

# Signal-Cooperative Multi-Layer Modulated VLC Systems For Automotive Applications

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**Abstract:** In visible light communication (VLC) systems for automotive applications, different kinds of traffic data with intended priority are usually demanded for transmission to meet an important requirement for both driving safety and the simplicity of a receiver mainly due to a time urgency issue. For this need, a concept called an additively uniquely decomposable constellation (signal amplitude set) group (AUDCG) is proposed in this paper to transmit multi-layer data for VLC through constellation cooperation. Then, an optimal AUDCG is designed by minimizing the average optical power subject to a fixed minimum Euclidean distance. One of significant advantages of this optimal design is fast demodulation of the sum signal from a noisy received signal as well as fast decoding individual signal from the estimated sum signal. Another important advantage is that this design allows each user-constellation to be flexibly assigned to meet different priority requirements. Computer simulations indicate that our proposed design has better error performance than currently available time orthogonal transmission scheme for this application.

**Index Terms:** Visible light communications, multi-layer modulation, additively uniquely decomposable constellation group, maximum likelihood (ML) detection.

## 1. Introduction

As a promising candidate to alleviate the strain on radio frequency spectrum, visible light communication (VLC) has drawn increasing attention, due to its known advantages of low cost, easy deployment, high security, freedom from spectral licensing issues and etc [1], [2]. As an additional advantage, VLC can provide one-way broadcast high-speed communication for numerous applications by using the already ubiquitously installed light emitting diodes (LEDs). It is desirable to transmit diverse kinds of information data under different service quality. A typical scenario is automotive use [3]–[6] of VLC, which utilizes traffic light, automotive headlights, rear lights, and brake lights, for data transmission. If an image sensor installed on a vehicle is used as a receiver, then, various applications can be integrated [5]–[9]. On one hand, the traffic safety is an important issue and thus, should be considered with a high priority. Specifically, the transmitter LED can transmit driving safety information such as lane change decision-aid service, adaptive cruise control, emergency brake warning, pedestrian detection, and providing range estimations for nearby vehicles. For driving safety application, the transmission robustness is critical and thus,

the detection of the corresponding signals should have low complexity with time urgency taken into consideration. On the other hand, some other applications such as the image data or video information for faraway traffic situation or the traffic jams of nearby roads may be more important. Therefore, these potential applications should have different priorities in terms of data rate or robustness.

For the need of the above-mentioned applications, overlay coding and double-layer modulation were proposed in [10]–[12]. These designs provide effective solutions to data transmission with two priority strategies and low-complexity detection. However, when the receiver image sensor installed on a vehicle is far from the transmitter, the high spatial-frequency for the lower layer fails to be detected [10], [11] because of image blur. As a result, only the one-bit upper-layer data with high priority can be recovered. In addition, since the channel coefficients are nonnegative for VLC, the well-developed beam-forming techniques for radio frequency (RF) are not applicable. Moreover, according to the experimental modelling results in [6], it may be proper for the receiver image sensor to choose a pixel with the largest intensity among all the projected images of the transmitter LED such that the received signal-to-noise ratio (SNR) is relatively robust to the distance between the moving vehicle and the transmitter LED. This channel characteristic may compromise the performance of the existing double-layer schemes [10]–[12]. Furthermore, the readout processes for CMOS image sensor are a bottleneck [6] and thus, the single pixel detection will increase the overall processing speed. For this reason, using one LED to simultaneously transmit multi-layer data information for multiple users may be demanded and appealing.

However, when the transmitter LED transmits the multi-layer data to an image sensor of each user, the received signals are additively mixed, leading to inter-layer interference. Generally, different users cannot uniquely and efficiently extract what is intended from the additive superimposition of multi-layer nonnegative signals. It is noticed that from the perspective of information theory, the concept of uniquely decodable codes [13]–[16] for RF two-access adder channel was proposed by confirming non-zero free distance between any designed distinct code pair. In addition, these designs are based on bipolar or complex valued numbers. However, the signals of VLC are required to be real-valued and nonnegative. Therefore, these uniquely decodable designs for two-access radio frequency are also not applicable to multi-layer VLC. Furthermore, the receiver for the designed codes or constellations (signal amplitude sets) has increasing decoding complexity of the resulted sum constellation. Although those schemes in [13]–[16] can be modified by adding proper direct current, the optical power efficiency can not be assured by means of adding direct current. In addition, the optimal maximum likelihood (ML) detection of the corresponding modified transmitted signals is based on the exhaustive search, whose complexity is exponentially increasing with respect to the average bit rate. It is known that the driving safety is time-urgent, implying the complexity of the receiver should be very low. It is also noticed that time division multiple access (TDMA) [2] is a possible orthogonal solution. However, TDMA implies time delay for the users to awaiting the assigned time slot for access and can not assure the energy efficiency, which may not meet the driving safety requirement of time urgency and robustness. In other words, the potential solution should be energy-efficient and has a low-complexity receiver for time urgency and robustness issues.

