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Energy-Efficient Space-Time Modulation for Indoor MISO Visible Light Communications

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We consider an indoor multi-input single-output (MISO) visible light communication (VLC) system without channel state information at the transmitter. For such a system, an energy-efficient time-collaborative modulation (TCM) constellation is first designed by minimizing a total optical power subject to a fixed minimum Euclidean distance. Then, a new space-time transmission scheme is proposed. Comprehensive computer simulations indicate that our proposed design always has better average error performance within illumination coverage area than the currently available schemes for this application.

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Visible light communication (VLC) operating on license-free visible spectrum has been considered to be a promising solution to indoor wireless access due to its cost-efficient and low-complexity applications [1, 2]. As an additional advantage, VLC uses ubiquitous light-emitting diodes (LEDs) as data transmitters, which are originally installed to provide sufficient illumination. These multiple LEDs and a popular built-in complementary metal-oxide semiconductor (CMOS) camera [3] in commonly used smart handsets naturally form a multi-input single-output (MISO) VLC link. In such a system, the feedback overhead for channel state information at the transmitter (CSIT) through radio frequency (RF) links has to place an additional strain on the almost saturated radio spectrum. For this reason, a robust MISO VLC system without CSIT may be appealing. Therefore, in this letter, we consider a MISO VLC system where CSIT is unknown. For such a system, it has been reported that currently available transmission schemes such as spatial modulation (SM) [4–6], repetition code (RC) [7] and modified orthogonal space-time block coding (OSTBC) [7] can be applicable. SM has high spectral efficiency and low-complexity detection, since only one transmitter LED is activated for any given instance. In addition, the activated transmitter LED of SM transmits symbols selected from a nonnegative pulse amplitude modulation (PAM) constellation. However, SM usually requires more than two photodiodes (PDs) to eliminate the error floor when channel coefficients are not remarkably different [6]. In addition, for MISO VLC without CSIT, the receiver PD may be located at any position. To provide robust error performance, a desired transmission for MISO VLC systems should have the ability to assure the reliable detection of signals at the receiver side for any given nonzero channel realization within the illumination area. According to the results in [7, 8], the space-repetitional (SR) structure that repeatedly transmits signals across space dimensions renders superior error performance for random channels. Hence, it should be suitable for MISO VLC when CSIT is unavailable. In fact, it was proved in [7] that RC outperforms the modified orthogonal Alamouti code, which is the best STBC for MISO RF wireless communication systems with two transmitter antennas and a single receiver antenna. However, it should be noticed here that RC actually utilizes all LEDs to transmit the same symbol selected from a nonnegative PAM constellation at each time slot, in which two symbols from any two time slots are independent and the optical power efficiency may be an issue. Despite the fact that for a single LED transmission, equally spaced nonnegative PAM is indeed the most energy-efficient in terms of the maximization of the minimum Euclidean distance under a power budget, it has been revealed by our recent results in [9] that this is no longer true even for a two-LEDs transmission system. Therefore, based on the SR structure, collaboratively designing the multiple time dimension signals will provide a more robust overall error performance for indoor MISO VLC without CSIT if a time-equivalent multi-dimensional constellation is properly designed in light of the energy efficiency. Motivated by the aforementioned factors, in this letter, we propose a new transmission scheme called *space-repetitional time-collaborative modulation (SRTCM)* for discussion convenience, by repeatedly transmitting signals across space dimensions as well as by collaboratively modulating the signals through multiple time dimensions.

To optimally generate such SRTCM, an energy-efficient multi-dimensional constellation is designed by minimizing the total optical power subject to a fixed minimum Euclidean distance.

