OFDM CHANNEL ESTIMATION IN THE PRESENCE OF ASYNCHRONOUS INTERFERENCE

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ABSTRACT

A frequency-domain channel estimation algorithm for a single-user orthogonal frequency division multiplexing (OFDM) wireless system in the presence of synchronous interference has recently been proposed in [1]. However, the interference cyclic prefix does not usually align with the desired user's cyclic prefix (i.e., the interferer is asynchronous) and thus a different approach is required. For an asynchronous interferer in a rich multipath environment, the received frequency-domain measurement is correlated in space with full-rank covariance matrix on each subcarrier. Therefore, the synchronous algorithms may give poor detection performance since they assume reduced rank interference and use smaller number of parameters. We overcome these problems by employing an appropriate structured model with properly defined number of covariance parameters. We estimate the interference covariance parameters using a residual method of moments (RMM) estimator and the mean (i.e., the desired user's channel) parameters by maximum likelihood (ML) estimation. Since the RMM estimates are invariant to the mean, we obtain simple non-iterative estimates of the covariance parameters while having optimal statistical efficiency. Numerical results illustrate the applicability of the proposed algorithm.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has received considerable interest over the last few years for its advantages in high-bit-rate transmissions over frequency selective fading channels. In OFDM systems, the input high-rate data stream is divided into many low-rate streams [7] that are transmitted in parallel, thereby increasing the symbol duration and reducing the intersymbol interference (ISI).

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Coherent OFDM transmission invariably requires estimation of the channel frequency response (i.e. the gains of the OFDM tones). Currently, there are three different types of channel parameter estimators: (i) blind, (ii) semi-blind, and (iii) pilot-aided. Blind estimation techniques, see [2], [3], try to estimate the channel without any knowledge of the transmitted data. They are attractive because of the possible savings in training overhead, however they are effective only when a large amount of data can be collected. Pilot-aided estimation, see [4], transmits training sequence of known data symbols (pilots) and the channel parameters are initially estimated using the received pilot signal. Semi-blind techniques try to reduce the size of the training sequence by exploiting both the known (training) and unknown (blind) portions of the data.

In this paper we present a frequency domain pilot-aided channel estimation algorithm in the presence of asynchronous interference. Similar problem in the time domain has recently been presented in [5]. Interference suppression is of utmost importance in high-rate wireless systems. In the presence of interference the received signal, in addition to multipath fading, may have heteroskedastic (non-constant variance) properties, such as frequency dependent covariance which means that channel estimators that ignore the interference are suboptimal. Sufficiently accurate estimation of the covariance matrices, necessary for efficient interference suppression, in such a model would impose significant training overhead. In [1], [6] we presented frequency domain channel estimation algorithms in the presence of synchronous interference. However, in many realistic scenarios the interference will be asynchronous. We demonstrated in [1] that in this case the performance of the synchronous estimator can deteriorate significantly. To overcome these problems, in this paper we present a structured model appropriate for estimating the channel and interference parameters for asynchronous interference. The proposed asynchronous model has larger number of parameters than the synchronous model presented in [1], [6] but gives better performance in the asynchronous case. We propose estimating the unknown parameters using: (i) maximum likelihood estimator (MLE) and (ii) an asymptotic MLE in which the non-linear covariance parameters are estimated using a method of moments (MM) estimator, which yields non-iterative algorithm.

In Section 2, we briefly describe the OFDM system with pilot-symbol-aided channel estimation and derive the statistical model. In Section 3, we present the MLE, asymptotic MLE and discuss numerical implementation of the proposed non-linear algorithms. Section 4 demonstrates the applicability of our results through numerical examples. Concluding remarks are given in Section 5.

2. THE OFDM MODEL

We briefly introduce the channel model for an OFDM system with pilot-symbol-aided channel estimator and describe the corresponding channel statistics.