Indeed, the aforementioned motivations lead us to extending the idea of uniquely decodable pair in [13]–[16] and then, proposing the concept of an additively uniquely decomposable constellation group (AUDCG) to turn additive inter-layer interference into uniquely identifiable signal components, which is the primary goal of this paper. We design an optimal AUDCG minimizing the optical power for a fixed minimum Euclidean distance. The proposed design has the property that the sum constellation is a commonly used pulse amplitude modulation (PAM) constellation. In addition, the receiver has a fast ML demodulation and decoding algorithm, whose complexity is robust to the sum constellation size and only depends on the number of data layers. Simulations show that the designed AUDCG has substantial performance gain over the currently available TDMA.

## 2. System Model

### 2.1. Transmitter

Let us consider a VLC broadcast link with one transmitter LED and  $N$  users with each equipped with a receiver image sensor. The LED transmits the symbol  $\sum_{i=1}^N x_i$ , where  $x_i$  denotes the  $i$ -th layer data intended for User  $n$ . Following the suggested model in [6], we assume that the image sensor only selects the received signal of the pixel with the strongest intensity, implying a single input single output system. Therefore, the received signal,  $y_n$  from User  $n$  can be represented by [10], [11]

$$y_n = \sum_{i=1}^N x_i + \zeta_n \quad (1)$$

where  $x_i$  is a nonnegative real number satisfying the unipolarity requirement of the intensity modulation and carries safety text information or traffic image data intended for User  $n$ .  $\zeta_n$  is the sum of the ambient shot noise induced by the background radiations and the thermal noise and modelled as an additive white Gaussian noise with zero mean and a variance  $\sigma^2$  [1]. We assume that  $x_i$  is chosen randomly, equally-likely and independently from a given constellation  $\mathcal{X}_i$  for  $i = 1, 2, \dots, N$ .

### 2.2. Receiver

At the receiver side of User  $n$ , the received signal is given by  $y_n$ . The receiver detection consists of two successive steps.

#### 2.2.1. ML Demodulation

Given the received signal  $y_n$ , the ML estimate of  $g = \sum_{i=1}^N x_i$  with  $x_i \in \mathcal{X}_i$  from User  $n$  is determined by

$$\hat{g}^{(n)} = \arg \min_g |y_n - g| \quad (2)$$

#### 2.2.2. Decoding

The task of the decoder is to determine the  $N$  signal components  $\hat{x}_i^{(n)}$  from the optimal sum estimate  $\hat{g}^{(n)}$ . The unique identification of  $(\hat{x}_1^{(n)}, \hat{x}_2^{(n)}, \dots, \hat{x}_N^{(n)})$  from  $\hat{g}^{(n)}$  is assured only if there exists a one-to-one mapping relationship between  $(\hat{x}_1^{(n)}, \hat{x}_2^{(n)}, \dots, \hat{x}_N^{(n)})$  and  $\hat{g}^{(n)}$  for any  $x_i^{(n)} \in \mathcal{X}_i$ . This necessary one-to-one relationship will be investigated in the ensuing section.

## 3. Additively Uniquely Decomposable Constellation Group

In this section, we first propose a concept called AUDCG, which allows  $N$  users to uniquely decode what is intended from the ML demodulator output, and then, design an optimal AUDCG with a low-complexity receiver.

### 3.1. Definition and Property of AUDCGs

To begin with, let us consider the following example, which motives us to propose the concept of AUDCG.