To this end, let us consider a MISO VLC system with N transmitter LEDs and one receiver PD. For such a system, at any time slot t, a received signal y_t in an equivalent discrete-time baseband channel model can be written as $y_t = \sum_{n=1}^{N} h_n x_{tn} + n_t$ for $t = 1, 2, \cdots, T$, where x_{tn} is the transmitted symbol through the n-th LED and n_t is the sum of the ambient shot noise induced by the background radiations and the thermal noise, which is modelled as white Gaussian noise with zero mean and variance σ^2 [1]. In addition, h_n denotes a channel coefficient between the n-th transmitter LED and the receiver PD. In this letter, we assume line-of-sight links and PD is located at distance d_n and angle ϕ_n with respect to the n-th transmitter LED. Then, the frequency-flat channel coefficient h_n is determined by [1]

$$h_n = \begin{cases} \frac{(\tau+1)A}{2\pi d_n^2} \cos^{\tau}(\phi_n)\cos(\psi_n), & 0 \le \psi_n \le \Psi; \\ 0, & \psi_n > \Psi. \end{cases} \quad \textbf{(1)}$$

where ψ_n is the angle of incidence from the n-th transmitter LED, Ψ is the field-of-view angle of the receiver PD, A denotes the PD detection area, and $\tau = -\log_2 \cos \Phi_{\frac{1}{2}}$ with $\Phi_{\frac{1}{2}}$ being defined by the half power angle of LEDs. Our SRTCM for such a system is made up of the following two major transmission components:

(a) Repetitional transmission in spatial dimensions. Let $\mathcal{S} \subseteq \mathbb{Z}_+^T$ denote a T-dimensional constellation to be designed, where notation $\mathbb{Z}_+^T$ denotes the set of all the $T \times 1$ nonnegative integer-valued vectors. First, randomly, independently and equally likely choose a $T \times 1$ signal vector $\mathbf{s} = [s_1, s_2, \cdots, s_T]^T$ from $\mathcal{S}$. Then, at the t-th time slot, all the N transmitter LEDs repeatedly transmit the same symbol s_t to the receiver PD, i.e., $x_{t1} = x_{t2} = \cdots = x_{tN} = s_t$. After total T channel uses, the corresponding T received signals form a $T \times 1$ vector $\mathbf{y} = [y_1, y_2, \cdots, y_T]^T$, which can be represented by

$$\mathbf{y} = \alpha\, \mathbf{s} + \mathbf{n} \quad \textbf{(2)}$$

where $\alpha = \sum_{n=1}^{N} h_n$ and $\mathbf{n} = [n_1, n_2, \cdots, n_T]^{\mathrm{T}}$ is a noise vector with zero mean and a covariance matrix being given by $\sigma^2 \mathbf{I}_{T\times T}$. One of significant advantages of such specially repetitional transmission for the VLC system is like OSTBC for a MISO RF system, to transform the original MISO channel into a scaled version of ideal MIMO channel Eq. (2), thereby significantly simplifying ML detection. Another advantage is to allow all the channel gains to be additively accumulated by fully making use of the feature of the VLC channel. It is for this reason that the transmitted signal $\mathbf{s}$ in Eq. (2) can be uniquely recovered in a noise-free case as well as reliably estimated in a noisy case if one of the channel gains is not zero. It is also for this reason that RC has better error performance than OSTBC when applied to the VLC systems [7].

(b) Collaborative transmission in temporal dimensions. The core in our SRTCM is the time-collaborative constellation $\mathcal{S}$, which will allow any two signals in any given $\mathbf{s} \in \mathcal{S}$ to be collaborated each other so as to efficiently and effectively combat against noise.

It is thanks to the above repetitional and collaborative transmissions in the respective space and time that we name our proposed modulation scheme as SRTCM. Here, a natural question is: how to design an energy-efficient TCM constellation $\mathcal{S} \subseteq \mathbb{Z}_+^T$? To answer this question, let us first recall a constellation design criterion for the maximum likelihood (ML) detector.

It is known that for any given nonzero α, the error performance of the ML detector for the channel model Eq. (2) in high signal-to-noise ratio (SNR) regimes is decided by the minimum Euclidean distance of $\mathcal{S}$, i.e., $\min_{\mathbf{s}\neq\tilde{\mathbf{s}},\mathbf{s},\tilde{\mathbf{s}}\in\mathcal{S}} \|\mathbf{s}-\tilde{\mathbf{s}}\|_2$. Hence, our primary task in this letter is to solve the following optimization problem:

Problem 1: *For arbitrarily given positive integers T and K, find a constellation $\mathcal{S} \subseteq \mathbb{Z}_+^T$ with size 2^K such that the total optical power $\sum_{\mathbf{s}\in\mathcal{S}} \mathbf{1}^{\mathrm{T}}\mathbf{s}$ is minimized subject to $\min_{\mathbf{s}\neq\hat{\mathbf{s}},\mathbf{s},\hat{\mathbf{s}}\in\mathcal{S}} \|\mathbf{s}-\hat{\mathbf{s}}\|_2 = 1$, where $\mathbf{1} = [1, 1, \cdots, 1]^{\mathrm{T}}$.*