2.1. Channel Model

Consider an OFDM system that consists of n_{s} subcarriers and n_{r} receiving antennas. Each transmission subcarrier is modulated by a data symbol x_{jk}, where j represents the subcarrier index number, and k the time slot number (OFDM symbol number). The received signal in the frequency domain can be written as

$$\boldsymbol{y}_{jk} = \boldsymbol{h}_j x_{jk} + \sum_{q=1}^{n_{\mathrm{i}}} \boldsymbol{u}_{jk}^q + \boldsymbol{e}_{jk} \qquad (1)$$

where $\boldsymbol{y}_{jk}$ is the n_{r}-dimensional received signal vector at the jth subcarrier and kth time slot, $\boldsymbol{h}_j$ the n_{r}-dimensional user channel vector at the jth subcarrier, n_{i} is the number of interferers, $\boldsymbol{u}_{jk}^q, q = 1, \ldots, n_{\mathrm{i}}$ the qth unknown interference vector at the jth subcarrier and kth time slot, and $\boldsymbol{e}_{jk}$ is additive white Gaussian noise. Observe that the subscript j denotes frequency dependence and k denotes time dependence. In the frequency domain, the user channel vector is the discrete Fourier transform of the user channel impulse response. In particular,

$$\boldsymbol{h}_j = \sum_{i=1}^{n_{\mathrm{m}}} \boldsymbol{\alpha}_l e^{-2\iota\pi B(j-1-n_{\mathrm{s}}/2)\tau_l/n_{\mathrm{s}}}, \qquad (2)$$

where n_{m} is the number of multipaths, $\{\boldsymbol{\alpha}_l\}$ are the path complex amplitudes, ι is the imaginary unit, B is the bandwidth, and τ_l are the path delays.

Unlike the synchronous case in [1], [6], the asynchronous interference cannot be represented as a product of the interferer's channel and interferer's data symbol sequence. Consequently, on the receiver side the interference becomes completely random although the data symbols come from a finite alphabet. Therefore, we combine the interference and additive noise into one random vector yielding

$$\boldsymbol{\epsilon}_{jk} = \boldsymbol{e}_{jk} + \sum_{q=1}^{n_{\mathrm{i}}} \boldsymbol{u}_{jk}^q. \qquad (3)$$

2.2. Statistical Model

In order to derive the MLE for the above OFDM model we first develop a statistical model of the received signal as a function of the channel and interference parameters. Our goal is to develop a model that will include the unknown random effect due to the presence of unknown interference and enable statistically efficient channel estimation using a small number of pilots.

We start by assuming that both the interference and ambient noise are zero-mean Gaussian wide-sense stationary (WSS) random processes. In addition, we assume that the ambient noise is uncorrelated in space and time according to:

$$\mathrm{E}[\boldsymbol{e}_{jk}\boldsymbol{e}_{j'k'}^H] = \sigma^2\delta(j-j')\delta(k-k')I_{n_{\mathrm{r}}} \qquad (4)$$

where the superscript "^{H}" denotes the Hermitian transpose. We assume that the interference is spatially correlated according to

$$\mathrm{E}[\boldsymbol{u}_{jk}^q(\boldsymbol{u}_{jk}^q)^H] = \vartheta_{j,q}\delta(q-q')\delta(j-j')\delta(k-k'). \qquad (5)$$

Observe that in the above model σ^2 and ϑ_j are not identifiable. Therefore, the second order statistical properties are completely described by the spatial covariances of the residual vectors i.e.

$$\Sigma_j = \mathrm{E}[\boldsymbol{\epsilon}_{jk}\boldsymbol{\epsilon}_{jk}^H] = \sigma^2 I + \sum_{q=1}^{n_{\mathrm{i}}} \vartheta_{j,q}. \qquad (6)$$

The asynchronous model (6) has $n_{\mathrm{s}}n_{\mathrm{r}}(n_{\mathrm{r}}+1)/2$ unknown covariance parameters which is larger than the number of parameters in the synchronous model presented in [1]. However, (6) has the full rank interference structure and is thus more adequate in the presence of asynchronous interference.

In order to further decrease the number of unknown parameters and linearize the model, we propose to exploit the structure of the channel vectors $\boldsymbol{h}_j$. We will model the user channel using a set of *a priori* known basis functions and unknown corresponding coefficients, i.e., we substitute the actual wireless system with an *equivalent* system in which the number of multipaths and corresponding delays are *a priori* known, i.e.

$$\boldsymbol{h}_j \approx \sum_{l=1}^{n_{\mathrm{b}}} \tilde{\boldsymbol{\alpha}}_l e^{-2\iota\pi B(j-1-n_j/2)\tilde{\tau}_l/n_{\mathrm{s}}}. \qquad (7)$$

where $n_{\rm b}$ is the number of basis functions, $\tilde{\boldsymbol{\alpha}}_l$ are the unknown multipath amplitudes in the approximate model, and $\tilde{\tau}_l$ are the known path delays $\tilde{\tau}_l = l\tau_{\max}/n_{\rm b}$ where $\tau_{\max}$ is the maximum delay.

