TRANSCEIVER PAIR DESIGNS FOR MULTIPLE ACCESS CHANNELS UNDER FIXED SUM MUTUAL INFORMATION USING MMSE DECISION FEEDBACK DETECTION

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ABSTRACT

In this paper, we consider the joint design of the transceivers for a multiple access Multiple Input and Multiple Output (MIMO) system having Inter-Symbol Interference (ISI) channels. The system we consider is equipped with the Minimum Mean Square Error (MMSE) Decision-Feedback (DF) detector. Traditionally, transmitter designs for this system have been based on constraints of either the transmission power or the signal-to-interference-and-noise ratio (SINR) for each user. Here, we explore a novel perspective and examine a transceiver design which is based on a fixed sum mutual information constraint and minimizes the arithmetic average of mean square error of MMSE-decision feedback detection. For this optimization problem, a closed-form solution is obtained and is achieved if and only if the averaged sum mutual information is uniformly distributed over each active subchannel. Meanwhile, the mutual information of the currently detected user is uniformly distributed over each individual symbol within the block signal of the user, assuming all the previous user signals have been perfectly detected.

1. INTRODUCTION

We consider a block-based synchronous multiple access frequency selective MIMO channel in which the users' data sequences are precoded separately and are transmitted over distinct ISI channels. If we denote the signal vector for the kth user as $\mathbf{x}_k$, $k = 1, \cdots, K$, then, the received signal vector can be represented by

$$\mathbf{y} = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{T}_k \mathbf{x}_k + \boldsymbol{\xi} \tag{1}$$

where $\mathbf{H}_k$ is a $P \times M$ block Toeplitz tall channel matrix corresponding to zero-padded modulation or an $M \times M$ square block diagonal channel matrix corresponding to Discrete Multiple Tone (DMT) modulation [1–4] for the kth user, and $\mathbf{T}_k$ is an $M \times N_k$ precoder matrix for the kth user, and $\boldsymbol{\xi}$ is the zero-mean Gaussian noise vector. Our task in this paper is to obtain an optimum design for all the K transceivers for such a system using MMSE-decision feedback detection.

In recent years, for the single-user MIMO system using linear receivers, there exist solutions to a large number of precoder design problems [5, 6], including the maximization of information rate [7] and of SNR [8], the minimization of the mean squared error [8] and of bit error probability for both zero-forcing [9] and MMSE equalization [10]. For a multiuser system using linear MMSE receivers, the joint design of transceivers minimizing the total MSE was efficiently implemented by solving a convex optimization problem [11, 12]. MIMO systems having DF detectors have also been studied. For single-user systems equipped with ZF-DF [13] and MMSE-DF [14, 15] detectors, closed-form optimal transceivers have been obtained using the equal diagonal QRS decomposition of a matrix [13]. However, these solutions for the single-user system cannot be directly generalized to a multiuser case.

In this paper, we examine the optimum design of transceivers of a MIMO system using DF detectors in a multiuser environment. We note that all existing transceiver designs thus far have been pursued based on solving some optimization problems subject to some power constraints or to some SINR constraints on each user. In this paper, we explore a novel perspective and examine the transceiver design by minimizing the mean square error for K users subject to a fixed sum mutual information constraint. Here, we focus our consideration on a block-by-block ISI multiple access MIMO system employing the MMSE-DF detector.

Notation: Matrices are denoted by uppercase boldface characters (e.g., $\mathbf{A}$), while column vectors are denoted by lowercase boldface characters (e.g., $\mathbf{b}$). The (i, j)-th entry of $\mathbf{A}$ is denoted by $A_{i,j}$. The i-th entry of $\mathbf{b}$ is denoted by b_i. The columns of an $M \times N$ matrix $\mathbf{A}$ are denoted by $\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_N$. Notation $\mathbf{A}_k$ denotes a matrix consisting of the first k columns of $\mathbf{A}$, i.e., $\mathbf{A}_k = [\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_k]$. By convention, $\mathbf{A}_0 = 1$. The j-th diagonal entry of a matrix $\mathbf{A}$ is denoted by $[\mathbf{A}]_j = A_j$. The Hermitian transpose of $\mathbf{A}$ (i.e., the conjugate and transpose of $\mathbf{A}$) is denoted by $\mathbf{A}^H$.