**Example 1:** Consider the model in (1) for  $N = 2$  and select the symbols  $x_1$  and  $x_2$  intended for User 1 and User 2 from  $\mathcal{X}_1 = \{0, 1\}$  and  $\mathcal{X}_2 = \{1, 2\}$ , respectively. This pair of constellations constitutes a sum constellation  $\mathcal{G} = \mathcal{X}_1 + \mathcal{X}_2 = \{1, 2, 3\}$ . If the ML output from User 1 is exactly accurate and specifically, given by  $\hat{g}^{(1)} = 2$ , then, the decoder of User 1 can not uniquely identify whether  $(0, 2)$  or  $(1, 1)$  is transmitted, even for a noise-free channel. Let alone the final unique identification of  $\hat{x}_1^{(1)}$  and  $\hat{x}_2^{(1)}$  from the received noisy signal  $y_1$ . ■

The above trivial example indeed leads us to introducing the following concept by generalizing the uniquely decodable pair concept in [13]–[16].

**Definition 1:** A group of constellations  $\mathcal{X}_i$  for  $i = 1, 2, \dots, N$  is said to constitute an additively uniquely decomposable constellation group (AUDCG) if there exist  $x_i, \tilde{x}_i \in \mathcal{X}_i$  such that  $\sum_{i=1}^N x_i = \sum_{i=1}^N \tilde{x}_i$ , then, we have  $x_i = \tilde{x}_i$  for any  $i = 1, 2, \dots, N$ . ■

By Definition 1, the  $N$  constellations  $\mathcal{X}_i$  in an AUDCG naturally result in a specific sum constellation,  $\{\sum_{i=1}^N x_i : x_i \in \mathcal{X}_i\}$ , which, for presentation convenience, is denoted by  $\mathcal{X}_1 \uplus \mathcal{X}_2 \uplus \dots \uplus \mathcal{X}_N$ . In addition, if each of them is finite, then, so is  $\mathcal{X}_1 \uplus \mathcal{X}_2 \uplus \dots \uplus \mathcal{X}_N$  and  $|\mathcal{X}_1 \uplus \mathcal{X}_2 \uplus \dots \uplus \mathcal{X}_N| = \prod_{i=1}^N |\mathcal{X}_i|$ , where  $|\mathcal{X}|$  denotes the number of the elements of a finite constellation  $\mathcal{X}$ . This statement, conversely, is also true. Hence, we have the following property:

**Property 1:** Given  $\mathcal{X}_i$  with finite size, let  $\mathcal{G} = \{\sum_{i=1}^N x_i : x_i \in \mathcal{X}_i, i = 1, 2, \dots, N\}$ . Then,  $\mathcal{G} = \mathcal{X}_1 \uplus \mathcal{X}_2 \uplus \dots \uplus \mathcal{X}_N$  if and only if  $|\mathcal{G}| = \prod_{i=1}^N |\mathcal{X}_i|$ . ■

Property 1 reveals the fact that an AUDCG can be constructed by uniquely decomposing a sizeable constellation into several constellations of small size.

**Example 2:** A pair of constellations  $\mathcal{X}_1 = \{0, 3\}$  and  $\mathcal{X}_2 = \{1, 2\}$  constitutes an AUDCG  $\mathcal{G} = \mathcal{X}_1 \uplus \mathcal{X}_2$ , since  $\mathcal{G} = \mathcal{X}_1 + \mathcal{X}_2 = \{1, 2, 4, 5\}$ . Given any  $g \in \mathcal{G}$ , we can uniquely decompose  $g$  into two numbers  $x_1 \in \mathcal{X}_1$  and  $x_2 \in \mathcal{X}_2$  such that  $g = x_1 + x_2$ . For instance,  $1 \in \mathcal{G}$ , uniquely giving us  $0 \in \mathcal{X}_1$  and  $1 \in \mathcal{X}_2$ . ■

**Example 3: TDMA.** For the  $N$  successive time slots, the  $i$ -th one is assigned to User  $i$  with a signal constellation  $\mathcal{S}_i = \{m\}_{m=0}^{2^{N K_i} - 1}$  with  $K_i \geq 1$ . Let  $\mathbf{e}_i$  be defined by a  $1 \times N$  vector with the  $i$ -th entry being one and the other  $(N - 1)$  entries being zeros. Then, the transmitted signal by the transmitter LED is given by  $\mathbf{x} = \frac{2^{\sum_{i=1}^N \mathbf{x}_i}}{2^{N K_i - N}}$ , where  $\mathbf{x}_i = s_i \mathbf{e}_i$  with  $s_i \in \mathcal{S}_i$ . Then, by Definition 1, such a group of constellations  $\mathcal{X}_i = \{s_i \mathbf{e}_i, s_i \in \mathcal{S}_i\}$  for  $i = 1, 2, \dots, N$  constitutes a vector version of AUDCG with  $\mathcal{G} = \mathcal{X}_1 \uplus \mathcal{X}_2 \uplus \dots \uplus \mathcal{X}_N = \{\sum_{i=1}^N s_i \mathbf{e}_i, s_i \in \mathcal{S}_i\}$ . ■

