\frac{1}{2}\text{trace}\{\Sigma^{-1}\sum_{i=1}^{n_k} \boldsymbol{e}_k \boldsymbol{e}_k^H\} - -\frac{n_s n_k}{2}|\Sigma| \tag{58}$$

$$\boldsymbol{e}_k = \boldsymbol{y}_k - H_k \boldsymbol{\theta} \tag{59}$$

The MLE of $\boldsymbol{\theta}$ and Σ are, respectively,

$$\hat{\boldsymbol{\theta}} = \frac{1}{n_k}(H_k^H H_k)^{-1} H_k^H \sum_{i=1}^{n_k} \boldsymbol{y}_k \tag{60}$$

$$\widehat{\Sigma} = \frac{1}{n_s n_k}\sum_{i=1}^{n_k} \boldsymbol{y}_k (I - H_k (H_k^H H_k)^{-1} H_k^H) \boldsymbol{y}_k^H \tag{61}$$

The above model does not assume structured covariance $\Sigma_j(\theta) = H_j \boldsymbol{\eta}\boldsymbol{\eta}^H H_j^H + \sigma^2 I$ and thus should perform better when the covariance differs significantly from this structure. However, since the number of unknown parameters is much larger in the unstructured model the approximate MLE may still give better results then MANOVA MLE for the same number of pilots.

5 Numerical Examples

In this section we demonstrate the performance of the proposed estimator by numerical examples. In all examples, unless otherwise stated, we assume $n_r = 4$ element antenna array and a 16-QAM signal occupying a 6.0MHz bandwidth i.e. $B = 6.0$MHz. The size of the DFT is set to 512 and the number of usable carriers is $n_s = 370$. The number of pilot bauds is $n_k = 3$ with pilot symbols on all subcarriers. The number of basis functions for both interpolation matrices is $n = 40$ (10 for each antenna). The maximum multipath delay is $\tau_{\max} = 8.0\mu$s. We define the signal-to-interference (SIR) as the ratio of signal power to interference power and set it to 15dB. The number of multipaths is set to $n_p = 100$ with an exponential power delay profile. The phase shift on each path is uniformly distributed over $[0, 2\pi)$.

Figure 1 illustrates the bit-error-rate (BER) performance of the proposed approximate MLE as a function of the number of basis functions n and SIR. It can be seen that for a particular SIR there exists a threshold above which increasing the number of basis functions will not yield significant improvement in the performance.

For comparison purposes in Figure 2 we present the performance of the least squares estimator, the approximate MLE, and the MANOVA MLE as a function of SIR for a single synchronous interferer scenario. The MANOVA MLE assumes that the received signal is correlated in frequency i.e. that the covariances are given by

$$\text{E}\left[\boldsymbol{y}_k \boldsymbol{y}_{k'}^H\right] = \Sigma\delta(k - k') \tag{62}$$

$$\boldsymbol{y}_k = \left[\boldsymbol{y}_{1k}^H s \boldsymbol{y}_{n_s k}^H\right]^H. \tag{63}$$

As expected the approximate MLE outperforms the other two estimators. Figure 4 illustrates the performance of these estimators for the same scenario but with two synchronous interferers. Figure 4 illustrates the same results for a single asynchronous interferer. For illustration purposes, we assume that the interferer is correlated in frequency with an exponentially decaying profile.

6 Possible Extensions

6.1 Multiple Users

It has been shown [5],[6] that the antenna arrays can increase the system capacity by allowing multiple users to share the same time-frequency resources, a practice called spatial division multiple access (SDMA). In this section we discuss possible extensions of our results to the multiple user scenario.

Assume there are n_c SDMA users in the system sharing the channel. The received signal in the frequency domain is

$$\boldsymbol{y}_{jk} = \sum_{l=1}^{n_c} \boldsymbol{h}_{j,l} x_{jk,l} + \boldsymbol{u}_{jk} + \boldsymbol{e}_{jk}. \tag{64}$$

We propose to model the the lth user channel using

$$\boldsymbol{h}_{j,l} = H_j \boldsymbol{\theta}_l. \tag{65}$$

Observe that we model different user channels with the same interpolation matrix H_j but different parameters $\boldsymbol{\theta}_l$. If the interpolation matrices are different the unknown linear parameters in the resulting model will be identifiable only when specific conditions are satisfied [7].

