Abstract

We develop a frequency-domain channel estimation algorithm for single-user orthogonal frequency division multiplex (OFDM) wireless systems in the presence of asynchronous interference. In this case, the received measurement is correlated in space with covariance matrix dependent on frequency. Hence, the commonly used least-squares algorithm is suboptimal. On the other hand, accurate estimation of the spatial covariance matrix in such a model using the unstructured model would impose significant computational overhead, since it would require large number of pilot symbols. To overcome these problems, we employ the structured model with smaller number of parameters. We estimate the interference covariance parameters using a residual method of moments (RMM) estimator and the mean (user channel) parameters by maximum likelihood (ML) estimation. Since our RMM estimates are invariant to the mean, this approach yields simple non-iterative estimates of the covariance parameters while having optimal statistical efficiency. Therefore, our algorithm outperforms the least-squares method in accuracy, and at the same time requires smaller number of pilots than the unstructured method and thus has smaller overhead. Numerical results illustrate the applicability of the proposed algorithm.

1 Introduction

Orthogonal frequency division multiplexing (OFDM) has received considerable interest over the last few years for its advantages in high-bit-rate transmissions over frequency selective fading channels. In OFDM systems, the input high-rate data stream is divided into many low-rate streams [3] that are transmitted in parallel, thereby increasing the symbol duration and reducing the intersymbol interference (ISI). These features have motivated the adoption of OFDM as a standard for digital audio broadcasting, digital video broadcasting, and broadband indoor wireless systems.

Coherent OFDM transmission invariably requires an estimation of the channel frequency response (i.e. the gains of the OFDM tones). Currently, there are two different types of channel parameter estimators: (i) blind and (ii) pilot-aided. Blind channel estimation techniques try to estimate the channel without any knowledge of the transmitted data. They are attractive because of the possible savings in training overhead, however they are effective only when a large amount of data can be collected (so that stochastic estimation can be made reliably).

This is clearly a disadvantage in the case of mobile wireless systems because of the time-varying nature of the channel. Pilot-aided channel estimation is the other type of approach in which training sequence consisting of known data symbols (pilots) is transmitted at the beginning of a session (or multiplexed into the user data stream at a later stage) and the initial estimation of the channel parameters is performed using the received pilot signal.

In this paper we present a frequency domain channel estimation in the presence of asynchronous interference using a pilot-aided algorithm. Interference suppression is of utmost importance in high-rate (high capacity) wireless systems. In the presence of interference the received signal, in addition to multipath fading, may have heteroskedastic properties, such as frequency dependent covariance which means that commonly used least-squares algorithms are suboptimal. Sufficiently accurate estimation of the covariance matrices, necessary for efficient interference suppression, in such a model would impose significant training overhead since the estimation requires large number of pilot symbols. In [1], [2] we presented frequency domain channel estimation algorithm in the presence of synchronous interference. We also demonstrated that when the asynchronous interference is present the performance of the proposed estimator can deteriorate significantly. To overcome these problems, in this paper we present the asynchronous structured model adequate for estimating channel and interference parameters in the presence of asynchronous interference. The asynchronous model has larger number of parameters than the synchronous model presented in [1], [2] but gives better performance in the asynchronous case. Similar to [1], [2] we propose estimating the unknown parameters using two estimators: (i) computationally intensive MLE and (ii) an asymptotic MLE in which the non-linear covariance parameters are estimated using a method of moments (MM) estimator.

In Section 2, we briefly describe the OFDM system with pilot-symbol-aided channel estimation and derive the statistical model. In Section 3, we present the MLE and asymptotic MLE and discuss numerical implementation of the proposed non-linear algorithms. Section 4 demonstrates the applicability of our results through numerical examples. Concluding remarks are given in Section 5.

2 The OFDM Model

In this section we briefly introduce the channel model for an OFDM system with pilot-symbol-aided channel estimator and describe the corresponding channel statistics. Our goal is to develop a model that will include unknown random effects due to a presence of unknown interference and enable statistically efficient channel estimation using sufficiently small number of pilots.

