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Optimal Diagonal Power Loading Minimizing Asymptotic Symbol Error Probability for Distributed MIMO Systems with ZF Receivers

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***Abstract*—In this paper, we consider a distributed multiple-input multiple-output (D-MIMO) system, where the channel is flat fading and may be correlated, and experiences both small and large-scale fading. We assume that full knowledge of channel state information (CSI) is available at the receiver and only the first-and second-order statistics of the channel are available at the transmitter. For such a system with square quadrature amplitude modulation (QAM), an asymptotic symbol error probability (SEP) is derived for the linear zero-forcing (ZF) receiver. Then, we propose an optimal diagonal power-loading (PL) strategy that minimizes the dominant term of the asymptotic SEP subject to either a total transmission power constraint when the total power normalization coefficient can be fed back to the transmitter from the receiver or an individual transmission power constraint. A simple closed-form solution is obtained. Computer simulations show that our presented optimal system attains significant performance gains over the currently available equal power-loading system.**

***Index Terms*—Distributed MIMO, large scale fading, zero-forcing (ZF), asymptotic symbol error probability (SEP), power-loading.**

I. Introduction

THE distributed MIMO system has become a promising candidate for future mobile communication systems thanks to its open architecture and flexible resource management [1]–[6]. Over the past several years, many research activities on distributed multiple-input multiple-output (D-MIMO) system have been intensified for the fast growing demand for high data-rate services [3]–[5]. A great deal of efforts have been made on the performance evaluation of D-MIMO system [5]–[19]. Contrary to conventional point-to-point MIMO systems, many radio antenna ports are geographically distributed over a large area and experience different path-loss and large-scale fading effects. Hence, each link experiences different propagation paths, as a result of the different access distances, along with different shadowing effects, which makes the performance analysis of D-MIMO systems a mathematically challenging problem [8]–[19].

In the analysis of composite fading channels, the Rayleigh/lognormal (RLN) model is the commonly-used model in the characterization of composite fading MIMO channels [15], [16], [19]. Since its joint probability density function (PDF) is not in closed-form, some most important figures of merit such as the ergodic capacity and SEP are difficult to evaluate [19]. On the other hand, due to the relatively-low implementation complexity of ZF receivers, researchers have continued seeking to evaluate its performance for such systems [19], [20].

In this paper, we are interested in the asymptotic average SEP analysis on the square q-QAM constellation as well as in optimal power allocation for the D-MIMO systems with composite fading MIMO channels employing the ZF receiver. The power allocation problem given channel statistics at the transmitter has been widely studied in literature under a variety of criteria including maximum ergodic capacity and minimum BER for point to point MIMO systems [21]–[23]. However, to the best of our knowledge, current work on this research topic for the DMIMO systems is focused on sum rate analysis. A few of studies closely related to our research have been recently reported in [15], [18], [20]. However, they considered an equal power allocation case and pulse amplitude modulation and phase-shift keying modulation. In addition, they proposed the use of the joint PDF of the ZF received SNR with respect to the composite random channels for their performance analysis and derived an exact and closed-form expression for the average SEP [15]. Unfortunately, the dominant term of the asymptotic average SEP is not easy to be extracted in a general case. In this paper, we fully make use of the transmitter structure as well as of the receiver structure and take an appropriate technical approach to the average SEP analysis by separating the joint expectation taken over both the small scale random fading and the large scale random fading into two successive individual expectations. With this idea, our principal task in this paper is to derive an asymptotic average SEP for the ZF receiver and that based on this, we propose

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an optimal diagonal power-loading strategy that minimizes its dominant term subject to either a total transmission power constraint and an individual transmission power constraint.

Notation: Matrices are denoted by uppercase boldface characters (e.g., $\mathbf{A}$), while column vectors are denoted by lowercase boldface characters (e.g., $\mathbf{b}$). The i-th entry of $\mathbf{b}$ is denoted by b_i. The (i,j)-th entry of $\mathbf{A}$ is denoted by $a_{i,j}$, and also denoted by $\left[\mathbf{A}\right]_{i,j}$. The columns of a $P\times K$ matrix $\mathbf{A}$ are denoted by $\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_K$. The transpose of $\mathbf{A}$ is denoted by $\mathbf{A}^T$. The Hermitian transpose of $\mathbf{A}$ (i.e., the conjugate and transpose of $\mathbf{A}$) is denoted by $\mathbf{A}^H$. An $L\times L$ identity matrix is denoted by $\mathbf{I}_L$.