In order to solve this problem, we first notice that for an arbitrarily given nonnegative integer ℓ, the Diophantine equation $\mathbf{1}^{\mathrm{T}}\mathbf{s} = \ell, \mathbf{s} \in \mathbb{Z}_+^T$ has exactly $\frac{(\ell+T-1)!}{\ell!(T-1)!}$ solutions. Therefore, when ℓ runs from $\ell = 0$ to $\ell = \bar{L}$, there are $\sum_{\ell=0}^{\bar{L}} \frac{(\ell+T-1)!}{\ell!(T-1)!}$ solutions in total. In addition, we note that $\sum_{\ell=0}^{\bar{L}} \frac{(\ell+T-1)!}{\ell!(T-1)!} = \frac{T!}{0!T!} + \frac{T!}{1!(T-1)!} + \cdots + \frac{(\bar{L}+T-1)!}{\bar{L}!(T-1)!} = \frac{(T+1)!}{T!} + \frac{(T+1)!}{2!(T-1)!} + \cdots + \frac{(\bar{L}+T-1)!}{\bar{L}!(T-1)!} = \frac{(T+\bar{L}-1)!}{(\bar{L}-1)!T!} + \frac{(T+\bar{L}-1)!}{\bar{L}!(T-1)!} = \frac{(T+\bar{L})!}{\bar{L}!T!}$. Let L to be the smallest positive integer satisfying $\frac{(T+L)!}{L!T!} \ge 2^K$. If we define

$$\mathcal{S}_\ell = \{\mathbf{s} \in \mathbb{Z}_+^T : \mathbf{1}^{\mathrm{T}}\mathbf{s} = \ell\} \quad \textbf{(3)}$$

for $\ell = 0, 1, \cdots, L-1$ and

$$\bar{\mathcal{S}}_L = \left\{\mathbf{s}_l \in \mathbb{Z}_+^T : \mathbf{1}^{\mathrm{T}}\mathbf{s}_l = L, l = 1, 2, \cdots, 2^K - \frac{(T+L-1)!}{(L-1)!T!}\right\} \quad \textbf{(4)}$$