2.1 Channel Model

Consider an OFDM system that consists of n_{s} subcarriers and n_{r} receiving antennas. Each transmission subcarrier is modulated by a data symbol x_{jk}, where j represents the subcarrier index number, and k the time slot number (OFDM symbol number).

Similar to [1], [2], the received signal in the frequency domain can be written as

$$\boldsymbol{y}_{jk} = \boldsymbol{h}_j x_{jk} + \sum_{q=1}^{n_{\mathrm{i}}} \boldsymbol{u}_{jk}^q + \boldsymbol{e}_{jk} \tag{2.1}$$

where $\boldsymbol{y}_{jk}$ is the n_{r}-dimensional received signal vector at the jth subcarrier and kth time slot, $\boldsymbol{h}_j$ the n_{r}-dimensional user channel vector at the jth subcarrier, n_{i} is the number of interferers, $\boldsymbol{u}_{jk}^q, q = 1, \ldots, n$ the qth unknown interference vector at the jth subcarrier and kth time slot, and $\boldsymbol{e}_{jk}$ is additive white Gaussian noise. Observe that we use the subscript j to denote frequency dependence and k to denote time dependence. In the frequency domain, the user channel vector is the Discrete Fourier Transform of the user channel impulse response, see [1]. In particular,

$$\boldsymbol{h}_j \;=\; \sum_{i=1}^{n_{\mathrm{m}}} \boldsymbol{\alpha}_l e^{-2\iota\pi B(j-1-n_{\mathrm{s}}/2)\tau_l/n_{\mathrm{s}}}, \tag{2.2}$$

where n_{m} is the number of multipaths, $\{\boldsymbol{\alpha}_l\}$ are the path complex amplitudes, ι is the imaginary unit, B is the bandwidth, and τ_l are the path delays.

Unlike the synchronous case discussed in [1], [2], the asynchronous interference cannot be represented as a product of the interferer channel and the interferer's data symbol sequence.

Consequently, on the receiver side the interference becomes completely random although the data symbols come from a finite alphabet. Therefore we combine interference and additive noise into one random vector yielding

$$\boldsymbol{y}_{jk} = \boldsymbol{h}_j x_{jk} + \boldsymbol{\epsilon}_{jk} \tag{2.3}$$

where

$$\boldsymbol{\epsilon}_{jk} = \boldsymbol{e}_{jk} + \sum_{q=1}^{n_{\mathrm{i}}} \boldsymbol{u}_{jk}^{q}. \tag{2.4}$$

2.2 Statistical Model

In order to derive the MLE for the above OFDM model we first develop a statistical model of the received signal as a function of the channel and interference parameters. Our goal is to develop a model which will include the unknown random effect due to the presence of unknown interference and enable statistically efficient channel estimation using a small number of pilots.

We start by assuming that both the interference and ambient noise are zero-mean Gaussian wide-sense stationary (WSS) random processes. In addition, we assume that the ambient noise is uncorrelated in space and time according to:

$$\mathrm{E}[\boldsymbol{e}_{jk}\boldsymbol{e}_{j'k'}^{H}] = \sigma^2 \delta(j-j')\delta(k-k') I_{n_{\mathrm{r}}} \tag{2.5}$$

where the superscript ^{H} denotes the Hermitian transpose. We assume that the interference is spatially correlated according to

$$\mathrm{E}[\boldsymbol{u}_{jk}^{q}(\boldsymbol{u}_{jk}^{q})^{H}] = \vartheta_j \delta(q-q')\delta(j-j')\delta(k-k') \tag{2.6}$$

Observe that in the above model σ^2 and ϑ_j are not identifiable. Therefore the second order statistical properties are completely described by the spatial covariances of the residual vectors i.e.

$$\Sigma_j = \mathrm{E}[\boldsymbol{\epsilon}_{jk}\boldsymbol{\epsilon}_{jk}^{H}] = \sigma^2 I + n_{\mathrm{i}} \vartheta_j. \tag{2.7}$$

The asynchronous model (2.7) has $n_{\mathrm{s}}n_{\mathrm{r}}(n_{\mathrm{r}}+1)/2$ unknown covariance parameters which is larger than the number of parameters in the synchronous model presented in [1]. However, the model (2.7) is the full rank interference model and thus is more adequate for channel estimation in the presence of asynchronous interference.