II. Distributed MIMO Model and ZF Receivers

In this section, we consider a single-cell D-MIMO communication system equipped with N receiver antennas at the base station (BS) and K users with each having M transmitter antennas and also require that $N \geq KM$. As it was previously mentioned, we assume that the BS has perfect CSI while only the first-and second-order statistics of the channel are available at the transmitter, and all users simultaneously transmit their data streams to the BS. Then, for such a system, an input-output relationship can be written as

$$\mathbf{y} = \mathbf{T}\mathbf{s} + \mathbf{n}, \tag{1}$$

where $\mathbf{y}$ is an $N\times 1$ received signal vector, $\mathbf{s}$ is a $KM\times 1$ transmitted symbol vector with unit average power per element. $\mathbf{n}$ is $N\times 1$ circularly symmetric complex Gaussian noise vector with zero-mean and covariance matrix $\mathrm{E}[\mathbf{n}\mathbf{n}^H] = N_0\mathbf{I}_N$. Here, channel matrix $\mathbf{T}$ is composed of three factors, i.e., $\mathbf{T} = \mathbf{H}\boldsymbol{\Xi}^{1/2}\mathbf{P}^{1/2}$. Small-scale fading is captured by an $N\times KM$ random matrix $\mathbf{H}$, which is assumed to follow a complex zero-mean Gaussian distribution with correlation among every row, and thus, we have $\mathbf{H} = \mathbf{Z}\mathbf{R}_T^{1/2}$, where the entries of $\mathbf{Z}$ are modeled as i.i.d $CN(0,1)$ random variables and $\mathbf{R}_T$ is the $KM\times KM$ transmitted positive definite covariance matrix. Note that small-scale fading correlation occurs only between the antennas of the one user, since the different users are, in general, geographically separated. Such a scenario can occur when several users, with multiple spatially correlated antennas, transmit to a common multiple-antennas BS. The transmitted correlation matrix can be constructed as $\mathbf{R}_T = \mathrm{diag}(\mathbf{R}_{T1}, \mathbf{R}_{T2}, \cdots, \mathbf{R}_{TK})$, where $\mathbf{R}_{Tk}$ is the correlation matrix between the antennas of the k-th user for $k = 1, 2, \cdots, K$. Its entries are modeled via the common exponential correlation model [24], i.e.,

$$[\mathbf{R}_{Tk}]_{i,j} = \rho_k^{|i-j|}, \tag{2}$$

where $|\rho_k| \in [0,1)$ for $i,j = 1,2,\cdots,M$ being the transmitter correlation coefficient.

The entries of the $KM\times KM$ diagonal matrix $\boldsymbol{\Xi}$ represent the large-scale effects, i.e., $\boldsymbol{\Xi} = \mathrm{diag}(\Xi_1/D_1^v\mathbf{I}_M, \Xi_2/D_2^v\mathbf{I}_M, \cdots, \Xi_K/D_K^v\mathbf{I}_M)$, where each D_k for $k = 1,2,\cdots,K$ denotes the distance between the k-th user and the BS and v is the path-loss exponent. We consider the lognormal shadowing model, which is the commonly-used model in the characterization of shadowing effects in various radar, optical and RF wireless channels. In this scenario, the probability density function (PDF) of the large-scale fading coefficients Ξ_k for $k = 1,2,\cdots,K$ is given by

$$f_{\Xi}(\xi_k) = \frac{\eta}{\xi_k\sqrt{2\pi}\delta_k} e^{-\frac{(\eta\ln\xi_k - \mu_k)^2}{2\delta_k^2}}, \qquad \text{for } \xi_k \geq 0, \tag{3}$$

where $\eta = 10/\ln 10$, μ_k and δ_k are the mean and standard deviation (both in dB) of the variable's natural logarithm, respectively.

The $KM\times KM$ diagonal power loading matrix $\mathbf{P}$ determines the available power for each data stream, i.e, $\mathbf{P} = \mathrm{diag}(\mathbf{P}_1, \mathbf{P}_2, \cdots, \mathbf{P}_K)$, where $\mathbf{P}_k = \mathrm{diag}(p_{k1}, p_{k2}, \cdots, p_{kM})$ with a total transmission power constraint $\sum_{k=1}^{K}\sum_{m=1}^{M} p_{km} = 1$ and $p_{km} \geq 0$.