Using the above model equation (64) can be written as:

$$\boldsymbol{y}_{jk} = H_j \Theta \boldsymbol{x}_{jk} + \boldsymbol{u}_{jk} + \boldsymbol{e}_{jk}. \tag{66}$$

where

$$\Theta = [\boldsymbol{\theta}_1 \cdots \boldsymbol{\theta}_{n_c}] \tag{67}$$
$$\boldsymbol{x}_{jk} = [x_{jk,1}, \ldots, x_{jk,n_u}]. \tag{68}$$

Next, let

$$Y_j = [\boldsymbol{y}_{j1} \cdots \boldsymbol{y}_{jn_k}] \tag{69}$$

be the collection of all received pilots, and

$$X_j = [\boldsymbol{x}_{j1} \cdots \boldsymbol{x}_{jn_k}] \tag{70}$$

be the collection of all pilot symbols at jth subcarrier. The received signal is described in the matrix form as:

$$Y_j = H_j \Theta X_j + U_j + E_j \tag{71}$$

where U_j represents the random interference matrix and E_j represents the random ambient noise matrix. We assume that the rows of E_j are i.i.d. according to a normal distribution, i.e.,

$$\mathrm{Vec}(E_j) \sim \mathcal{N}(0, \sigma^2 I). \tag{72}$$

We continue to assume a synchronous interferer case i.e.,

$$U_j = U_j \boldsymbol{\eta} \otimes \boldsymbol{\psi}_j \tag{73}$$
$$\boldsymbol{\psi}_j = [\psi_{j1}, \ldots, \psi_{jn_k}]^T \tag{74}$$

which results in the following distribution:

$$\mathrm{Vec}(\mathrm{U_j}) \sim \mathcal{N}(0, (\mathrm{U_j} \boldsymbol{\eta}\boldsymbol{\eta}^{\mathrm{H}} \mathrm{U_j^H}) \otimes \mathrm{I}). \tag{75}$$

Computing the MLE of Θ and $\boldsymbol{\eta}$ in the above model requires an iterative algorithm. However, we can further simplify the model assuming that the pilot symbols sent on different carriers are the same i.e. $X_j = X_i$ for all i, j. In this case the distribution of the received signal will be given by

$$\mathrm{Vec} Y \sim \mathcal{N}(0, [H(\boldsymbol{\eta}\boldsymbol{\eta}^H \otimes I)] \otimes I) \tag{76}$$

where

$$H = \mathrm{bdiag}[H_1, \ldots, H_{n_s}] \tag{77}$$
$$Y = \begin{bmatrix} Y_1 \\ \vdots \\ Y_{n_s} \end{bmatrix} \tag{78}$$

The extension of the approximate MLE presented in Section 4.2 to the above model is straightforward.

6.2 Time-varying Channel

The channel estimators discussed so far assume a constant channel over the block of received pilot symbols. These estimators work well only when the channel has not changed significantly in time. In this section we discuss means for interpolating in time between blocks of channel estimates at different times in order to find channel estimates at all times of interest.

We propose to model the channel time variations using a set of appropriately chosen temporal basis functions $\{p_i(k)\}$ where $i = 1, \ldots, m$. In particular we model the channel $\boldsymbol{h}_{jk}$ (we introduce subscript k to denote the time dependence) as

$$\boldsymbol{h}_{jk} = \boldsymbol{f}_j^H \Theta \boldsymbol{p}_k \tag{79}$$

where Θ is $n \times m$ matrix of unknown parameters and

$$\boldsymbol{p}_k = [p_1(k), \ldots, p_m(k)]^T. \tag{80}$$

Next let $P_j = [x_{j1}\boldsymbol{p}_1, \ldots, x_{jn_k}\boldsymbol{p}_{n_k}]$. Then, using the matrix notation from the previous section the received signal can be written as

$$Y_j = H_j \Theta P_j + U_j \Phi Q_j + E_j \tag{81}$$

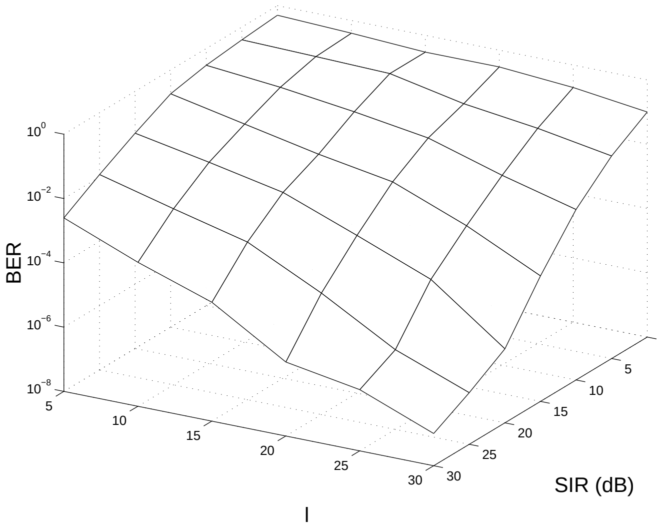

Figure 1: BER as a function of n and SIR.

where Φ is matrix of time-frequency parameters modeling the interferer channel and Q_j is a matrix of interferer basis functions (analogous to P_j). It is straightforward to show that the distribution of the received signal is:

$$\mathrm{Vec}Y_j \sim \mathcal{N}(\mathrm{Vec}(H_j \Theta P_j), H_j \Phi \tilde{Q} \Phi^H H_j^H) \tag{82}$$

where

$$\tilde{Q} = \mathrm{bdiag}[\boldsymbol{q}_1 \boldsymbol{q}_1^H, \ldots, \boldsymbol{q}_{n_k} \boldsymbol{q}_{n_k}^H]. \tag{83}$$

Due to a complex structure of the covariance matrix the RMM estimator of Φ similar to one proposed in Section 4.2 may not exist. In this case the most reasonable choice will most likely be an iterative MLE.

