BLIND UNIQUE IDENTIFICATION OF ALAMOUTI SPACE-TIME CODED CHANNEL VIA SIGNAL DESIGN AND TRANSMISSION TECHNIQUE

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ABSTRACT

In this paper, we present a simple signal design and transmission technique to uniquely and blindly identify Alamouti space-time coded channels under both noise-free and complex Gaussian noise environments in which pth-order and qth-order statistics (p and q are co-prime) of the received signals are available. A closed-form solution to determine the channel coefficients is obtained by exploiting specific properties of the Alamouti space-time code and the linear Diophantine equation theory. When only finite received data are given, we propose using the semi-definite relaxation algorithm to approximate maximum likelihood (ML) detection so that the joint estimation of the channel and symbols can be efficiently implemented. Simulation results show that our signal design and transmission method yields lower mean-square error in the estimation of the channel when compared to other existing methods and that the average symbol error rate approaches that of the coherent detector which needs perfect channel information at the receiver.

1. INTRODUCTION

In recent development of wireless communications, multiple antennas and space-time block codes (STBCs) technologies [1–4] have been employed to improve the spectral efficiency, while maintaining satisfactory performance over fading channels. In particular, orthogonal STBCs [4, 5] have attracted much attention because they can achieve maximum diversity and need only linear processing for the coherent ML receiver. To decode OSTBC effectively, however, perfect channel state information (CSI) is required at the receiver which, in practice, is not easily attainable since wireless channels usually change constantly. Although CSI can be obtained using training symbols, a substantial penalty has to be paid for the loss in bandwidth efficiency [6, 7]. This loss can be saved by using differential STBC (DSTBC) in flat fading channels [8, 9] which, unfortunately, involve an approximate loss of 3dB in performance compared to coherent detection. To arrive at a more satisfactory solution, blind channel estimation and decoding algorithms were proposed for OSTBC [12–14]. However, for the Alamouti space-time coded channel, there still exists the ambiguity issue [12–14] for which the transmitted symbols may not be determined uniquely.

In this paper, we propose a novel blind channel identification technique for the Alamouti space-time coded channel by properly designing the transmitted signals. Using this new strategy we prove that a) in the noise-free case, only two distinct pairs of symbols are needed to *uniquely* determine the channel coefficients, and b) in the case for which complex Gaussian noise are added and for which the pth-order and qth-order statistics on the received signals are available, the channel coefficients can also be uniquely determined. In both cases, simple closed-form solutions are obtained. In practice, when only finite received data are available, we propose to use the semi-definite relaxation (SDR) algorithm to approximate ML detection so that the joint estimation of the channel and symbols can be efficiently implemented.

Let us first examine the Alamouti space-time coded channel: Consider a wireless communication system with two transmitter antennas and a single receiver antenna in a flat-fading environment. For the Alamouti space-time coding scheme, each set of two transmitted symbols spans over two consecutive time slots which is designated a *frame*. Thus, at the ith frame, during the first time slot, $[s_{pi}, s_{qi}]$ are transmitted simultaneously from Antennas 1 and 2, respectively. During the second time slot of the ith frame, we transmit $-s_{qi}^*$ and s_{pi}^* respectively from Antennas 1 and 2. Therefore, at the receiver antenna, the received signal in the two consecutive time slots of the ith frame can be written as

$$\begin{bmatrix} z_i(1) \\ z_i(2) \end{bmatrix} = \begin{bmatrix} s_{pi} & s_{qi} \\ -s_{qi}^* & s_{pi}^* \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} \xi_i(1) \\ \xi_i(2) \end{bmatrix} \qquad (1)$$

where s_{pi} and s_{qi} are the two symbols chosen to be transmitted at the ith frame, h_1 and h_2 denote the respective channel coefficients from the transmitter Antenna 1 and 2 to the receiver antenna, and $\xi_i(1)$ and $\xi_i(2)$ denote complex circular white Gaussian noises with zero-mean and variance σ^2 in the two time slots. We assume that $|h_1|^2 + |h_2|^2 \neq 0$ and h_1, h_2 are constant within T time slots.