III. Performance Analysis and Optimization

Our main purpose in this section is to first derive an asymptotic SEP formula for the D-MIMO system as described in Section 1 using the ZF receiver and then, to optimize its dominant term subject to a total transmission power constraint.

A. Asymptotic SEP Analysis

To do that, we notice that the received signal after the ZF equalizer $\mathbf{G} = \left(\mathbf{T}^H\mathbf{T}\right)^{-1}\mathbf{T}^H$, which is used to recover the spatially multiplexed data streams, becomes

$$\widehat{\mathbf{s}} = \mathbf{G}\mathbf{y} = \mathbf{s} + \mathbf{G}\mathbf{n}. \tag{4}$$

Then, the received signal is decomposed into KM parallel data streams with the instantaneous received SNR at the m-th datastream of the k-th user ZF ($1\leq k\leq K, 1\leq m\leq M$) output being equal to

$$\gamma_{km} = \frac{\mathrm{SNR}}{\left[(\mathbf{T}^H\mathbf{T})^{-1}\right]_{km,km}} = \frac{\mathrm{SNR}\left[\boldsymbol{\Xi}\mathbf{P}\right]_{km,km}}{\left[(\mathbf{H}^H\mathbf{H})^{-1}\right]_{km,km}}, \tag{5}$$

where $\mathrm{SNR} = \frac{1}{N_0}$. Therefore, the SEP of the mth datastream of kth user for the square q-ary QAM constellation for the above given SNR can be expressed by

$$\begin{aligned}\mathrm{SEP}_{km} =& 4\left(1-\frac{1}{\sqrt{q}}\right)Q\left(d\sqrt{\frac{\gamma_{km}}{2}}\right)\\ &-4\left(1-\frac{1}{\sqrt{q}}\right)^2 Q^2\left(d\sqrt{\frac{\gamma_{km}}{2}}\right),\end{aligned}$$

where $d = \sqrt{\frac{3}{2(q-1)}}$ is the unit-energy factor for the square q-ary QAM constellation [25]. Therefore, an arithmetic mean SEP for the given channel realization is determined by

$$\mathrm{SEP}(\mathbf{H},\boldsymbol{\Xi}) = \frac{1}{KM}\sum_{k=1}^{K}\sum_{m=1}^{M}\mathrm{SEP}_{km}. \tag{6}$$

In order to evaluate the expectation of $\mathrm{SEP}(\mathbf{H},\boldsymbol{\Xi})$ over the random channels $\mathbf{H}$ and $\boldsymbol{\Xi}$, we use two alternative formulas for the Q-funcation and Q^2-function below:

$$Q(t) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{-\frac{t^2}{2\sin^2\theta}}d\theta, \quad Q^2(t) = \frac{1}{\pi}\int_0^{\frac{\pi}{4}} e^{-\frac{t^2}{2\sin^2\theta}}d\theta$$

so that each SEP_{km} can be represented by

$$\mathrm{SEP}_{km} = A_1 \int_0^{\frac{\pi}{2}} e^{-\frac{d^2\gamma_{km}}{4\sin^2\theta}} d\theta - A_2 \int_0^{\frac{\pi}{4}} e^{-\frac{d^2\gamma_{km}}{4\sin^2\theta}} d\theta, \quad (7)$$

where $A_1 = \frac{4}{\pi}(1-\frac{1}{\sqrt{q}})$, $A_2 = \frac{4}{\pi}(1-\frac{1}{\sqrt{q}})^2$. In addition, in order to further simplify the expectation, we also need the joint probability density function (PDF) of γ_{km} in terms of random variables $\mathbf{H}$ and $\mathbf{\Psi}$, which was the original idea from [15] dealing with PAM and PSK constellations. Here, we will take a significantly different approach. We separate the joint expectation of $\mathrm{SEP}(\mathbf{H},\mathbf{\Xi})$ taken over both $\mathbf{H}$ and $\mathbf{\Xi}$ into two successive individual expectations: the one being the conditional expectation of $\mathrm{SEP}(\mathbf{H},\mathbf{\Xi})$ taken over the random matrix $\mathbf{H}$ first given $\mathbf{\Xi}$ and the other being the expectation of the resulting conditional expectation taken over $\mathbf{\Xi}$. More clearly, that is:

$$\mathrm{E}_{\mathbf{H},\mathbf{\Xi}}\big[\mathrm{SEP}(\mathbf{H},\mathbf{\Xi})\big] = \mathrm{E}_{\mathbf{\Xi}}\Big[\mathrm{E}_{\mathbf{H}}\big[\mathrm{SEP}(\mathbf{H},\mathbf{\Xi})|\mathbf{\Xi}\big]\Big]. \quad (8)$$

The essential reason of why this approach successfully works is because $\mathbf{H}$ and $\mathbf{\Xi}$ are independent. Note that the second equality in equation (5) follows from the fact that both $\mathbf{\Xi}$ and $\mathbf{P}$ are diagonal, and $\mathbf{R}_T$ is block diagonal. To fulfill our idea, we notice γ_{km} can be rewritten as

$$\begin{aligned}\gamma_{km} &= \frac{\mathrm{SNR}\mathbf{\Xi}_{km,km}\mathbf{P}_{km,km}}{\big[\mathbf{R}_T^{-1}\big]_{km,km}} \times X_{km} \\ &= \frac{\mathrm{SNR}\Xi_k p_{km}}{D_k^v\big[\mathbf{R}_{Tk}^{-1}\big]_{m,m}} \times X_{km},\end{aligned} \quad (9)$$

where the random small-scale counterpart X_{km} is defined by

$$X_{km} = \frac{[\mathbf{R}_{Tk}^{-1}]_{m,m}}{[(\mathbf{H}^H\mathbf{H})^{-1}]_{km,km}}. \quad (10)$$

It is known that X_{km} follows a complex Chi-square distribution with its PDF being given by [26]

$$f_X(x_{km}) = \frac{1}{\Gamma(N-KM+1)} x_{km}^{N-KM} e^{-x_{km}}, \quad x_{km} > . \quad (11)$$

For notational simplicity, let

$$C_{km} = \frac{\mathrm{SNR}d^2\Xi_k p_{km}}{4[\mathbf{R}_{Tk}^{-1}]_{m,m} D_k^v \sin^2\theta} \quad (12)$$

so that equation (7) can be rewritten as

$$\mathrm{SEP}_{km} = A_1 \int_0^{\frac{\pi}{2}} e^{-C_{km}X_{km}} d\theta - A_2 \int_0^{\frac{\pi}{4}} e^{-C_{km}X_{km}} d\theta. \quad (13)$$

Using equation (11), we have

$$\begin{aligned}\mathrm{E}_X[e^{-C_{km}X_{km}}] &= \int_0^\infty e^{-C_{km}x_{km}} f_X(x_{km}) dx_{km} \\ &= \frac{1}{(N-KM)!}\int_0^\infty x_{km}^{N-KM} e^{-(1+C_{km})x_{km}} dx_{km} \\ &= \frac{1}{(1+C_{km})^{N-KM+1}},\end{aligned} \quad (14)$$

where the last line followed by applying the identity [27]. We now analyze the SER performance in the high-SNR regime. Consider the definition given by (12) and we can obtain the following asymptotic results when SNR tends to infinity:

$$\begin{aligned}\mathrm{E}_X[e^{-C_{km}X_{km}}] &= \frac{1}{(1+C_{km})^{N-KM+1}} \\ &= \frac{1}{C_{km}^{N-KM+1}} + O\big(\mathrm{SNR}^{-(\mathrm{N-KM+2})}\big) \\ &= \left(\frac{4[\mathbf{R}_{Tk}^{-1}]_{m,m}D_k^v}{\mathrm{SNR}d^2 p_{km}}\sin^2\theta\right)^{N-KM+1} \frac{1}{\Xi_k^{N-KM+1}} \\ &\quad + O\big(\mathrm{SNR}^{-(\mathrm{N-KM+2})}\big).\end{aligned} \quad (15)$$