In this paper, we are interested in uniquely decomposing a one-dimensional unipolar constellation into the sum of  $N$  constellations, since the signals in VLC are required to be nonnegative. A unipolar constellation  $\mathcal{G}$  is a finite subset of  $\mathbb{R}_+$ ,  $\mathcal{G} \subseteq \mathbb{R}_+$ , where notation  $\mathbb{R}_+$  denotes the set of all the nonnegative real numbers. Generally speaking, the existence of a uniquely decomposable constellation  $\mathcal{G}$  is not unique. In other words, there may have several ways to uniquely decompose a given constellation into the sum of some sub-constellations. For example,  $\{0, 1, 2, 3\} = \{0\} \uplus \{0, 1, 2, 3\} = \{0, 1\} \uplus \{0, 2\}$ . Here, a natural question is how to find an optimal AUDCG ?

### 3.2. Optimal AUDCG With A Low-Complexity Receiver

In this subsection, we give a specific design criterion and then, design an optimal AUDCG.

#### 3.2.1. Optimal AUDCG

Given the received signal  $y_n$  defined in (1) and an AUDCG  $\mathcal{G}$ , the error performance of the ML demodulation is determined by the minimum Euclidean distance. To optimize the ML demodulation performance in (2), the optimal AUDCG  $\mathcal{G}$  should maximize the minimum Euclidean distance between any two distinct signals  $g$  and  $\tilde{g}$ , say,  $\min_{g \neq \tilde{g}, g, \tilde{g} \in \mathcal{G}} |g - \tilde{g}|$ , under an optical power budget. Hence, the design problem of an optimal AUDCG is formally stated as follows.

**Problem 1:** Let  $K$  be an arbitrarily given integer not less than  $N$ , i.e.,  $K \geq N$ . Then, for any given positive integers  $K_1, K_2, \dots, K_N$  satisfying  $\sum_{i=1}^N K_i = K$ , find an AUDCG  $\mathcal{G} = \mathcal{X}_1 \uplus \mathcal{X}_2 \uplus \dots \uplus \mathcal{X}_N \subseteq \mathbb{R}_+$  such that 1) the size of  $\mathcal{X}_i$  is given by  $|\mathcal{X}_i| = 2^{K_i}$  and 2) the total optical power  $2^{-K} \sum_{g \in \mathcal{G}} g$  is minimized subject to a fixed the minimum Euclidean distance  $\min_{g \neq \tilde{g}, g, \tilde{g} \in \mathcal{G}} |g - \tilde{g}| = 1$ . ■

We are now in a position to formally state our main result in this paper.

**Design 1:** The optimal solution to Problem 1 is given by  $\mathcal{G} = \{m\}_{m=0}^{2^K - 1}$ . Furthermore,  $\mathcal{X}_1 = \{m\}_{m=0}^{2^{K_1} - 1}$  and  $\mathcal{X}_n = \{m \times 2^{\sum_{i=1}^{n-1} K_i}\}_{m=0}^{2^{K_n} - 1}$  for  $2 \leq n \leq N$ . ■

The optimality proof of Design 1 is postponed into Appendix. Notice that the sub-constellations in Design 1 can have different rate and different average optical power. This flexibility can meet the following different priority requirements in applications:

- (a) *Strategy 1: equal rate with different power.* Two examples are given as follows. **1)**  $K_1 = K_2 = 1$  and  $N = 2$ .  $\mathcal{G} = \{0, 1, 2, 3\}$  with  $\mathcal{X}_1 = \{0, 1\}$  and  $\mathcal{X}_2 = \{0, 2\}$ . **2)**  $N = 4$  and  $K_1 = K_2 = K_3 = K_4 = 1$ .  $\mathcal{G} = \{m\}_{m=0}^{m=15}$  with  $\mathcal{X}_1 = \{0, 1\}$ ,  $\mathcal{X}_2 = \{0, 2\}$ ,  $\mathcal{X}_3 = \{0, 4\}$  and  $\mathcal{X}_4 = \{0, 8\}$ .
- (b) *Strategy 2: low rate with more power.* In this case, we provide the following two examples. **1)**  $N = 2$ ,  $K_1 = 3$  and  $K_2 = 1$ .  $\mathcal{G} = \{m\}_{m=0}^{m=15}$  with  $\mathcal{X}_1 = \{m\}_{m=0}^{m=7}$  and  $\mathcal{X}_2 = \{0, 8\}$ . **2)**  $N = 3$ ,  $K_1 = 3$ ,  $K_2 = 2$  and  $K_3 = 1$ .  $\mathcal{G} = \{m\}_{m=0}^{m=63}$  with  $\mathcal{X}_1 = \{m\}_{m=0}^{m=7}$ ,  $\mathcal{X}_2 = \{0, 8, 16, 24\}$  and  $\mathcal{X}_3 = \{0, 32\}$ .
- (c) *Strategy 3: high rate with more power.* Two specific examples are given as follows. **1)**  $N = 2$ ,  $K_1 = 1$  and  $K_2 = 3$ .  $\mathcal{G} = \{m\}_{m=0}^{m=15}$  with  $\mathcal{X}_1 = \{0, 1\}$  and  $\mathcal{X}_2 = \{2m\}_{m=0}^{m=7}$ . **2)**  $N = 3$ ,  $K_1 = 1$ ,  $K_2 = 2$  and  $K_3 = 3$ .  $\mathcal{G} = \{m\}_{m=0}^{m=63}$  with  $\mathcal{X}_1 = \{0, 1\}$ ,  $\mathcal{X}_2 = \{0, 2, 4, 6\}$  and  $\mathcal{X}_3 = \{8m\}_{m=0}^{m=7}$ .

### 3.2.2. Low-Complexity Receiver

Design 1 tells us that for any constellation  $\mathcal{G} \subseteq \mathbb{R}_+$ , the optimal AUDCG is given by a  $2^K$ -ary equally spaced pulse amplitude modulation (PAM) constellation. One of significant advantages for this constellation is fast demodulation for the sum signal from  $\mathcal{G} = \{m\}_{m=0}^{m=2^K-1}$ .

**Algorithm 1:** (Fast ML demodulation) Given the received signals  $y_n$  for  $n = 1, 2, \dots, N$  defined by (1), the optimal ML estimate of the sum signal  $g = \sum_{i=1}^N x_i$  can be attained by

$$\hat{g}^{(n)} = \begin{cases} 0, & \text{if } y_n \leq 0; \\ \lfloor y_n + \frac{1}{2} \rfloor, & \text{if } 0 < y_n \leq 2^K - 1; \\ 2^K - 1, & \text{if } y_n > 2^K - 1. \end{cases}$$

When the optimal ML estimate  $\hat{g}^{(n)}$  of  $g$  from User  $n$  has been obtained by Algorithm 1, now by Design 1, there exists a unique  $\hat{x}_i^{(n)} \in \mathcal{X}_i$  such that  $\hat{g}^{(n)} = \sum_{i=1}^N \hat{x}_i^{(n)}$ . By fully taking advantage of the properties of  $\mathcal{X}_i$ , we know  $\hat{x}_1^{(n)} = \hat{g}^{(n)} \bmod 2^{K_1}$  and  $\hat{x}_2^{(n)} = 2^{K_1} \left( \frac{\hat{g}^{(n)} - \hat{g}^{(n)} \bmod 2^{K_1}}{2^{K_1}} \bmod 2^{K_2} \right)$ . Then, following this process gives us a fast decoding algorithm below:

**Algorithm 2:** (Fast decoding) Let  $\hat{g}^{(n)}$  be the optimal estimate of  $g$  by Algorithm 1 from User  $n$ . Then, the estimate of  $x_i$  from User  $n$ , say,  $\hat{x}_i^{(n)}$ , is given by  $\hat{x}_1^{(n)} = \hat{g}^{(n)} \bmod 2^{K_1}$  and  $\hat{x}_i^{(n)} = 2^{\sum_{\ell=1}^{i-1} K_\ell} \left( \frac{\hat{g}^{(n)} - \hat{g}^{(n)} \bmod 2^{\sum_{\ell=1}^{i-1} K_\ell}}{2^{\sum_{\ell=1}^{i-1} K_\ell}} \bmod 2^{K_i} \right)$  for  $2 \leq i \leq N$ .