2. THE INVERSE WATER-FILLING SOLUTION

We first provide the inverse water-filling solution on which our transceiver design is based. Consider a general single-user MIMO system such that

$$\mathbf{y} = \mathbf{H}\mathbf{T}\mathbf{x} + \boldsymbol{\xi} \tag{2}$$

where $\mathbf{H}$ is an $P \times M$ complex matrix, $\mathbf{T}$ is an $M \times N$ matrix and $\boldsymbol{\xi}$ is an $P \times 1$ Gaussian noise vector with a covariance matrix $\boldsymbol{\Xi}$. It is well known that if the channel matrix $\mathbf{H}$ in (2) is known at both the transmitter and the receiver, then, the Gaussian mutual information of model (2) is given by $\mathcal{I} = \log\det(\mathbf{I} + \mathbf{T}^H\mathbf{H}^H\boldsymbol{\Xi}^{-1}\mathbf{H}\mathbf{T})$ [16]. Thus, subject to a power constraint $\operatorname{tr}(\mathbf{T}^H\mathbf{T}) \leq p$, the channel capacity is achieved when the optimal transmitter $\widetilde{\mathbf{T}}$ is the water-filling solution [17]. In the inverse water-filling problem, instead of constraining the power, the total transmission power is minimized subject to a fix Gaussian mutual information, which can be stated as

Problem 1 *Find an optimal transmitter* $\mathbf{T}$ *such that*

$$\begin{array}{ll} \min\limits_{\mathbf{T}} & tr(\mathbf{T}^H\mathbf{T}) \\ \text{s.t.} & \log\det(\mathbf{I}+\mathbf{T}^H\mathbf{H}^H\mathbf{H}\mathbf{T}) = \mathcal{I} \end{array}$$

where, for simplicity, we have assumed the channel noise is uncorrelated. If the eigenvalue decomposition of $\mathbf{H}^H\mathbf{H}$ is $\mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$ with eigenvalues λ_i arranged in non-increasing order, then, the optimal solution to the Problem 1 is given by $\mathbf{T} = \mathbf{U}_r\mathbf{\Gamma}^{1/2}\mathbf{S}$, where $\mathbf{S}$ is an arbitrary unitary matrix, $\mathbf{U}_r$ consists of the first r columns of the unitary matrix $\mathbf{U}$, $\mathbf{\Gamma} = \mathrm{diag}(\gamma_1, \gamma_2, \cdots, \gamma_r)$ with each γ_n determined by

$$\gamma_n = \left(\frac{2^{\mathcal{I}}}{\prod_{i=1}^{r}\lambda_i}\right)^{1/r} - \lambda_n^{-1} \tag{3}$$

and r is the largest integer index, not exceeding M, for the eigenvalues such that

$$\lambda_k > \left(\frac{\prod_{i=1}^{r}\lambda_i}{2^{\mathcal{I}}}\right)^{1/r} \qquad \text{for } k = 1, 2, \cdots, r \tag{4}$$

Similar research work to Problem 1 can be found in [18, 19]

3. JOINT DESIGN OF TRANSCEIVER FOR MULTIPLE ACCESS CHANNELS

With the inverse water-filling solution in mind, we now consider the joint design of the transceivers for a multiple access ISI MIMO system equipped with the MMSE-DFE. The goal of our design is to minimize the arithmetic MSE for the K users subject to a fixed sum mutual information constraint.