Similarly, utilizing equation (3) yields

$$\begin{aligned}\mathrm{E}_{\Xi}[\frac{1}{\Xi_k^{N-KM+1}}] &= \int_0^\infty \frac{1}{\xi_k^{N-KM+1}} f_{\Xi}(\xi_k) d\xi_k \\ &= e^{\frac{(N-KM+1)^2\delta_k^2 - 2\eta(N-KM+1)\mu_k}{2\eta^2}}.\end{aligned} \quad (16)$$

In addition, we write

$$\begin{aligned}&\int_0^{\frac{\pi}{2}} (\sin^2\theta)^{N-KM+1} d\theta = \frac{\pi}{2}\prod_{k=1}^{N-KM+1}\frac{2k-1}{2k} = B_1, \\ &\int_0^{\frac{\pi}{4}} (\sin^2\theta)^{N-KM+1} d\theta \\ &= \frac{\pi}{4}\prod_{k=1}^{N-KM+1}\sum_{j=1}^{k}\frac{(2k)!!(j-1)!!}{(2k-)!!(j)!!}\big(\frac{1}{\sqrt{2}}\big)^{2(k-j+1)} = B_2.\end{aligned} \quad (17)$$

Therefore, substituting the equations (15), (16), and (17) into (13) gives

$$\mathrm{E}\big[\mathrm{SEP}_{km}\big] = \overline{\mathrm{SEP}}_{km}^{\infty} + O\big(\mathrm{SNR}^{-(N-KM+2)}\big), \quad (18)$$

where notation $\overline{\mathrm{SEP}}_{km}^{\infty}$ denotes the dominant term of $\mathrm{E}\big[\mathrm{SEP}_{km}\big]$, i.e.,

$$\overline{\mathrm{SEP}}_{km}^{\infty} = \frac{\beta_{km} p_{km}^{-(N-KM+1)}}{\mathrm{SNR}^{(N-KM+1)}}$$

with β_{km} given by

$$\begin{aligned}\beta_{km} =& (A_1B_1 - A_2B_2)\left(\frac{4[\mathbf{R}_{Tk}^{-1}]_{m,m}D_k^v}{d^2}\right)^{N-KM+1} \\ &\times e^{\frac{(N-KM+1)^2\delta_k^2 - 2\eta(N-KM+1)\mu_k}{2\eta^2}}.\end{aligned} \quad (19)$$

Hence, the overall average SEP is represented by

$$\begin{aligned}\mathrm{E}\big[\mathrm{SEP}(\mathbf{H},\mathbf{\Xi})\big] =& \frac{1}{KM}\sum_{k=1}^{K}\sum_{m=1}^{M}\mathrm{E}\big[\mathrm{SEP}_{km}\big] \\ =& \overline{\mathrm{SEP}}^{\infty} + O\big(\mathrm{SNR}^{-(N-KM+2)}\big),\end{aligned} \quad (20)$$

where $\overline{\mathrm{SEP}}^{\infty}$ is defined by

$$\begin{aligned}\overline{\mathrm{SEP}}^{\infty} =& \frac{1}{KM}\sum_{k=1}^{K}\sum_{m=1}^{M}\overline{\mathrm{SEP}}_{km}^{\infty} \\ =& \frac{1}{KM}\sum_{k=1}^{K}\sum_{m=1}^{M}\frac{\beta_{km}p_{km}^{-(N-KM+1)}}{\mathrm{SNR}^{(N-KM+1)}}.\end{aligned} \quad (21)$$

Now, it can be observed from (20) and (21) that the ZF receiver achieves the diversity gain of $N-KM+1$ for the DMIMO systems.

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B. High-SNR Power Loading

In this section, we provide an optimal power-loading scheme in the high-SNR regime by minimizing the dominant term of the average SEP subject to either a total transmission power constraint or an individual transmission power constraint.