Using (7) we can model the desired user's channel as

$$\boldsymbol{h}_j = H_j\boldsymbol{\theta} \tag{8}$$

where $\boldsymbol{\theta}$ is the unknown coefficients vector, and

$$\begin{aligned} H_j &= \boldsymbol{f}_j^H \otimes I_{n_{\rm r}} \\ \boldsymbol{f}_j &= [e^{i2\pi\tau_1 jB/n_{\rm s}}, e^{i2\pi\tau_2 jB/n_{\rm s}}, \ldots, e^{i2\pi\tau_l jB/n_{\rm s}}]^H \end{aligned}$$

where $\otimes$ is the Kronecker product.

3. FREQUENCY DOMAIN CHANNEL ESTIMATION

The maximum likelihood (ML) estimates are given by

$$\begin{aligned} \hat{\boldsymbol{\theta}} &= \left(\sum_{j=1}^{n_{\rm s}} H_j^H \hat{\Sigma}_j^{-1} H_j\right)^{-1} \sum_{j=1}^{n_{\rm s}} H_j^H \hat{\Sigma}_j^{-1} \sum_{k=1}^{n_{\rm p}} x_{jk}^* \boldsymbol{y}_{jk} \\ \hat{\Sigma}_j &= \frac{1}{n_{\rm p}} \sum_{k=1}^{n_{\rm p}} \boldsymbol{e}_{jk}(\hat{\boldsymbol{\theta}}) \boldsymbol{e}_{jk}^H(\hat{\boldsymbol{\theta}}) \\ \boldsymbol{e}_{jk}(\boldsymbol{\theta}) &= \boldsymbol{y}_{jk} - x_{jk} H_j \boldsymbol{\theta} \end{aligned} \tag{9}$$

To obtain the estimates (9) we implement an iterative algorithm. At the step $n = -1, 0, 1, 2, \ldots$ we compute

$$\begin{aligned} \boldsymbol{\theta}^{(n)} &= \left(\Psi^{(n)}\right)^{-1} \sum_{j=1}^{n_{\rm s}} H_j^H ({\Sigma_j}^{(n)})^{-1} \sum_{k=1}^{n_{\rm p}} x_{jk}^* \boldsymbol{y}_{jk} \\ \Psi^{(n)} &= \left(\sum_{j=1}^{n_{\rm s}} H_j^H ({\Sigma_j}^{(n)})^{-1} H_j\right)^{-1} \\ {\Sigma_j}^{(n+1)} &= \frac{1}{n_{\rm p}} \sum_{k=1}^{n_{\rm p}} \boldsymbol{e}_{jk}(\boldsymbol{\theta}^{(n)}) \boldsymbol{e}_{jk}^H(\boldsymbol{\theta}^{(n)}). \end{aligned} \tag{10}$$

The above algorithm can be initialized with the ordinary least squares estimate $\boldsymbol{\theta}^{(-1)} = \hat{\boldsymbol{\theta}}_{\rm OLS}$ Alternatively, we can obtain a better initial estimate using

$$\Sigma_j^{(-1)} = \frac{1}{n_{\rm p}} Y_j' [I - H_j(H_j^H H - j) H_j^H] Y_j \tag{11}$$

$$Y_j = [\boldsymbol{y}_{j1}, \cdots, \boldsymbol{y}_{1n_{\rm p}}] \tag{12}$$

which is the MLE of Σ_j when only the pilot measurements from the jth frequency slot are used.

Since our interest lies primarily in drawing inference about $\boldsymbol{\theta}$ we propose a simpler estimation scheme based on the MLE for $\boldsymbol{\theta}$ and σ^2, and MM estimator for $\Sigma_j, j = 1, \ldots, n_{\rm s}$. This approach provides a non-iterative estimates of the covariance parameters without sacrificing any efficiency in the estimation of $\boldsymbol{\theta}$, see [9].