3.1. Problem Statement and Formulation

In general, for a DF receiver, signals are detected in the reverse order of the user index, i.e., we first detect the signal from User K, then that from User $K-1$, and so on. Based on this detection order, we thus re-write the received signal of (1) as

$$\begin{aligned} \mathbf{y} - \sum_{i=k+1}^{K}\mathbf{H}_i\mathbf{T}_i\mathbf{x}_i &= \mathbf{H}_k\mathbf{T}_k\mathbf{x}_k + \underbrace{\sum_{\ell=1}^{k-1}\mathbf{H}_\ell\mathbf{T}_\ell\mathbf{x}_\ell + \boldsymbol{\xi}}_{\boldsymbol{\zeta}_k} \\ &= \mathbf{H}_k\mathbf{T}_k\mathbf{x}_k + \boldsymbol{\zeta}_k, \quad k = K, \cdots, 1. \end{aligned} \tag{5}$$

where $\boldsymbol{\zeta}_k$ is the kth interference-plus-noise vector. In Eq. (5), the MMSE-DFE is used to detect $\mathbf{x}_k$ from the received signal $\mathbf{y}$ by successively cancelling the previously detected user signals.

Let $\mathbf{B}$ and $\mathbf{F}$ be the feedback and feedforward matrices of the MMSE-DFE respectively. Exploiting the orthogonality principle [20] and using the Matrix Inversion Lemma [21] leads the optimum $\mathbf{F}_{MMSE,k}$ which results in the error covariance matrix for User k being given by [14, 22–24]

$$\begin{aligned} \mathbf{A}_k &= E[\mathbf{e}_k\mathbf{e}_k^H] \\ &= \mathbf{W}_k(\mathbf{J}_k)^{-1}(\mathbf{W}_k)^H \\ &= \mathrm{diag}([\mathbf{R}_k]_1^{-2}, [\mathbf{R}_k]_2^{-2}, \cdots, [\mathbf{R}_k]_{N_k}^{-2}). \end{aligned} \tag{6}$$

where

$$\begin{aligned} \mathbf{J}_k &= \mathbf{I} + (\mathbf{H}_k\mathbf{T}_k)^H(\mathbf{\Sigma}_k)^{-1}\mathbf{H}_k\mathbf{T}_k \\ \mathbf{\Sigma}_k &= E[\boldsymbol{\zeta}_k\boldsymbol{\zeta}_k^H] = \mathbf{I} + \sum_{\ell=1}^{k-1}\mathbf{H}_\ell\mathbf{T}_\ell(\mathbf{H}_\ell\mathbf{T}_\ell)^H \\ & \quad \text{for } k = 1, 2, \cdots, K \text{ and } \mathbf{\Sigma}_1 = \mathbf{I} \\ \mathbf{W}_k &= \mathbf{B}_k + \mathbf{I} \end{aligned} \tag{7}$$

and $\mathbf{R}$ is the upper triangular matrix obtained by the QR-decompostion of $\mathbf{J}^{1/2}$. We note that $\mathbf{W}_k$ is an upper-triangular matrix with unit diagonal entries. If we define the average MSE of the K users of the successive cancellation detector as

$$\mathcal{E} \triangleq \frac{1}{N}\sum_{k=1}^{K}\mathrm{tr}\left(\mathrm{E}[\mathbf{e}_k\mathbf{e}_k^H]\right) = \frac{1}{N}\sum_{k=1}^{K}\mathrm{tr}\left(\mathbf{A}_k\right) \tag{8}$$

where $N = \sum_{k=1}^{K}N_k$, our optimization problem can be formally stated as follows:

Problem 2 *Let* $\mathrm{rank}(\mathbf{H}_k) = L_k$, $k = 1, 2, \cdots, K$. *Then, given* K *non-negative integers* $N_1, N_2, \cdots, N_K$ *with* $N_k \leq L_k$, *find the matrix sequence* $\{\mathbf{T}_k\}_{k=1}^{K}$ *such that*