Problem 1: Let $\overline{\text{SEP}}^{\infty}$ be defined by (21). Then, find an optimal distribution of power p_{km} for $k = 1, 2, \cdots, K$ and $m = 1, 2, \cdots, M$ such that

$$\{\widetilde{p}_{km}\}_{k=1,m=1}^{K,M} = \arg\min \overline{\text{SEP}}^{\infty}$$

subject to a total transmission power constraint,

$$\sum_{k=1}^{K}\sum_{m=1}^{M} p_{km} = 1. \tag{22}$$

■

In order to solve this optimization problem, we note that function $g(t) = t^{-(N-KM+1)}$ is convex. Hence, the objective function $\overline{\text{SEP}}^{\infty}$ in terms of design variables p_{km} is convex [28]. Since the constraint (22) is linear, the overall optimization Problem 1 is convex. Let $L(\mathbf{P}, \lambda)$ denote its Lagrange multiplier function. Then,

$$L(\mathbf{P}, \lambda) = \overline{\text{SEP}}^{\infty} + \lambda\Big(\sum_{i=1}^{K}\sum_{j=1}^{M} p_{ij} - 1\Big)$$

with $\mathbf{P} = (p_{11}, \cdots, p_{KM})$. By letting all its first-order derivatives be equal to zeros, i.e.,

$$\frac{\partial L}{\partial \lambda} = \sum_{i=1}^{K}\sum_{j=1}^{M} p_{ij} - 1 = 0,$$

$$\frac{\partial L}{\partial p_{ij}} = \frac{-(N-KM+1)\beta_{ij}}{\text{SNR}^{(N-KM+1)}} p_{ij}^{-(N-KM+2)} + \lambda = 0,$$

we can obtain the optimal solution $\widetilde{p}_{km}$ below:

$$\widetilde{p}_{km} = \frac{\beta_{km}^{\frac{1}{N-KM+2}}}{\sum_{i=1}^{K}\sum_{j=1}^{M} \beta_{ij}^{\frac{1}{N-KM+2}}}. \tag{23}$$

Problem 2: Let $\overline{\text{SEP}}^{\infty}$ be defined by (21). Then, find an optimal distribution of power p_{km} for $k = 1, 2, \cdots, K$ and $m = 1, 2, \cdots, M$ such that

$$\{\widetilde{p}_{km}\}_{k=1,m=1}^{K,M} = \arg\min \overline{\text{SEP}}^{\infty}$$

subject to an individual transmission power constraint

$$\sum_{m=1}^{M} p_{km} = p_k,$$

■

where p_k is fixed for $k = 1, 2, \cdots, K$.

Following the discussion similar to solving Problem 1, we can attain the optimal solution $\widetilde{p}_{km}$ as follows:

$$\widetilde{p}_{km} = \frac{p_k \beta_{km}^{\frac{1}{N-KM+2}}}{\sum_{j=1}^{M} \beta_{kj}^{\frac{1}{N-KM+2}}}. \tag{24}$$

All the above discussions can be summarized as the following theorem:

Theorem 1: The average SEP for correlated RLN MIMO channels with square q-QAM using the ZF receiver has the following asymptotic formula:

$$\text{E}\big[\text{SEP}(\mathbf{H}, \mathbf{\Xi})\big] = \frac{1}{KM\text{SNR}^{N-KM+1}} \sum_{k=1}^{K}\sum_{m=1}^{M} \frac{\beta_{km}}{\widetilde{p}_{km}^{N-KM+1}} + O\big(\text{SNR}^{-(N-KM+2)}\big),$$

where β_{km} is given by (19), and the optimal power loading $\widetilde{p}_{km}$ is provided by (23) for the total transmission power constraint and by (24) for the individual power constraint. ■

We would like to make the following three comments on this theorem:

1) Theorem 1 provides us with a new and simple power allocation scheme for both the transmitter and the receiver so that the dominant error performance is minimized under two kinds of the power constraints.

2) Under the total transmission power constraint and the assumption that the first and the second order statistics are available at the transmitter, the BS computes the power normalization coefficient $\bar{\beta} = \sum_{i=1}^{K}\sum_{j=1}^{M} \beta_{ij}^{\frac{1}{N-KM+2}}$ and feeds back this information to every RP. Then, each RP utilizes (23) to optimally distribute the power to its each individual subchannel.

3) Under the individual transmission power constraint and the assumption that the first and the second order statistics are available at the transmitter, each RP uses (24) directly to optimally allocate its total power to its each individual subchannel without any need of the feedback information from the receiver.