2. SIGNAL DESIGN AND JOINT ESTIMATION

We now develop a novel signal design technique for the Alamouti space-time coded channel so that the channel coefficients can be uniquely identified under noise-free and noisy conditions.

2.1. Noise-free case

We now show that two distinct received signal vectors can determine the channel coefficients and the transmitted symbols uniquely in a noise-free environment. Using the Alamouti scheme of transmission, for the ith frame, we assign the symbols s_{pi} and s_{qi} in Eq. (1) such that $s_{pi} \in \mathcal{S}_p$ and $s_{qi} \in \mathcal{S}_q$, where $\mathcal{S}_p$ and $\mathcal{S}_q$ are p-PSK and q-PSK constellations with p and q being co-prime positive integers. Therefore, for two consecutive frames, say i and $i+1$, we will send out the symbols $\{s_{pi}, s_{qi}\}, \{-s_{qi}^*, s_{pi}^*\}$ and $\{s_{p(i+1)}, s_{q(i+1)}\}, \{-s_{q(i+1)}^*, s_{p(i+1)}^*\}$ for the four consecutive time slots. At the receiver, if there is no noise, we can solve Eq. (1) for the four consecutive time slots in frames i and $i+1$,

$$\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} s_{pi}^* & -s_{qi} \\ s_{qi}^* & s_{pi} \end{bmatrix} \begin{bmatrix} z_i(1) \\ z_i(2) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} s_{p(i+1)}^* & -s_{q(i+1)} \\ s_{q(i+1)}^* & s_{p(i+1)} \end{bmatrix} \begin{bmatrix} z_{i+1}(1) \\ z_{i+1}(2) \end{bmatrix}$$

From this we can further obtain

$$\begin{bmatrix} z_{i+1}(1) \\ z_{i+1}(2) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} s_{p(i+1)} & s_{q(i+1)} \\ -s^*_{q(i+1)} & s^*_{p(i+1)} \end{bmatrix} \begin{bmatrix} s^*_{pi} & -s_{qi} \\ s^*_{qi} & s_{pi} \end{bmatrix} \begin{bmatrix} z_i(1) \\ z_i(2) \end{bmatrix} \quad (2)$$

Let $a = \frac{1}{2}(s_{p(i+1)} s^*_{pi} + s_{q(i+1)} s^*_{qi})$, $b = \frac{1}{2}(s_{pi} s_{q(i+1)} - s_{qi} s_{p(i+1)})$. Then, taking conjugates, Eq. (2) can be rewritten as

$$\begin{bmatrix} z_{i+1}(1) \\ z^*_{i+1}(2) \end{bmatrix} = \begin{bmatrix} z_i(1) & z_i(2) \\ z^*_i(2) & -z^*_i(1) \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \quad (3)$$

from which we obtain

$$\begin{aligned} a &= (z^*_i(1) z_{i+1}(1) + z_i(2) z^*_{i+1}(2))/(|z_i(1)|^2 + |z_i(2)|^2) \\ b &= (z^*_i(2) z_{i+1}(1) - z_i(1) z^*_{i+1}(2))/(|z_i(1)|^2 + |z_i(2)|^2) \quad (4) \end{aligned}$$

Substituting the definitions of a and b into (4) results in

$$\begin{aligned} s_{pi} s_{qi} a + b &= s_{pi} s_{q(i+1)} \quad (5a) \\ s_{pi} s_{qi} a - b &= s_{qi} s_{p(i+1)} \quad (5b) \end{aligned}$$

Now, the key problem is whether the quadratic equations (5a) and (5b) have a unique solution with respect to symbol variables s_{pn} and s_{qn}, $n = i,\ i+1$, for the given a and b in (4). The following theorem provides the answer:

Theorem 1 *Let s_{pi}, s_{qi} and $s_{p(i+1)}, s_{q(i+1)}$ be symbols selected to be transmitted in the time frames i and $i+1$ respectively such that $s_{pn} \in \mathcal{S}_p$ and $s_{qn} \in \mathcal{S}_q$, $n = i,\ i+1$, where $\mathcal{S}_p$ and $\mathcal{S}_q$ are p-PSK and q-PSK constellations with p and q being co-prime positive integers. Let $\mathbf{z}_i = [z_i(1), z_i(2)]^T$ and $\mathbf{z}_{i+1} = [z_{i+1}(1), z_{i+1}(2)]^T$ be two distinct received signal vectors from the Alamouti space-time coded channel within the two consecutive time frames (four time slots). Let a and b be given by Eqs. (4). Then, in noise-free case, there exists a unique pair of positive integers ℓ and k with $0 \le \ell \le p-1$ and $0 \le k \le pq-1$ such that*

$$\frac{b}{a - \exp\left(j\frac{2\pi\ell}{p}\right)} = \exp\left(j\frac{2\pi k}{p\,q}\right), \quad a \neq \exp\left(j\frac{2\pi\ell}{p}\right) \quad (6)$$

Furthermore, two pairs of the transmitted symbols $[s_{pi}, s_{qi}]$ and $[s_{p(i+1)}, s_{q(i+1)}]$ can be uniquely determined as follows:

$$s_{pi} = \exp\left(j\frac{2\pi}{p}\left(\frac{k}{q}\left(1 - p^{\varphi(q)}\right) + p\left\lceil\frac{kp^{\varphi(q)-1}}{q}\right\rceil\right)\right) \quad (7a)$$

$$s_{qi} = \exp\left(j\frac{2\pi}{q}\left(kp^{\varphi(q)-1} - q\left\lceil\frac{kp^{\varphi(q)-1}}{q}\right\rceil\right)\right) \quad (7b)$$

$$s_{p(i+1)} = \exp\left(j\frac{2\pi\ell}{p}\right) s_{pi} \quad (7c)$$

$$s_{q(i+1)} = \frac{2a - \exp\left(j\frac{2\pi\ell}{p}\right)}{a - \exp\left(j\frac{2\pi\ell}{p}\right)} b \cdot s^*_{pi} \quad (7d)$$

where $\varphi(q)$ is the Euler function [17], and $\lceil x \rceil$ denotes the greatest integer not exceeding x. In addition, the channel coefficients h_1 and h_2 can be further uniquely determined by

$$\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} s^*_{pi} & -s_{qi} \\ s^*_{qi} & s_{pi} \end{bmatrix} \begin{bmatrix} z_i(1) \\ z_i(2) \end{bmatrix} \quad (8)$$

We would like to make the following remarks on Theorem 1.

1. Theorem 1 not only tells us that the channel coefficients can be uniquely identified by transmitting two distinct symbol pairs from two co-prime constellations in the four time slots, but also provides simple and close-form solutions to both the channel coefficients and transmitted symbols.
2. In the noise-free case, two different received signal vectors are the smallest number of data requirement for the unique identification of the Alamouti space-time coded channel and symbols. In other words, from (1), one received signal vector will not enable us to determine the transmitted symbols $[s_{pi}, s_{qi}]$ or the channel coefficients h_1 and h_2.
3. In [12–14], it has been shown that if the transmitted symbols are chosen from the same constellation for transmission through the Alamouti space-time coded channel, then, information symbols and channel coefficients cannot be uniquely determined no matter how many data sets are sent.

2.2. Complex Gaussian noise case

Let us now consider the Alamouti space-time coded channel model with white complex Gaussian noises. We assume that at any time frame i, the two symbols s_{pi} and s_{qi} sent over the two transmitter antennas are independent and are equally likely chosen respectively from the p-PSK and q-PSK constellations, p and q being co-prime positive integers. In this case, the received signal vector can be expressed as

$$\begin{bmatrix} z_i(1) \\ z_i(2) \end{bmatrix} = \begin{bmatrix} s_{pi} & s_{qi} \\ -s^*_{qi} & s^*_{pi} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} \xi_i(1) \\ \xi_i(2) \end{bmatrix}, \qquad i = 1, 2, \cdots \quad (9)$$

Now we formally state our second main result without proof.