1. *the MMSE for the* K *users of the MMSE-DF detection is first minimized, subject to a fixed sum mutual information constraint, i.e.,*

$$\{\widetilde{\mathbf{T}}_k\}_{k=1}^{K} = \arg\min\ \mathcal{E} \tag{9}$$

s.t.

$$\mathcal{I} = \log\det\left(\mathbf{I} + \sum_{k=1}^{K}\mathbf{H}_k\mathbf{T}_k\mathbf{T}_k^H\mathbf{H}_k^H\right) \tag{10}$$

2. *then, with respect to all the remaining free parameters, the transmission power for each of the* k*-th user is minimized respectively.*

3.2. Closed-form optimal solution

To solve Problem 2, we employ the inequality relationship between the trace and determinant of a square matrix so that the total system error of the MMSE-DFE in (8) is lower-bounded by

$$\begin{aligned} \mathcal{E} &\geq \frac{1}{N}\sum_{k=1}^{K}N_k\det\left(\mathbf{J}_k^{-1/N_k}\right) \quad (11) \\ &= \frac{1}{N}\sum_{k=1}^{K}N_k\det\Big(\mathbf{I}+\mathbf{T}_k^H\mathbf{H}_k^H(\mathbf{\Sigma}_k)^{-1}\mathbf{H}_k\mathbf{T}_k\Big)^{\frac{-1}{N_k}} \\ &= \frac{1}{N}\sum_{k=1}^{K}N_k\frac{\det(\mathbf{\Sigma}_k)^{1/N_k}}{\det(\mathbf{\Sigma}_{k+1})^{1/N_k}} \\ &\geq \det(\mathbf{\Sigma}_{K+1})^{-1/N} = 2^{-\frac{\mathcal{I}}{N}} \quad (12) \end{aligned}$$

Equality in (11) holds if and only if matrices $\mathbf{J}_k^{1/2}$ have equal diagonal R-factors, i.e., in the DF receiver, the mutual information of the currently detected user is uniformly distributed over each individual symbol within the block signal of the user when all the previous user signals have been perfectly detected. Equality in (12) holds if and only if $\det(\mathbf{\Sigma}_k)$ constitutes a geometrical sequence, i.e.,

$$\left(\frac{\det(\mathbf{\Sigma}_1)}{\det(\mathbf{\Sigma}_2)}\right)^{1/N_1} = \cdots = \left(\frac{\det(\mathbf{\Sigma}_K)}{\det(\mathbf{\Sigma}_{K+1})}\right)^{1/N_K} \tag{13}$$

which means the averaged sum mutual information is uniformly distributed over each active subchannel and is equivalent to

$$\det(\mathbf{J}_k) = 2^{\frac{N_k}{N}\mathcal{I}} \tag{14}$$

Therefore, solving Problem 2 using the principle of the Inverse Water-filling solution is finally reduced to solving the following optimization problem:

Formulation 1 *For any given K non-negative integers $N_1, N_2, \cdots, N_K$ with $N_k \leq L_k$, find a sequence of matrices $\{\mathbf{T}_k\}_{k=1}^{K}$ such that*

1. *the total power for the kth user is minimized subject to the constraints that the mutual information for User k is $\mathcal{I}_k = \log\det(\mathbf{J}_k) = \frac{N_k}{N}\mathcal{I}$.*
2. *within the space of the remaining parameters, (11) holds with equality.*

Now, parallel to the result on the inverse water-filling for the single-user system [**?**], we can obtain the following closed-form solution to Problem 2 yielding the following:

Theorem 1 *Given any K non-negative integers $N_1, N_2, \cdots, N_K$ with $N_k \leq L_k$, let*

$$\mathcal{A}_k = \mathbf{H}_k^H(\mathbf{\Sigma}_k)^{-1}\mathbf{H}_k \qquad \text{for } k = 1, 2, \cdots, K$$

and let the eigen value decomposition of $\mathcal{A}_k$ be $\mathcal{A}_k = \mathbf{U}_k\mathbf{\Lambda}_k(\mathbf{U}_k)^H$ with the diagonal elements in $\mathbf{\Lambda}_k$ arrange in non-increasing order. Then, the optimal solution to Problem 2 is given by

$$\widetilde{\mathbf{T}}_k = \mathbf{U}_{N_k,k}(\mathbf{\Gamma}_k)^{1/2}\mathbf{S}_k, \qquad k = 1, 2, \cdots, K \tag{15}$$

where $\mathbf{U}_{N_k,k}$ is the first N_k columns of $\mathbf{U}_k$, $\mathbf{S}_k$ is an $N_k \times N_k$ unitary matrix denoting the S-factors of the QRS decomposition of $\mathbf{J}_k^{1/2}$, and N_k is a pre-assigned subchannel number for the kth user. For the k-th user, let r_k be the maximal positive integers such that

$$\lambda_{n,k} > \left(\frac{\prod_{i=1}^{r_k}\lambda_{i,k}}{2^{\mathcal{I}_k}}\right)^{1/r_k} \qquad \text{for } n = 1, 2, \cdots, r_k. \tag{16}$$

If $N_k \leq r_k$, the diagonal entries of $\mathbf{\Gamma}_k$ are determined by

$$\gamma_{n,k} = \left(\frac{2^{\mathcal{I}_k}}{\prod_{i=1}^{N_k}\lambda_{i,k}}\right)^{1/N_k} - (\lambda_{n,k})^{-1} \tag{17}$$

for $n = 1, 2, \cdots, N_k$. If $N_k > r_k$, the diagonal entries of Γ_k are assigned by

$$\gamma_{n,k} = \begin{cases} \left(\frac{2^{\mathcal{I}_k}}{\prod_{i=1}^{r_k}\lambda_{i,k}}\right)^{1/r_k} - (\lambda_{n,k})^{-1} & n = 1, \cdots, r_k \\ 0 & n = r_k + 1, \cdots, N_k \end{cases}$$

4. SIMULATIONS

In this section, we verify the performance of our optimal transceiver design using computer simulations. Three examples are shown in the following:

1. Fig. 1 shows the scenario of a two-user system. In this example, each user employs a DMT modulation having 32 subcarriers and a channel length of 10. Fig. 1 shows the BER against the sum mutual information averaged over 100 channel realizations. Three cases are simulated: $a)$ The number of subcarriers assigned to User 1 and User 2 is 16 each ($N_k = 16, k = 1, 2$); $b)$ $N_1 = 16$ and $N_2 = 17$, i.e., there is at least one shared subchannel; $c)$ $N_1 = 17$ and $N_2 = 17$, i.e., there are at least two subchannels shared by these two users.
2. Fig. 2 shows a three-user scenario. In this example, each user employs a DMT modulation having 32 subcarriers. Again, the channels have a memory size of 10. Fig. 2 shows the BER against the sum mutual information averaged over 100 channel realizations. Three cases are simulated with 100 channel realizations: $a)$ $N_1 = 11$, $N_2 = 11$, and $N_3 = 10$; $b)$ $N_1 = 11$, $N_2 = 11$, and $N_3 = 11$, i.e., at least one subchannel is shared; $c)$ $N_1 = 12$, $N_2 = 11$, and $N_3 = 11$, i.e., at least two subchannels are shared.
3. Fig. 3 shows a two-user scenario. Here, our transceiver design is compared with the linear transceiver design discussed in [12]. In this example, the simulation environment is the same as Example 1. To ensure a fair comparison, the sum information and numbers of subcarriers assigned in our design are calculated from the counterpart algorithm in [12]. 200 channel realizations are simulated and taken average over information. Figure 4 shows the average bit error rate according to sum mutual information, while Fig. 4 shows transmission power vs information.