To deeply appreciate the optimally designed AUDCG, the following three points should be addressed:

- 1) Although the proposed concept of AUDCG is similar to the uniquely decodable pair in [13]–[16], the bipolar or complex-valued designs for RF in [13]–[16] are only for two-user and not applicable for VLC since the signals of intensity modulation are required to be nonnegative. Our work is for arbitrarily given number of users allowing very flexible allocations of signal set and optical power.
- 2) In [13]–[16], the main design target is to assure the non-zero free distance of the received signals. In this paper, our attained optimal design is via analytical optimization by minimizing the average optical power subject to a fixed minimum Euclidean distance between any two distinct signal points of the sum signal. Thus, the highest energy efficiency of our optimal AUDCG is assured.
- 3) The proposed design in this paper is proved to be the commonly used PAM constellation without any assumption on the feasible domain of nonnegative signal set. This property of the optimal design allows us to demodulate and decode the received signals with a very low complexity.

## 4. Simulations

In this section, we examine the error performance of our designed AUDCGs with TDMA described in Example 3. For TDMA, the transmitted signal by the transmitter LED is given by  $\mathbf{x} =$

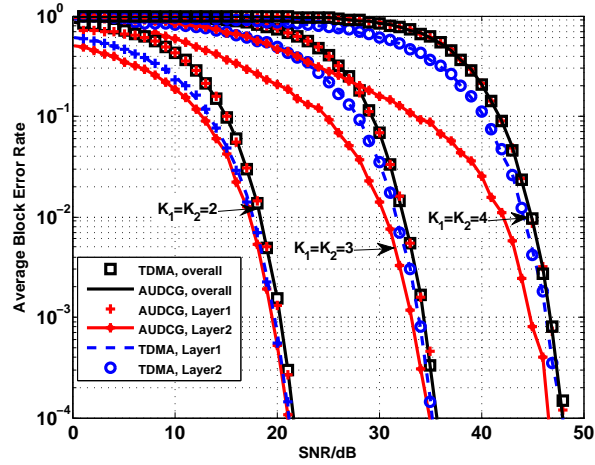


Fig. 1. Average block error rate comparisons for  $N = 2$  under *Strategy 1*.

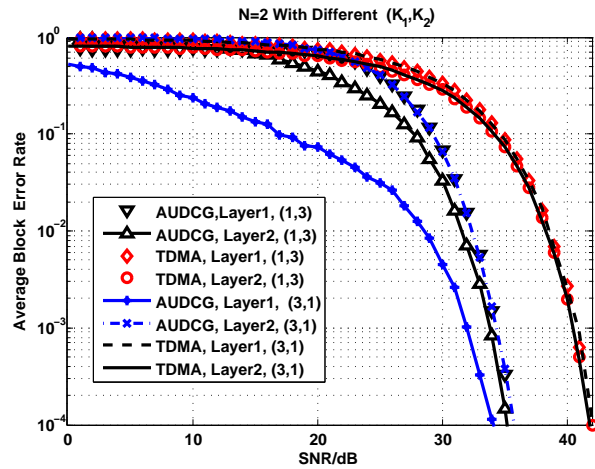


Fig. 2. Average block error rate performances under *Strategy 2* and *Strategy 3*.

$\frac{2 \sum_{i=1}^N \mathbf{x}_i}{\sum_{i=1}^N 2^{N K_i} - N}$ , where  $\mathbf{x}_i = s_i \mathbf{e}_i$  with  $s_i \in \{m\}_{m=0}^{2^{N K_i} - 1}$ . In addition, the transmitted signal from the transmitter LED for our optimal AUDCG is given by  $\mathbf{x} = \frac{2 \sum_{i=1}^N \mathbf{x}_i}{N 2^K - N}$ , where  $\mathbf{x}_i = [x_{1i}, x_{2i}, \dots, x_{N_i}]$  with  $x_{ni} \in \mathcal{X}_i$  and  $|\mathcal{X}_i| = 2^{K_i}$ . With the normalized optical power, the SNR is defined by  $\frac{1}{\sigma^2}$ . To make all comparisons fair, the ML demodulator is used for both schemes. More details about our computer simulations are provided as follows.