In order to further decrease the number of unknown parameters and linearize the model we propose to exploit the structure of channel vectors $\boldsymbol{h}_j$. Following the approach of [1] we model the user channel using set of *a priori* known basis functions and the unknown corresponding coefficients. To choose an adequate basis-function model we observe that the main difficulty with estimating the unknown parameters in models (2.2) is the unknown, possibly large, number of multipaths and the corresponding multipath delays. Also, note that if several multipaths have the same delay they will be represented with only one term in the summation (2.2). Therefore, we propose to substitute the *real* wireless system with an *equivalent* system in which the number of multipaths and corresponding delays are *a priori* known, i.e.

$$\boldsymbol{h}_j \approx \sum_{l=1}^{n_{\mathrm{b}}} \tilde{\boldsymbol{\alpha}}_l e^{-2\iota\pi B(j-1-n_j/2)\tilde{\tau}_l/n_{\mathrm{s}}}. \tag{2.8}$$

where n_{b} is the number of basis functions, $\tilde{\boldsymbol{\alpha}}_l$ are the unknown multipath amplitudes in the approximate model, and $\tilde{\tau}_l$ are the known path delays $\tilde{\tau}_l = l\tau_{\max}/n_{\mathrm{b}}$ where $\tau_{\max}$ is the maximum delay.

Using (2.8) we can model the user channel as:

$$\boldsymbol{h}_j = H_j\boldsymbol{\theta} \tag{2.9}$$

where $\boldsymbol{\theta}$ are the unknown coefficients vector, and

$$\begin{aligned} H_j &= \boldsymbol{f}_j^H \otimes I_{n_{\mathrm{r}}} \\ \boldsymbol{f}_j &= [e^{i2\pi\tau_1 jB/n_{\mathrm{s}}}, e^{i2\pi\tau_2 jB/n_{\mathrm{s}}}, \ldots, e^{i2\pi\tau_l jB/n_{\mathrm{s}}}]^H \end{aligned}$$

where $\otimes$ is the Kronecker product.

3 Frequency Domain Channel Estimation

The likelihood function for the above model is given by

$$L(\boldsymbol{y}|\boldsymbol{\theta},\Sigma) \;=\; -\frac{1}{2}\sum_{j=1}^{n_{\rm s}}\sum_{k=1}^{n_{\rm p}}\left[\Big(\boldsymbol{y}_{jk}-x_{jk}H_j\boldsymbol{\theta}\Big)^H\Sigma_j^{-1}\Big(\boldsymbol{y}_{jk}-x_{jk}H_j\boldsymbol{\theta}\Big)+\log|\Sigma_j|\right] \tag{3.10}$$

The maximum likelihood (ML) estimates are given by

$$\hat{\boldsymbol{\theta}} \;=\; \left(\sum_{j=1}^{n_{\rm s}}H_j^H\hat{\Sigma}_j^{-1}H_j\right)^{-1}\sum_{j=1}^{n_{\rm s}}H_j^H\hat{\Sigma}_j^{-1}\sum_{k=1}^{n_{\rm p}}x_{jk}^*\boldsymbol{y}_{jk} \tag{3.11}$$

$$\hat{\Sigma}_j \;=\; \frac{1}{n_{\rm p}}\sum_{k=1}^{n_{\rm p}}(\boldsymbol{y}_{jk}-H_j\hat{\boldsymbol{\theta}}x_{jk})(\boldsymbol{y}_{jk}-H_j\hat{\boldsymbol{\theta}}x_{jk})^H \tag{3.12}$$