4) Here, it should be clearly pointed out that our designed optimal precoder Theorem 1 completely relies on the perfect knowledge of the first and second order statistics of the channel at the transmitter. As we know, on one hand, in spite of the fact that these information can be first estimated by the receiver using some training signals provided by the transmitter, and then, fed back to the transmitter, the resulting estimation error, in practice, is unavoidable. On the other hand, despite the fact that sending more training signals can improve accurate estimation of the channel, it, meanwhile, will reduce the necessary information rate [29]. Specifically for the distributed multi-user MIMO uplink system considered in our paper, each user has to send orthogonal pilot sequences in order to accurately estimate the channel. In practice, a simple scheme is the unit matrix $\mathbf{I}_{KM}$, which takes up KM transmission time slots, and then, send information data within a transmission period not longer than the fading coherence time [29]. More specifically, if we assume that the total number of time slots is T in the period of a transmission cycle, then, a rate loss is $\frac{KM}{T}$.

Therefore, how to effectively and efficiently design the optimal precoder robust to the estimation error while maintaining a reasonable information rate that minimizes SEP with the ZF detector is under our investigation in future.

5) Regarding the total complexity of our proposed optimal precoder in this paper, it includes the complexity of ZF detection and that of the optimal diagonal power loading designed by Theorem 1. Since it is difficult to calculate the exact number

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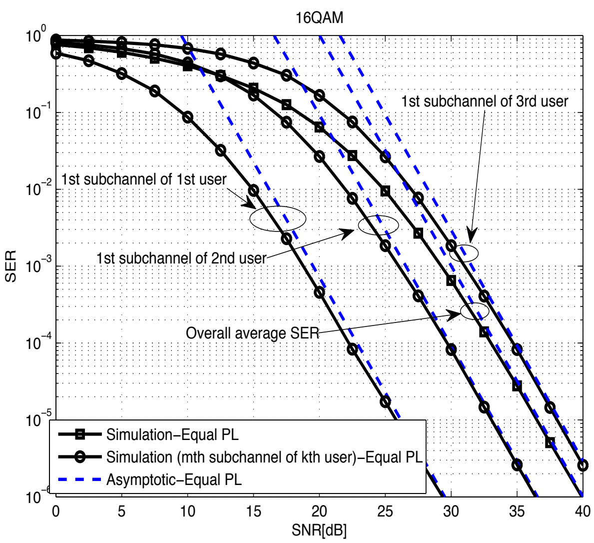

Fig. 1. Average SER of mth subchannel of kth user in a distributed MIMO system with 16-QAM modulation ($N = 8, K = 3, M = 2, \rho_k = 0.6, v = 4, \mu_k = 4, \delta_k = 2, D_1 = 1km, D_2 = 1.5km, D_3 = 2km$)

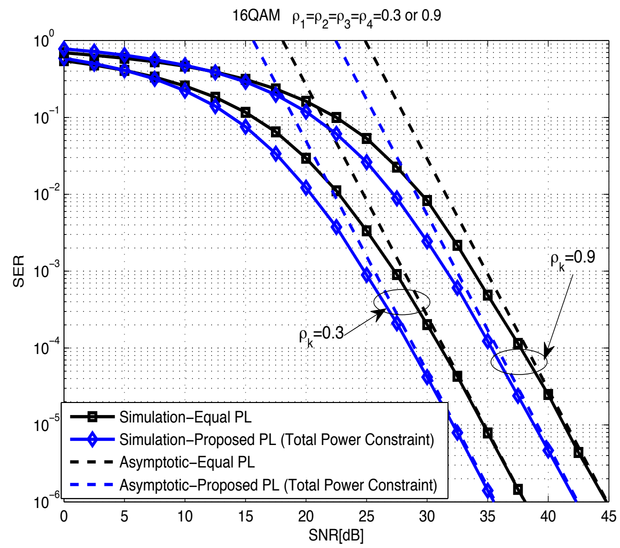

Fig. 2. Overall average SER comparison of power loading schemes in a distributed MIMO system against SNR ($N = 10, K = 4, M = 2, \rho_k = 0.3$ or $0.9, v = 4, \mu_k = 4, \delta_k = 2, D_1 = 0.5km, D_2 = 1km, D_3 = 1.5km, D_4 = 2km$)

of operations for various schemes, we compute the complexity in terms of the required floating point operations (flops). For ZF detection in the system model, its main computation complexity is the pseudo inverse of channel matrix. According to [30], the required flops of inversion of a $KM \times KM$ matrix using Gauss-Jordan elimination is about $\frac{4}{3} \times (KM)^3$.