We first consider the case with  $N = 2$ , where  $x_1$  and  $x_2$  are intended for User 1 and User 2, respectively. For a fixed  $K$ , we assign the sub-constellations  $\mathcal{X}_1$  and  $\mathcal{X}_2$  using different priority strategies. Under *Strategy 1*, the constellations of two users have the same rate, but different robustness for intended application. For example, if User 2 needs some emergent brake data, then, more power is allocated to  $\mathcal{X}_2$ . As illustrated by Fig. 1, under *Strategy 1* with  $K_1 = K_2$ , the overall block, say,  $\mathbf{x}$ , error performances of both the optimally designed AUDCG and TDMA are the same, since when  $K$  is fixed, their block signals are equal. Meanwhile, the block, say,  $\mathbf{x}_2$ , error performance of User 2 is slightly better than that of User 1, and when  $K$  becomes larger, the corresponding performance gap between two users becomes noticeable, since more power is allocated to User 2. When  $K_1 \neq K_2$  for a given  $K$ , the error performances for two users of

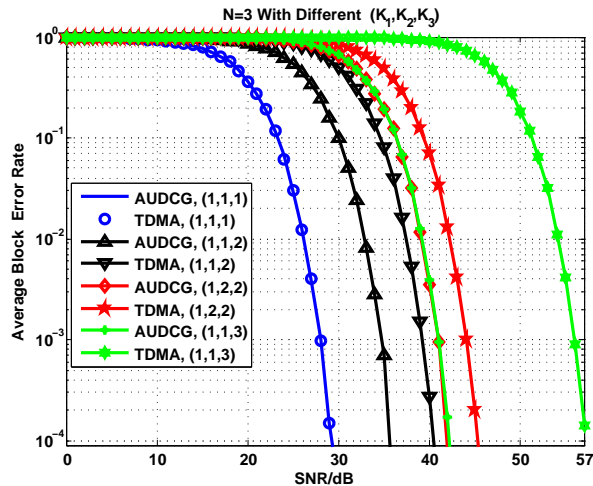


Fig. 3. Overall average block error rate comparisons under Strategy 3.

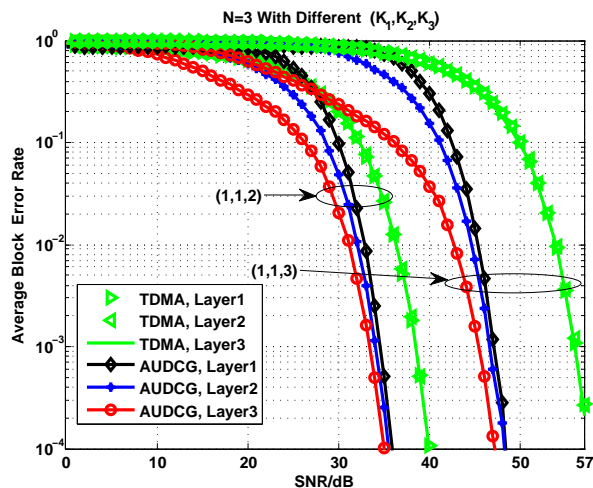


Fig. 4. Average block error rate performances under Strategy 3.

AUDCG can remarkably outperform those of TDMA as shown by Fig. 2. The main reason for this is that the error performance is dominated by the minimum block Euclidean distance term. By computations, the attained gain in terms of the ratio of the minimum distances of AUDCG and TDMA is given by  $20 \log \frac{\sum_{i=1}^N 2^{N K_i} - N}{N 2^K - N}$ . For  $K = 4$  with  $K_1 = 3$  and  $K_2 = 1$  under Strategy 2 or  $K_1 = 1$  and  $K_2 = 3$  under Strategy 3, the attained gain of AUDCG over TDMA is given by  $20 \log \frac{2^6 + 2^2 - 2}{2 \times 2^4 - 2} \approx 6.8485$  dB, which confirms with the numerical results in Fig. 2. From Fig. 2, it is noticed that under Strategy 3, the block, say,  $\mathbf{x}_2$ , error performance of User 2 is better than that of User 1, even when the rate of User 2 is three times that of User 1. Strategy 2 may be useful in such a scenario where the lane-change aid information is intended for User 2 requiring robustness and rate priority, and User 1 may needs the warning signal of the traffic light changing from green to red. To put the error performance of Strategy 3 into perspective, we examine the block error performance for AUDCG and TDMA with  $N = 3$ . As illustrated by Fig. 3 and Fig. 4, the gain of AUDCG over TDMA is substantial. For example, when  $K_1 = 1$ ,  $K_2 = 1$  and  $K_3 = 3$ , the SNR gain of the overall block error performance is about  $20 \log \frac{2^3 + 2^3 + 2^9 - 3}{3 \times 2^5 - 3} \approx 15.0335$  dB. Thus, a constellation assignment is important for the system performance under different priority

strategies. Therefore, our optimal AUDCG is more suitable under Strategy 2 and Strategy 3. When the number of data layer becomes larger, the more performance gain over TDMA will be attained.