then, the cardinalities of $\mathcal{S}_\ell$ and $\bar{\mathcal{S}}_L$ are given by $\frac{(\ell+T-1)!}{\ell!(T-1)!}$ for $\ell = 0, 1, \cdots, L-1$ and $2^K - \frac{(T+L-1)!}{(L-1)!T!}$, respectively. Now, let us consider any constellation $\tilde{\mathcal{S}} \subseteq \mathbb{Z}_+^T$ such that the cardinality of $\tilde{\mathcal{S}}$ is 2^K and $\min_{\mathbf{s}\neq\tilde{\mathbf{s}},\mathbf{s},\tilde{\mathbf{s}}\in\tilde{\mathcal{S}}} \|\mathbf{s}-\tilde{\mathbf{s}}\|_2 = 1$. It is also noticed that we can always decompose $\tilde{\mathcal{S}}$ into a union of its $(L+1)$ disjoint subconstellations such that $\tilde{\mathcal{S}} = \cup_{\ell=0}^{L} \tilde{\mathcal{S}}_\ell$, where $\tilde{\mathcal{S}}_\ell = \{\tilde{\mathbf{s}}_{\ell,1}, \cdots, \tilde{\mathbf{s}}_{\ell,\frac{(\ell+T-1)!}{\ell!(T-1)!}}\}$ with $\mathbf{1}^{\mathrm{T}}\tilde{\mathbf{s}}_{\ell,i} \le \mathbf{1}^{\mathrm{T}}\tilde{\mathbf{s}}_{\ell,i+1}$ for $i = 1, \cdots, \frac{(\ell+T-1)!}{\ell!(T-1)!} - 1$, $\mathbf{1}^{\mathrm{T}}\tilde{\mathbf{s}}_{\ell,\frac{(\ell+T-1)!}{\ell!(T-1)!}} \le \mathbf{1}^{\mathrm{T}}\tilde{\mathbf{s}}_{\ell+1,1}$ for $\ell = 1, \cdots, L-1$, and $\mathbf{1}^{\mathrm{T}}\tilde{\mathbf{s}}_{L,i} \le \mathbf{1}^{\mathrm{T}}\tilde{\mathbf{s}}_{L,i+1}$, $i = 1, \cdots, 2^K - \frac{(T+L-1)!}{(L-1)!T!} - 1$. Then, we claim that $\mathbf{1}^{\mathrm{T}}\tilde{\mathbf{s}}_{\ell,1} \ge \ell$ for $0 \le \ell \le L$. We prove this claim by induction. When $\ell = 0$, it is indeed true. Let us assume that $\mathbf{1}^{\mathrm{T}}\tilde{\mathbf{s}}_{\ell,1} \ge \ell$ is true for $\ell = J$ with $1 \le J \le L-1$. For $\ell = J+1$, if $\mathbf{1}^{\mathrm{T}}\tilde{\mathbf{s}}_{\ell,1} \ge \ell$ is not true, then, our assumption that $J \le \mathbf{1}^{\mathrm{T}}\tilde{\mathbf{s}}_{J,1} \le \mathbf{1}^{\mathrm{T}}\tilde{\mathbf{s}}_{J,2} \le \cdots \mathbf{1}^{\mathrm{T}}\tilde{\mathbf{s}}_{J,\frac{(J+T-1)!}{J!(T-1)!}} \le \mathbf{1}^{\mathrm{T}}\tilde{\mathbf{s}}_{J+1,1} < J+1$ and $\tilde{\mathcal{S}} \subseteq \mathbb{Z}_+^T$ gives us that $\mathbf{1}^{\mathrm{T}}\tilde{\mathbf{s}}_{J,1} = \mathbf{1}^{\mathrm{T}}\tilde{\mathbf{s}}_{J,2} = \cdots \mathbf{1}^{\mathrm{T}}\tilde{\mathbf{s}}_{J,\frac{(J+T-1)!}{J!(T-1)!}} = \mathbf{1}^{\mathrm{T}}\tilde{\mathbf{s}}_{J+1,1} = J$. As a result, we attain $\frac{(J+T-1)!}{J!(T-1)!} + 1$ nonnegative integer solutions to $\mathbf{1}^{\mathrm{T}}\mathbf{s} = J$, contradicting with the fact that $\mathbf{1}^{\mathrm{T}}\mathbf{s} = J, \mathbf{s} \in \mathbb{Z}_+^T$ has at most $\frac{(J+T-1)!}{J!(T-1)!}$ solutions. Therefore, $\mathbf{1}^{\mathrm{T}}\tilde{\mathbf{s}}_{\ell,1} \ge \ell$ is also true for $\ell = J+1$. In addition, by our assumption that $\mathbf{1}^{\mathrm{T}}\tilde{\mathbf{s}}_{\ell,i} \le \mathbf{1}^{\mathrm{T}}\tilde{\mathbf{s}}_{\ell,i+1}$ for $\ell = 0, \cdots, L-1, i = 1, \cdots, \frac{(\ell+T-1)!}{\ell!(T-1)!} - 1$ and $\mathbf{1}^{\mathrm{T}}\tilde{\mathbf{s}}_{L,i} \le \mathbf{1}^{\mathrm{T}}\tilde{\mathbf{s}}_{L,i+1}$ for $i = 1, \cdots, 2^K - \frac{(T+L-1)!}{(L-1)!T!} - 1$, we attain $\sum_{\tilde{\mathbf{s}}\in\tilde{\mathcal{S}}} \mathbf{1}^{\mathrm{T}}\tilde{\mathbf{s}} \ge \sum_{\mathbf{s}\in\mathcal{S}} \mathbf{1}^{\mathrm{T}}\mathbf{s}$. Therefore, the optimal solution to Problem 1 is given below:

Optimal Solution to Problem 1: *For any fixed positive integers T and K, let L be the smallest positive integer satisfying $\frac{(T+L)!}{L!T!} \ge 2^K$. Then, the optimal solution to Problem 1 is given by $\mathcal{S} = \cup_{\ell=0}^{L-1} \mathcal{S}_\ell \cup \bar{\mathcal{S}}_L$, where $\mathcal{S}_\ell$ and $\bar{\mathcal{S}}_L$ are determined by the respective* Eq. (3) *and* Eq. (4) .