The new objective function is the same as for the MLE except that Σ_j is replaced by any consistent estimator of Σ_j. In the remainder of the paper we will refer to this method as asymptotic MLE. The asymptotic MLE of $\boldsymbol{\theta}$ is then given by

$$\hat{\boldsymbol{\theta}} = \left(\sum_{j=1}^{n_{\rm s}} H_j^H \hat{\Sigma}_j^{-1} H_j\right)^{-1} \sum_{j=1}^{n_{\rm s}} H_j^H \hat{\Sigma}_j^{-1} \sum_{k=1}^{n_{\rm p}} x_{jk}^* \boldsymbol{y}_{jk} \tag{13}$$

which is equivalent to the true MLE whenever $\hat{\Sigma}_j$ is the MLE. Furthermore, since $\hat{\Sigma}_j \to \Sigma_j$ as $n_{\rm p} \to \infty$ the asymptotic MLE can be shown to be asymptotically equivalent to the MLE.

We derive a non-iterative algorithm by exploiting the fact that $H_j = I \otimes f_j^H$ which yields

$$\begin{aligned} \widehat{\Sigma}_{j\rm RMM} &= S - \frac{1}{n_{\rm s} n_{\rm k}} \sum_{k=1}^{n_{\rm p}} \left[1 - \boldsymbol{f}_j^H (H^H H)^{-1} \boldsymbol{f}_j\right] \\ H &= [\boldsymbol{f}_1 \ldots \boldsymbol{f}_{n_{\rm s}}]^H \\ S &= Y\left(I - H(H^H H)H^H)\right) Y^H. \end{aligned} \tag{14}$$

A major drawback of this estimation method is that on occasion, it can result in a non-positive semi-definite estimate of Σ_j. To ensure having a positive semi-definite estimate of Σ_j we suggest using a modified asymptotic MLE

$$\tilde{\Sigma}_{j\rm RMM} = S - \min(1, \lambda) \sum_{k=1}^{n_{\rm p}} \left[1 - \boldsymbol{f}_j^H (H^H H)^{-1} \boldsymbol{f}_j\right] \tag{15}$$

where λ is the smallest root of

$$|S - \lambda \sum_{k=1}^{n_{\rm p}} \left[1 - \boldsymbol{f}_j^H (H^H H)^{-1} \boldsymbol{f}_j\right]| = 0. \tag{16}$$

Observe that we derived above estimators assuming $H_j = I \otimes f_j^H$. However, the method can be easily extended to the general case by simply noting that for an arbitrary H_j there exists vector $\boldsymbol{f}_j$ and an incidence matrix Q of zeroes and ones such that$H_j = (I \otimes \boldsymbol{f}_j)Q$. Then it can be shown that, see [1], the RMM estimator (14) will remain the unbiased estimator of Σ_j and that it is invariant to $\boldsymbol{\theta}$.

4. NUMERICAL EXAMPLES

We demonstrate the performance of the proposed estimators by numerical examples. In all examples, unless otherwise stated, we assume $n_{\rm r} = 4$ element antenna array and a 16-QAM signal occupying a $B = 6.0$MHz bandwidth. The