## 5. Conclusion and Future Work

In this paper, we have developed the concept on AUDCG to deal with the inter-layer interference of multi-layer modulated VLC by transforming the additive interference into constellation-cooperative signal components. An optimal AUDCG has been proved to be the unique decomposition of a unipolar PAM constellation into a group of constellations with flexible sizes. The optimal ML fast demodulation and decoding algorithms for such design have been provided. Compared with TDMA, our proposed design has better error performance and has been numerically verified to be applicable under different priority strategies. However, our work in this paper is just initiative. Since the SNR of vehicle communications may not be constant especially for the case when either the transmitter or the receiver may be moving in a high speed, the adaptive rate-robustness tradeoff for the varying VLC automobile links will be considered in future. In addition, our theoretical results in this paper are numerically shown to have encouraging error performance. The verification of the performance of our proposed AUDCG through experimental setup is under investigation.

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## Appendix

### Optimality Proof of Design 1

It can be verified by calculation that  $|\mathcal{X}_i| = 2^{K_i}$  and thus,  $|\mathcal{G}| = 2^K = \prod_{i=1}^N |\mathcal{X}_i|$ . On the other hand, for any  $x_i \in \mathcal{X}_i$ , we know  $0 \leq \sum_{i=1}^N x_i \leq \sum_{i=1}^N (\max_{x \in \mathcal{X}_i} x) = 2^{K_1} - 1 + 2^{K_2+K_1} - 2^{K_1} + \dots + 2^{\sum_{i=1}^N K_i} - 2^{\sum_{i=1}^{N-1} K_i} = 2^K - 1$  and thus,  $\sum_{i=1}^N x_i \in \mathcal{G}$ . Therefore,  $\mathcal{X}_1 + \mathcal{X}_2 + \dots + \mathcal{X}_N \subseteq \mathcal{G}$ . Combining this with  $|\mathcal{G}| = \prod_{i=1}^N |\mathcal{X}_i|$  gives us that  $\mathcal{X}_1 + \mathcal{X}_2 + \dots + \mathcal{X}_N = \mathcal{G}$ . Then, by Property 1, we have  $\mathcal{G} = \mathcal{X}_1 \uplus \mathcal{X}_2 \uplus \dots \uplus \mathcal{X}_N$ . Let us consider any  $\tilde{\mathcal{G}} = \tilde{\mathcal{X}}_1 \uplus \tilde{\mathcal{X}}_2 \uplus \dots \uplus \tilde{\mathcal{X}}_N \subseteq \mathbb{R}_+$  satisfying the constraint that the minimum Euclidean distance between any two distinct signal points of  $\tilde{\mathcal{G}}$  is normalized, say,  $\min_{g \neq \hat{g}, g, \hat{g} \in \tilde{\mathcal{G}}} |g - \hat{g}| = 1$  and  $|\tilde{\mathcal{X}}_i| = 2^{K_i}$ . Without loss of generality, we assume that the  $2^K$  elements of  $\tilde{\mathcal{G}}$  satisfy  $0 \leq g_0 < g_1 < \dots < g_{2^K-1}$ . Then, to ensure  $\min_{g \neq \hat{g}, g, \hat{g} \in \tilde{\mathcal{G}}} |g - \hat{g}| = 1$ , it holds that  $g_{i+1} - g_i \geq 1$  for any  $0 \leq i \leq 2^K - 2$ . This observation gives us that  $\sum_{g \in \tilde{\mathcal{G}}} g \geq \sum_{i=1}^{2^K-1} i + g_0 \geq \sum_{i=1}^{2^K-1} i$ , where the equality holds if and only if  $\tilde{\mathcal{G}} = \mathcal{G}$ . Therefore, the optimality proof of Design 1 is complete. ■

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