We would like to make the following four observations on the above solution.

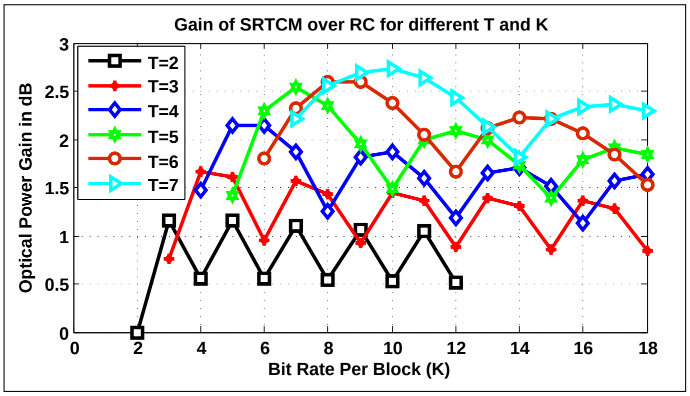

Fig. 1. Optical power gains of SRTCM over RC for different T and K.

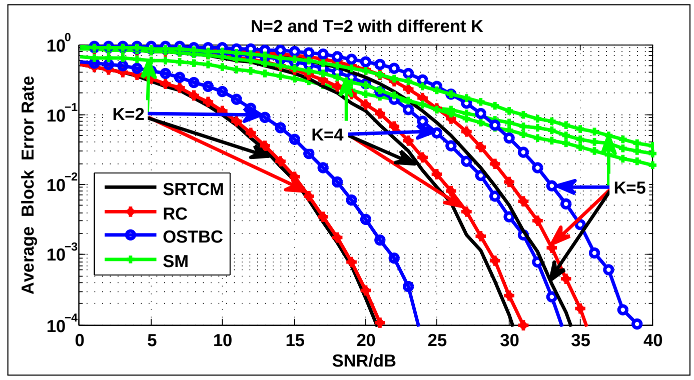

Fig. 2. Error performance comparisons for $T = 2$ with LED 1 and LED 2.

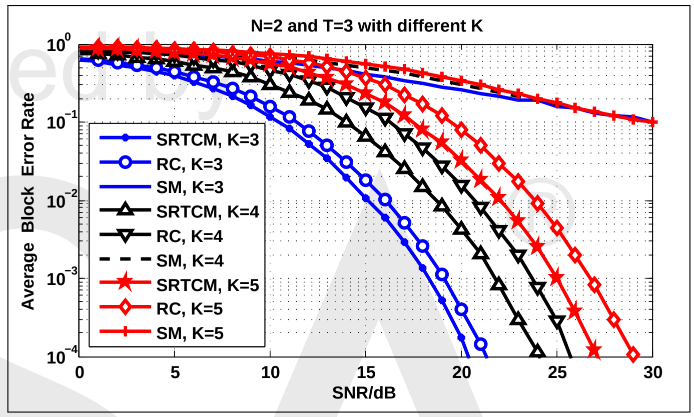

Fig. 3. Error performance comparisons for $T = 3$ with LED 1 and LED 2.

1) *Is STBC necessary for VLC?* Up to now, STBC has been well understood to improve the spectral efficiency of a multi-antennas MIMO RF wireless communication system. In particular, orthogonal STBC is an attractive example to enable full diversity with a simple ML detector. However, when applied to an optical channel with direct detection, it has worse error performance than RC [7]. This strong evidence seems to demonstrate that STBC may not be necessary for the VLC systems [7]. Fortunately, our designed SRTCM is actually a specific STBC which performs better than RC.

2) *Coded modulation*. The essence of SRTCM is to allow the transmitted signals to be cooperative in multiple time dimensions for increasing channel reliability. At this point, TCM designed by Eq. (3) and Eq. (4) can be viewed as a new kind of coded modulation for the specific VLC systems, which is different from the trellis coded modulation proposed by Ungerboeck in [10, 11] for the RF digital communication systems.