To obtain the estimates (3.11) and (3.12) we implement an iterative algorithm

$$\boldsymbol{\theta}^n \;=\; \left(\sum_{j=1}^{n_{\rm s}}H_j^H\left({\Sigma_j}^{(n)}\right)^{-1}H_j\right)^{-1}\sum_{j=1}^{n_{\rm s}}H_j^H\left({\Sigma_j}^{(n)}\right)^{-1}\sum_{k=1}^{n_{\rm p}}x_{jk}^*\boldsymbol{y}_{jk} \tag{3.13}$$

$${\Sigma_j}^{(n+1)} \;=\; \frac{1}{n_{\rm p}}\sum_{k=1}^{n_{\rm p}}(\boldsymbol{y}_{jk}-H_j\boldsymbol{\theta}^{(n)}x_{jk})(\boldsymbol{y}_{jk}-H_j\boldsymbol{\theta}^{(n)}x_{jk})^H \tag{3.14}$$

The above algorithm can be initialized with $\Sigma_j^{(-1)}=0$, for $j-1,\ldots,n_{\rm s}$ implying that $\boldsymbol{\theta}^{(=1)}$ is simply the ordinary least squares estimate. Alternatively, we can obtain a better initial estimate using

$$\Sigma_j^{(-1)} \;=\; \frac{1}{n_{\rm p}}Y_j'[I-H_j(H_j^HH-j)H_j^H]Y_j \tag{3.15}$$

$$Y_j=[\boldsymbol{y}_{j1},\cdots,\boldsymbol{y}_{1n_{\rm p}}] \tag{3.16}$$

which is the ML estimate of Σ_j when only the pilot measurements from the jth frequency slot are used.

Since our interest lies primarily in drawing inference about $\boldsymbol{\theta}$ (and subsequently x_{jk}) we propose a simpler estimation scheme based on the maximum likelihood method for $\boldsymbol{\theta}$ and

σ^2, and method of moments estimator (MM) for $\boldsymbol{\eta}$. This approach provides a simple non-iterative estimates of the variance-covariance parameters without sacrificing any efficiency in the estimation of $\boldsymbol{\theta}$, see [5].

The new objective function is the same as for the MLE except that Σ_j is replaced by any consistent estimator of Σ_j. In the remainder of the paper we will refer to this method as asymptotic ML. The asymptotic MLE of $\boldsymbol{\theta}$ is then given by

$$\hat{\boldsymbol{\theta}} = \left(\sum_{j=1}^{n_\mathrm{s}} H_j^H \hat{\Sigma}_j^{-1} H_j \right)^{-1} \sum_{j=1}^{n_\mathrm{s}} H_j^H \hat{\Sigma}_j^{-1} \sum_{k=1}^{n_\mathrm{p}} x_{jk}^* \boldsymbol{y}_{jk} \tag{3.17}$$

which is equivalent to the MLE whenever $\hat{\Sigma_j}$ is the MLE. Furthermore, since $\hat{\Sigma_j} \to \Sigma_j$ as $n_\mathrm{p} \to \infty$ the asymptotic MLE can be shown to be asymptotically equivalent to the MLE.

Similar to [1], by exploiting the fact that $H_j = I \otimes f_j^H$ we obtain the unknown parameters using a non-iterative the RMM estimate [1]

$$\widehat{\Sigma_j}_{\mathrm{RMM}} = S - \frac{1}{n_\mathrm{s} n_\mathrm{k}} \sum_{k=1}^{n_\mathrm{p}} \left[1 - \boldsymbol{f}_j^H (H^H H)^{-1} \boldsymbol{f}_j \right] \tag{3.18}$$

where

$$H = [\boldsymbol{f}_1 \dots \boldsymbol{f}_{n_\mathrm{s}}]^H, \tag{3.19}$$

$$S = Y \left(I - H(H^H H) H^H \right) Y^H \tag{3.20}$$

A major drawback with this method of estimation is that on occasion, it can result in a non-positive semi-definite estimate of Σ_j. To ensure having a positive semi-definite estimate of Σ_j we suggest using the modified asymptotic MLE

$$\tilde{\Sigma_j}_{\mathrm{RMM}} = S - \min(1, \lambda) \sum_{k=1}^{n_\mathrm{p}} \left[1 - \boldsymbol{f}_j^H (H^H H)^{-1} \boldsymbol{f}_j \right] \tag{3.21}$$

where λ is the smallest root of

$$|S - \lambda \sum_{k=1}^{n_\mathrm{p}} \left[1 - \boldsymbol{f}_j^H (H^H H)^{-1} \boldsymbol{f}_j \right]| = 0. \tag{3.22}$$