Theorem 2 *Let two positive integers p and q be co-prime. Then, we have*

$$\begin{cases} h_1^p = \mathrm{E}[z_i^p(1)] \\ h_1^q = (-1)^q \mathrm{E}[z_i^q(2)] \end{cases} \qquad \begin{cases} h_2^p = \mathrm{E}[z_i^p(2)] \\ h_2^q = \mathrm{E}[z_i^q(1)] \end{cases} \quad (10)$$

where $\mathrm{E}[\cdot]$ denotes the expectation operator. From this, the channel coefficients h_1 and h_2 can be uniquely determined by

$$h_1 = |\mathrm{E}[z_i^p(1)]|^{1/p} e^{j\theta_1},\ h_2 = |\mathrm{E}[z_i^p(2)]|^{1/p} e^{j\theta_2} \quad (11)$$

with

$$\begin{aligned} \theta_1 &= \frac{\arg(\mathrm{E}[z_i^p(1)]) + 2n_1\pi}{p} = \frac{\arg((-1)^q \mathrm{E}[z_i^q(2)]) + 2m_1\pi}{q} \\ \theta_2 &= \frac{\arg(\mathrm{E}[z_i^p(2)]) + 2n_2\pi}{p} = \frac{\arg(\mathrm{E}[z_i^q(1)]) + 2m_2\pi}{q} \end{aligned}$$

Here, $0 \le m_1, m_2 < q$ and $0 \le n_1, n_2 < p$. $\arg(\cdot)$ denotes the phase angle in an interval $0 < \arg(\cdot) \le 2\pi$. Integers m_1, m_2 and n_1, n_2 can be uniquely obtained by solving the following Diophantine equation using the Euclid algorithm,

$$\begin{aligned} \frac{\arg((-1)^q \mathrm{E}[z_i^q(2)])p - \arg(\mathrm{E}[z_i^p(1)])q}{2\pi} &= n_1 q - m_1 p \\ \frac{\arg(\mathrm{E}[z_i^q(1)])p - \arg(\mathrm{E}[z_i^p(2)])q}{2\pi} &= n_2 q - m_2 p \end{aligned}$$

with $0 \le m_1, m_2 < q$ and $0 \le n_1, n_2 < p$.

Theorem 2 shows that even in complex Gaussian noise case, our signal design approach is also able to provide a unique closed-form solution to the channel coefficients if the pth- and qth-order statistics of the received signals are available. However, in practice, the channel coefficients of a wireless communication system will change randomly from the one observation period to the next. Thus, only a limited number of samples will be available during one observation period. Under such conditions, it is well known that the optimal solution is the joint estimation of the channel and signals based on maximum likelihood (ML) detection. Unfortunately, in general, this is a *nondeterministic polynomial-time* (NP) hard problem, i.e., the globally optimal solution cannot be found in polynomial-time complexity. In the next section, taking advantage of the orthogonality of the Alamouti space-time code, we will exploit the semi-definite relaxation (SDR) method [10, 11] to efficiently approximate the ML decoder for our designed constellation.

2.3. Semi-definite relaxed maximum likelihood decoding

In this section, we will use ML detection to jointly estimate the channel coefficients and signals. Suppose we have received L time frames in which we receive L signal vectors $\{\mathbf{z}_i\},\ i = 1, 2, \cdots, L$ during $2L$ observable time slots where $\mathbf{z}_i = [z_i(1)\ \ z_i(2)]^T$. Let

$$\mathbf{X}_i = \begin{bmatrix} s_{pi} & s_{qi} \\ -s_{qi}^* & s_{pi}^* \end{bmatrix}, \quad i = 1, \cdots, L \tag{12}$$