3) *Optimal multi-dimensional constellation*. The TCM constellation constructed by Eq. (3) and Eq. (4) is optimal only within $\mathbb{Z}_+^T$ and may not be optimal within the positive orthants of a multi-dimensional real space. Attaining a general optimal solution is as hard as solving a parallel and long-standing well known optimization problem in modern RF digital communications, which, to the best knowledge of the authors, still remains unsolved thus far [12–15].

4) *Optical power gain*. To further put the performance gains of SRTCM over RC into perspective, we compute the optical power gain by $20\log_{10}\frac{\sum_{\mathbf{s}\in\mathcal{P}}\mathbf{1}^T\mathbf{s}}{\sum_{\mathbf{s}\in\mathcal{S}}\mathbf{1}^T\mathbf{s}}$ for different T and K as shown in Fig. 1, from which we can observe that with T increasing, the substantial gains for various K are attained accordingly. These results can also be generalized to MIMO VLC systems. It can be expected that more gains will be obtained.

To examine the average error performance of our proposed SRTCM in this letter, we carry out comprehensive computer simulations to compare our design with the currently available SM [4, 5], RC and OSTBC [7], in a 4.0m × 4.0m × 3.0m room. We assume that $\Phi_{\frac{1}{2}} = \Psi = 60^\circ$, $A = 1\text{cm}^2$ and $N = 4$. The locations of the four transmitter LEDs are given by $(1.0\text{m}, 1.0\text{m}, 3.0\text{m})$, $(3.0\text{m}, 3.0\text{m}, 3.0\text{m})$, $(1.0\text{m}, 3.0\text{m}, 3.0\text{m})$ and $(3.0\text{m}, 1.0\text{m}, 3.0\text{m})$, and for notational simplicity, designated by LED 1, LED 2, LED 3 and LED 4, respectively. The receiver PD is located at $(x\text{m}, y\text{m}, 0\text{m})$. To evaluate the average error performance within the illumination coverage area, we assume that both x and y are uniformly chosen from the interval $(0, 4)$. The channel coefficients of each channel realization are computed based on Eq. (1) and normalized by the maximum channel coefficient. With the average optical power per transmission being normalized, SNR is defined by $\frac{1}{\sigma^2}$. To make all comparisons as fair as possible, the receivers for all the transmission schemes are the optimal ML detectors. In addition, specifically for RC, the time-equivalent constellation $\mathcal{P} = \{m\}_{m=0}^{m=2^{K_1}-1} \times \cdots \times \{m\}_{m=0}^{m=2^{K_T}-1}$ is optimally determined by properly selecting positive integer K_i such that $\sum_{\mathbf{s}\in\mathcal{P}}\mathbf{1}^T\mathbf{s}$ is minimized. More details on simulations are described as follows.

Figs. 2 and 3 show the average block error rate when the transmitters are LED 1 and LED 2. From Figs. 2 and 3, we can see that SRTCM has better error performances than SM, RC and OSTBC. Notice that the performance of SM reaches an error floor, since when h_1 and h_2 are equal, signals can not be uniquely identified, even for noise-free channels. For this reason, SM requires more than two PDs to establish a viable link [6]. In addition, from Fig. 2, when $T = K = 2$, the error performance of SRTCM is almost the same as that of RC. The reason for this phenomenon to occur is that when $T = K = 2$, our designed TCM constellation $\mathcal{S}$ has the same energy efficiency as the time-equivalent constellation $\mathcal{P}$ of RC. However, when $K = 4, 5$, SRTCM has a respective SNR advantage of about 0.5 dB and 1 dB over RC at the error rate of 10^{-4}. When $T = 3$, as shown in Fig. 3, the attained SNR gains of SRTCM over RC for $K = 3, 4, 5$ at a target error rate 10^{-4} are about 0.7 dB, 1.6 dB and 1.5 dB, respectively. These performance gains are dependent on