Observe that we derived above estimators assuming $H_j = I \otimes f_j^H$. However, the method can be easily extended to the general case by simply noting that for an arbitrary H_j there exists vector $\boldsymbol{f}_j$ and an incidence matrix Q of $0's$ and $1's$ such that

$$H_j = (I \otimes \boldsymbol{f}_j)Q. \tag{3.23}$$

Then it can be shown that, see [1], the RMM estimator (3.18) will remain the unbiased estimator of Σ_j and that it is invariant with respect to $\boldsymbol{\theta}$.

4 Numerical Examples

We demonstrate the performance of the proposed estimator by numerical examples. In all examples, unless otherwise stated, we assume $n_{\mathrm{r}} = 4$ element antenna array and a 16-QAM signal occupying a $B = 6.0$MHz bandwidth. The number of pilot bauds is $n_{\mathrm{p}} = 3$ with pilot symbols on all subcarriers. The number of basis functions for both interpolation matrices is $n_{\mathrm{b}} = 40$ (10 for each antenna). The maximum multipath delay is $\tau_{\max} = 8.0\mu$s. We define the signal-to-interference (SIR) as the ratio of signal power to interference and additive noise power and set it to 15dB. The number of multipaths is set to $n_{\mathrm{p}} = 100$ with an exponential power delay profile. The phase shift on each path is uniformly distributed over $[0, 2\pi)$.

Figure 1 illustrates the bit-error-rate (BER) performance of the proposed asymptotic MLE and MLE as a function of SIR. We observe that the difference in the detection performance of these two estimators is acceptable considering the difficulties with embedded software implementation of iteratively computed MLE. In Figure 2 we illustrate the mean-square-error (MSE) of the asymptotic MLE estimator as a function of n_{p} computed using 5000 trials. We define MSE as

$$\frac{1}{n_{\mathrm{se}}} \sum_{j=1}^{n_{\mathrm{s}}} \|\boldsymbol{h}_j - \hat{\boldsymbol{h}}_j. \tag{4.24}$$

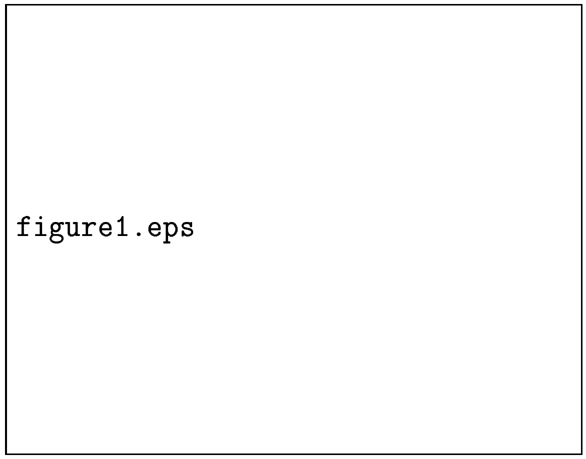

Figure 1:

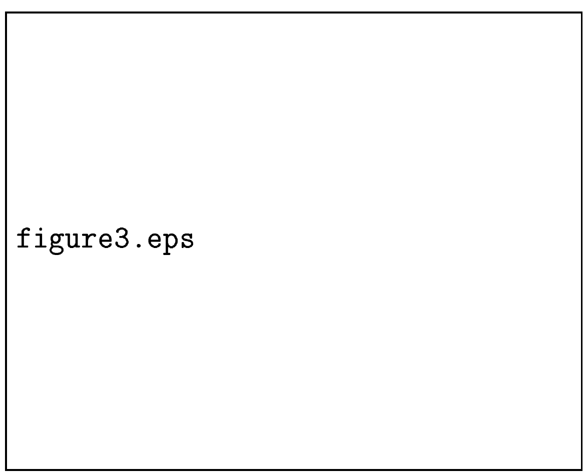

Figure 2:

5 Conclusions

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