In addition, regarding the computational cost of our proposed optimal diagonal power loading scheme, its main computation complexity is to calculate the optimal power allocation factor according to (23). Its computational complexity is linearly increasing with the number of the transmitter antenna, i.e., $O(M)$, which can be ignored compared to the complexity of ZF detection.

IV. Numerical Results

In this section, we carry out some computer simulations to verify our analysis and to compare our proposed power-loading algorithm with the equal power-loading scheme considered in literature. For simulation simplicity, some parameters of the small and large-scale fading are fixed throughout our experiments.

In order to examine our proposed criterion on minimizing the dominant term of the average SEP, we carry out computer simulations to compare the simulated average symbol error rate result with the theoretical expression of the dominant term. Fig 1 shows the SER of the mth subchannel of the kth user and overall average SER in the distributed MIMO system over the correlated MIMO channels with $\rho_k = 0.6$ when 16-QAM modulation is used. The results show that when SNR is large, they match very well.

In addition, to show superiority of our proposed optimal power-loading scheme, we compare the SER performance of the proposed system with that of the system using the equal power scheme. All numerical results are shown in

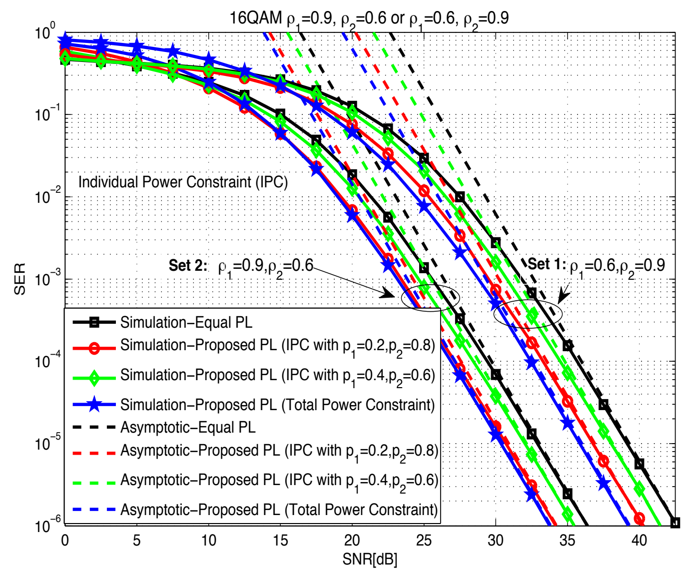

Fig. 3. Overall average SER comparison of power loading schemes in a distributed MIMO system against SNR ($N = 10, K = 2, M = 4, v = 4, \mu_k = 4, \delta_k = 2, D_1 = 0.5km, D_2 = 1.5km$)

Figs. 2 and 3. Fig. 2 shows the overall average SER of the proposed scheme over the RLN MIMO channels with a total transmission power constraint under various correlation scenarios (i.e., Set 1: $\rho_k = 0.3$ and Set 2: $\rho_k = 0.9$). Fig. 3 shows the overall average SER of the proposed scheme over the RLN MIMO channels with the individual transmission power constraint (i.e., $\{p_1 = 0.2, p_2 = 0.8\}$ and $\{p_1 = 0.4, p_2 = 0.6\}$) under various correlation scenarios (i.e., Set 1: $\{\rho_1 = 0.6, \rho_2 = 0.9\}$ and Set 2: $\{\rho_1 = 0.9, \rho_2 = 0.6\}$) . Specifically, we can observe from these figures that the performance gain also depends on the channel correlation coefficient

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ρ. The more performance gain can be obtained with larger $|\rho|$. Therefore, the efficient utilization of the statistics of the channels and the feedback information from the receiver at the transmitter can lead to significant performance enhancement.

V. Conclusion

In this paper, we have considered the D-MIMO system, in which the channel experiences both small and large-scale fading. For such a system, we have developed a new technical approach to deriving the asymptotic average SEP for the square QAM constellation using the ZF receiver. Based on this, we have proposed the optimal diagonal power-loading scheme minimizing the dominant term of the asymptotic SEP subject to either a total transmission power constraint or an individual transmission power constraint. A simple closed-form solution has been attained. Comprehensive computer simulations have verified our theoretic analysis and demonstrated that our presented optimal system attains significant performance gains over the currently available equal power-loading system.

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