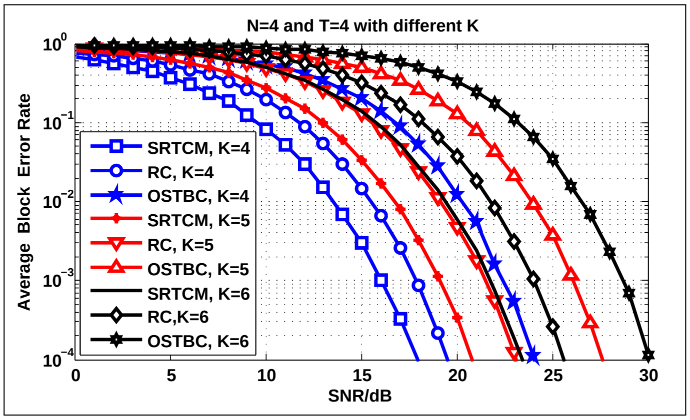

Fig. 4. Error performance comparisons for $N = 4$ and $T = 4$.

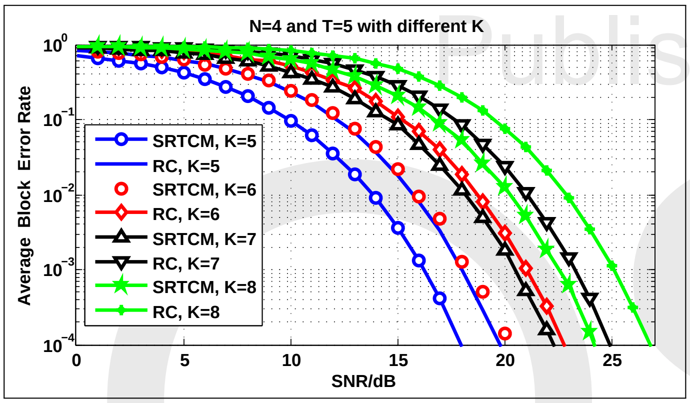

Fig. 5. Error performance comparisons for $N = 4$ and $T = 5$.

T and K as well as on the energy-efficient TCM constellation. For $T = 4, 5, 6$, the respective error performance comparisons are shown in Figs. 4, 5 and 6. It can be noticed that substantial gains are attained by our proposed SRTCM when compared with RC and OSTBC. For instance, as shown in Fig. 6, when $T = 6$ and the target error rate is 10^{-4} , the SNR gains by SRTCM compared with RC are almost 1.6 dB, 2.3 dB, 2.6 dB and 2.5 dB for $K = 6, 7, 8, 9$, respectively. Therefore, for indoor MISO VLC, when CSIT is unknown at the transmitter LEDs, SRTCM has superior error performance to RC. However, the tradeoff between performance and rate remains unknown. Therefore, we propose the use of SRTCM for MISO VLC systems.

To summarize, with an optimally designed multi-dimensional constellation, a novel energy-efficient transmission scheme called SRTCM has been proposed for the MISO VLC system without CSIT by repeatedly transmitting signals across space dimensions as well as by collaboratively modulating the signals through multiple time dimensions. Comprehensive computer simulations have shown that SRTCM attains substantial performance gains over the currently available schemes. Here, it should be mentioned that despite the fact that our proposed SRTCM utilizing both space and time dimensions can be viewed as the first example of STBC providing better error performance gains than RC for any nonnegative channel, we have to perform an exhaust search to implement ML detection for demodulating the resulting highly non-linear TCM, whose complexity is $\mathcal{O}(2^K)$. Hence, a fast ML demodulation algorithm is under our future investigation.

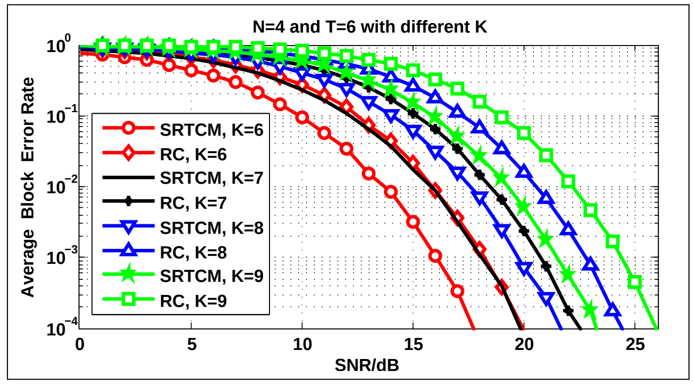

Fig. 6. Error performance comparisons for $N = 4$ and $T = 6$.

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