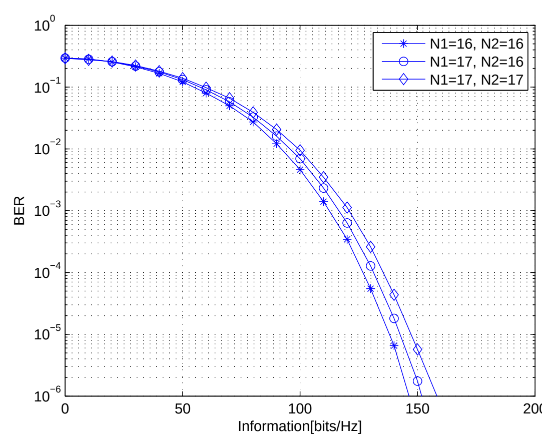

Fig. 1. BER vs information in two-user scenario

5. CONCLUSION

In this paper, the design of the transceivers for an ISI multiple-access MIMO communication system using MMSE-DF detection has been considered. The design goal is to minimize the MSE under a fixed sum mutual information. The optimal closed-form solution is obtained when the sum mutual information is uniformly distributed over each active subchannel and each individual symbol within the

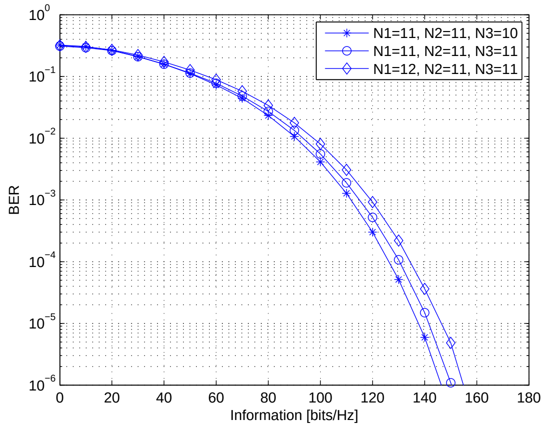

Fig. 2. BER vs information in three-user scenario

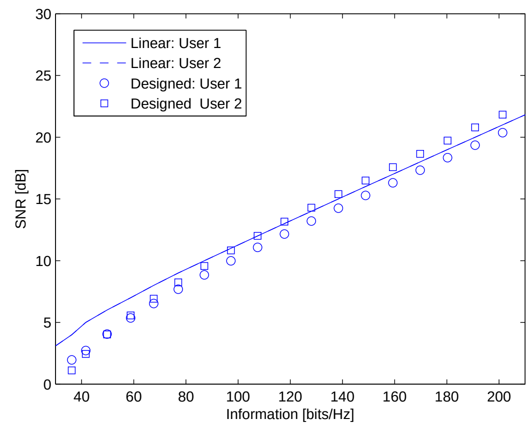

Fig. 4. SNR vs information: compared with linear MMSE detection

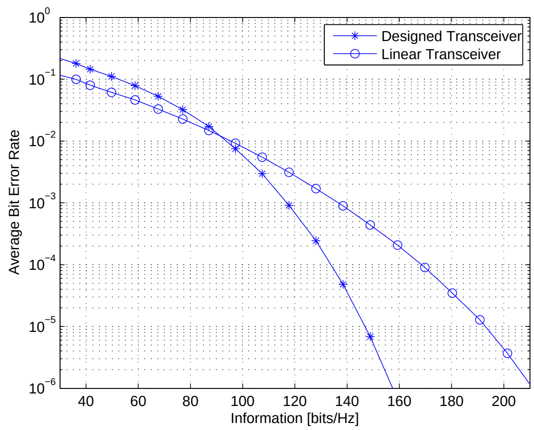

Fig. 3. BER vs information: compared with linear MMSE detection

block signal of the user. The latter condition is achieved by applying the QRS decomposition on the mutual information matrix of each user, rendering the symbol mutual information uniformly distributed among all the subcarriers.

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