Then, the received signal can be represented in a compact form as:

$$\mathbf{z} = \mathbf{X}\mathbf{h} + \boldsymbol{\xi} \tag{13}$$

where $\mathbf{z} = [\mathbf{z}_1^T, \mathbf{z}_2^T, \ldots, \mathbf{z}_L^T]^T$, $\mathbf{X} = [\mathbf{X}_1^T, \mathbf{X}_2^T, \cdots, \mathbf{X}_L^T]^T$, $\mathbf{h} = [h_1, h_2]^T$ and $\boldsymbol{\xi} = [\boldsymbol{\xi}_1^T, \boldsymbol{\xi}_2^T, \cdots, \boldsymbol{\xi}_L^T]^T$ with $\boldsymbol{\xi}_i^T = [\xi_i(1)\ \ \xi_i(2)]^T$. Then, our problem can be formulated as

$$\{\hat{\mathbf{X}}, \hat{\mathbf{h}}\} = \arg\min_{\mathbf{X},\mathbf{h}} \ ||\mathbf{z} - \mathbf{X}\mathbf{h}||^2 \tag{14}$$

Differentiating the object function w.r.t. $\mathbf{h}$, and using the orthogonality of Alamouti code, we have:

$$\mathbf{h} = (\mathbf{X}^H\mathbf{X})^{-1}\mathbf{X}^H\mathbf{z} \ = \frac{1}{2L}\mathbf{X}^H\mathbf{z}. \tag{15}$$

Substituting (15) into (14) yields

$$\{\hat{\mathbf{X}}\} = \arg\min_{\mathbf{X}}\{\mathbf{z}^H\mathbf{z} - \frac{1}{2L}\mathbf{z}^H\mathbf{X}\mathbf{X}^H\mathbf{z}\}. \tag{16}$$

Since the term $\mathbf{z}^H\mathbf{z}$ is constant, the above optimization problem is equivalent to

$$\{\hat{\mathbf{X}}\} = \arg\max_{\mathbf{X}} \ (\mathbf{X}^H\mathbf{z})^H(\mathbf{X}^H\mathbf{z}). \tag{17}$$

Note that $\mathbf{X}^H\mathbf{z} = \sum_{i=1}^L \mathbf{X}_i^H\mathbf{z}_i$ with

$$\mathbf{X}_i^H\mathbf{z}_i = \begin{bmatrix} 0 & -z_i(2) \\ z_i(2) & 0 \end{bmatrix}\begin{bmatrix} s_{pi} \\ s_{qi} \end{bmatrix} + \begin{bmatrix} z_i(1) & 0 \\ 0 & z_i(1) \end{bmatrix}\begin{bmatrix} s_{pi}^* \\ s_{qi}^* \end{bmatrix}. \tag{18}$$

Hence, $\mathbf{X}^H\mathbf{z}$ can be represented as

$$\mathbf{X}^H\mathbf{z} = \begin{bmatrix} -\sum_{i=1}^L z_i(2)s_{qi} + \sum_{i=1}^L z_i(1)s_{pi}^* \\ \sum_{i=1}^L z_i(2)s_{pi} + \sum_{i=1}^L z_i(1)s_{qi}^* \end{bmatrix}.$$

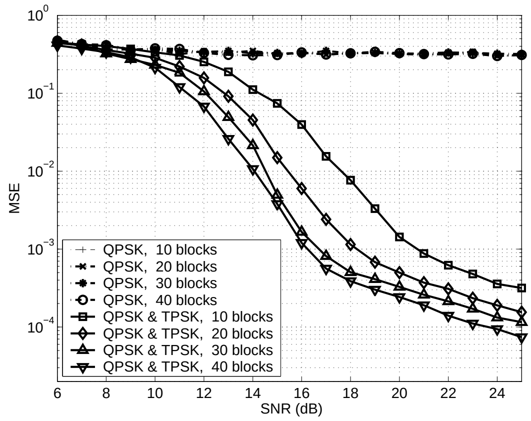

Figure 1: Channel MSE vs. SNR and the block length. The randomization number of the SDR-ML is set to be 40.