7 Conclusions

We have proposed pilot-aided channel estimation algorithms for estimating OFDM wireless channel in the presence of interference. The algorithms included the MLE and the approximate MLE. We have shown that the approximate MLE can achieve desired performance with a sufficiently small number of pilots unlike the unstructured MANOVA MLE. We have also compared performance of three algorithms: LS, approximate MLE, and MANOVA MLE. We have demonstrated that even when the asynchronous or multiple synchronous interferers are present, the performance of the approximate MLE is comparable to the performance of the MANOVA MLE. An effort will be made to examine the applicability of our approach to the proposed extensions: time-varying channel and multiple users scenarios.

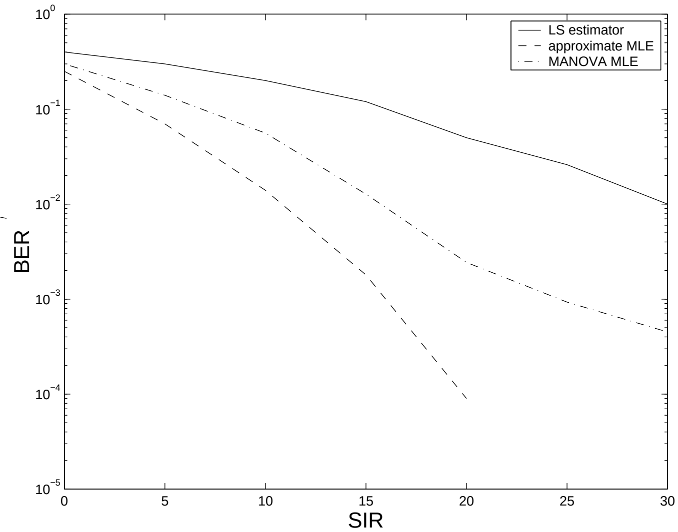

Figure 2: BER performance comparison - single interferer.

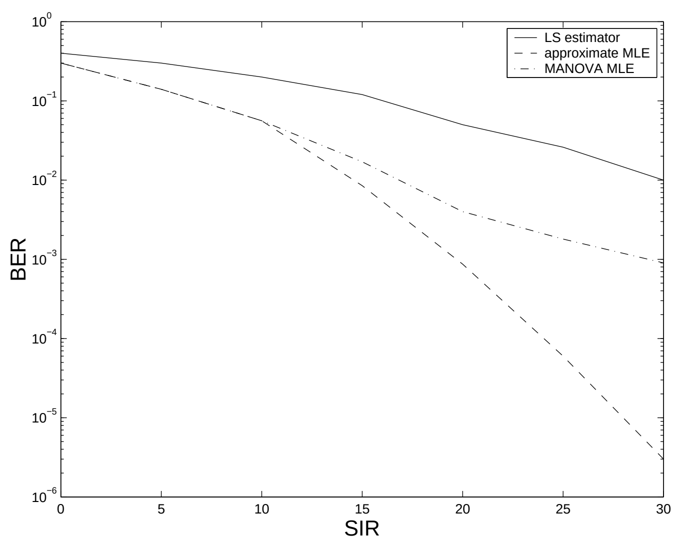

Figure 3: BER performance comparison - two interferers.

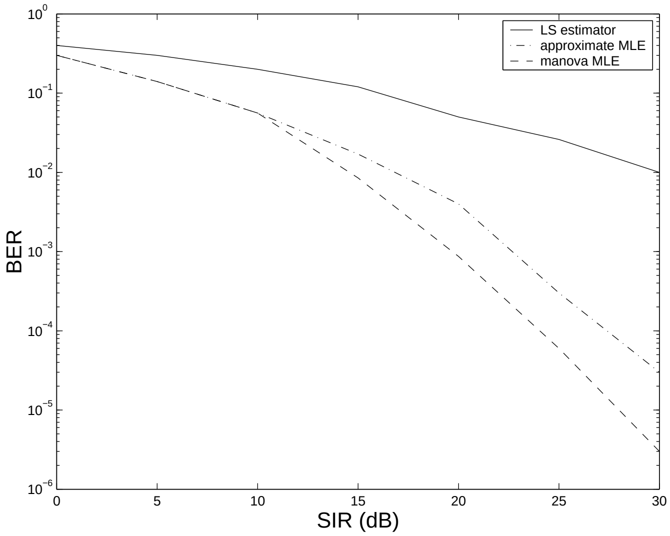

Figure 4: BER performance comparison - two interferers.

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