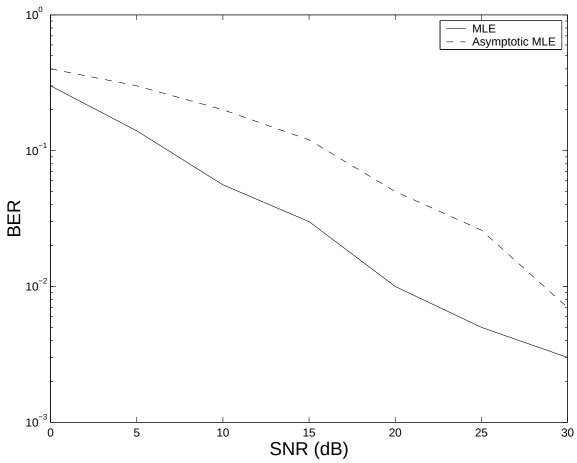

Fig. 1. BER performance comparison

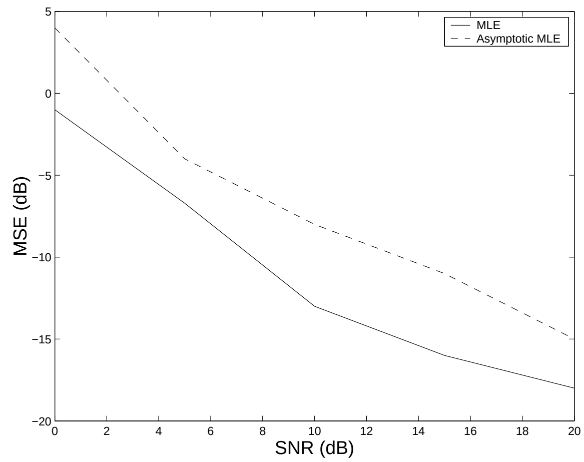

Fig. 2. MSE performance comparison

number of pilot bauds is $n_{\rm p} = 11$ with pilot symbols on all subcarriers ($n_{\rm s} = 400$). The number of basis functions for both interpolation matrices is $n_{\rm b} = 40$ (10 for each antenna). The maximum multipath delay is $\tau_{\rm max} = 8.0\mu$s. We define the signal-to-interference (SIR) as the ratio of signal power to interference and additive noise power and set it to 5dB. The number of multipaths is set to $n_{\rm p} = 100$ with an exponential power delay profile. The phase shift on each path is uniformly distributed over $[0, 2\pi)$.

Figure 1 illustrates the bit-error-rate (BER) performance of the proposed asymptotic MLE and MLE as a function of SNR. We observe that the difference in the detection performance of these two estimators is acceptable considering the difficulties with embedded software implementation of iteratively computed MLE. In Figure 2 we illustrate the mean-square-error (MSE) of the MLE and the asymptotic MLE estimator as a function of SNR computed using 5000 trials. We define MSE as $\frac{1}{n_{\rm se}} \sum_{j=1}^{n_{\rm s}} ||\boldsymbol{h}_j - \hat{\boldsymbol{h}}_j||$.

5. CONCLUSIONS

We have proposed pilot-aided channel estimation algorithms for estimating OFDM wireless channels in the presence of asynchronous interference. To reduce the number of unknown parameters (and keep the detection efficiency) we modeled the statistical properties of the interference using full-rank subcarrier spatial covariance matrices. We have derived MLE and asymptotic MLE algorithms for estimating channel parameters and subcarrier spatial covariances. The asymptotic MLE estimates the unknown interference parameters using a residual method of moments estimator invariant to the user channel parameters. We also compared the performance of the asymptotic MLE and ordinary MLE, and showed that the performance of the asymptotic MLE is comparable to the performance of ordinary MLE.

6. REFERENCES

[1] A. Jeremic, T. Thomas, and A. Nehorai, "OFDM channel estimation in the presence of interference," *submitted.*

[2] C. Tepedelenlioglu, and G. B. Giannakis, "Trasmitter redundancy for blind estimation and equalization of time- and frequency-selective channels," *IEEE Transactions on Signal Processing,* Vol. 48, July 2000.

[3] B. Muquet, M. de Courville, and P. Duhamel, "Subspace-based blind and semi-blind channel estimation for OFDM systems," *IEEE Transactions on Signal Processing,* Vol. 50, July 2002.

[4] J. van de Beek, O. Eldfors, M. Sandell, S. K. Wilson, and P. O. Borjesson, "On channel estimation in OFDM systems," *IEEE VTC-1995,* pp. 815-819, Chicago, IL.

[5] Y. Song, and S. D. Blostein, "Data detection in MIMO systems with co-channel interference," *IEEE VTC*, Vancouver, 2002.

[6] A. Jeremic, T. Thomas, and A. Nehorai, "OFDM channel estimation in the presence of interference using an approximate maximum likelihood approach," *Workshop on Signal Processing, Communications, Chaos and Systems*, Newport, RI, June 2002, pp. 107-116.

[7] S. B. Weinstein and P. M. Ebert, "Data transmission by frequency-division multiplexing using the discrete fourier transform," *IEEE Trans. Commun.,* vol. COMM-19, pp. 628–634, Oct. 1971.

[8] T. A. Thomas, F. W. Wook, and K. L. Baum, "Least-squares multi-user frequency-domain channel estimation for broadband wireless systems," *Proc. 37th Annual Allerton Conference on Comm., Control and Computing,* pp. 1026–1035, Monticello, IL, Sept. 1999.

[9] E. F. Vonesh and R. L. Carter, "Effi cient inference for a random coeffi cient growth curve model with unbalanced data," *Biometrics,* Vol. 43, pp. 617-628, 1987.