It can be verified that problem (17) can be reformulated as a homogenous quadratic problem

$$\max \quad \tilde{\mathbf{s}}^H\mathbf{U}\mathbf{U}^H\tilde{\mathbf{s}} \tag{19a}$$
$$\text{s.t.} \quad |\tilde{s}_i| = 1, \qquad \text{for} \quad i = 1, \cdots, 2L. \tag{19b}$$

where

$$\mathbf{U} = \begin{bmatrix} z_1^*(1), & \cdots & z_L^*(1), & -z_1^*(2), & \cdots & -z_L^*(2) \\ z_1(2), & \cdots & z_L(2), & z_1(1), & \cdots & z_L(1) \end{bmatrix}^T,$$

and $\tilde{\mathbf{s}}^T = [s_{p1}^*, \cdots, s_{pL}^*, s_{q1}, \cdots, s_{qL}]$. Since $\tilde{\mathbf{s}}^H\mathbf{Q}\tilde{\mathbf{s}} = \mathrm{Trace}(\tilde{\mathbf{s}}\tilde{\mathbf{s}}^H\mathbf{Q})$, problem (19) can be further reduced to

$$\max_{\mathbf{S}\in\mathbb{C}^{2L\times 2L}} \quad \mathrm{tr}(\mathbf{S}\mathbf{Q}) \tag{20a}$$
$$\text{s.t.} \quad \mathbf{S} = \tilde{\mathbf{s}}\tilde{\mathbf{s}}^H \tag{20b}$$
$$S_{ii} = 1, \ \ i = 1, \cdots, 2L \tag{20c}$$

where $\mathbf{Q} = \mathbf{U}\mathbf{U}^H$, and $\mathbf{Q} = \mathbf{Q}^H \in \mathbb{C}^{2L\times 2L}$. Relaxing this rank-1 constraint (20b) , problem (20) can be reformulated to

$$\max_{\mathbf{S}\in\mathbb{C}^{2L\times 2L}} \quad \mathrm{tr}(\mathbf{S}\mathbf{Q}) \tag{21a}$$
$$\text{s.t.} \quad \mathbf{S} \succeq \mathbf{0} \tag{21b}$$
$$S_{ii} = 1, \ \ i = 1, \cdots, 2L \tag{21c}$$

where $\mathbf{S} \succeq \mathbf{0}$ means that $\mathbf{S}$ is symmetric and positive semi-definite (PSD). Thus, this complex-valued SDR problem can be efficiently solved using the interior-point method [15]. Once the solution of the SDR problem (21) has been obtained, we use the Goemans-Williamson randomization technique [10, 11, 16] to obtain the approximate solution of the original problem (19).

3. SIMULATIONS

We now carry out two simulations where $p = 4$ and $q = 3$, i.e., QPSK and TPSK constellation. The first simulation compares the mean square error performance of channel estimation using our proposed signal design method with that of the method transmitting symbols from only one constellation. The results are shown in Fig. 1. We see that the method transmitting only one constellation

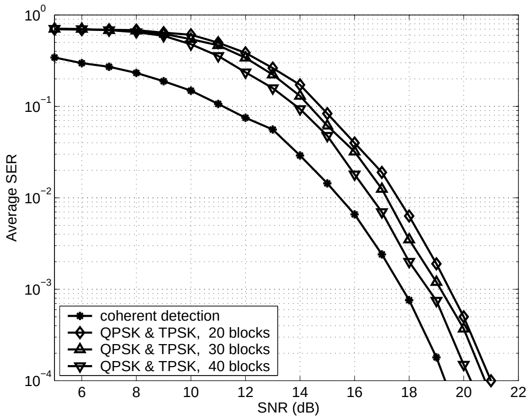

Figure 2: SER vs. SNR for coherent detection and our proposed method with the increasing block length. The randomization number of the SDR-ML is set to be 40.

has very poor performance of channel estimation no matter how large the data block size and no matter how high the signal-to-noise ratio (SNR) is. This poor performance is due to the rotational ambiguity with the signal constellation at the receiver end [12]. Our signal design which employs two signal constellations eliminates such rotational ambiguity and thus, offers substantially better MSE performance without using any pilot symbols. The second simulation compares the average symbol error rate of our joint estimation method to that of coherent detection that has perfect channel knowledge at the receiver. The result is shown in Fig. 2, in which we can observe that the performance of our method is close to that of coherent detection (which necessitate full CSI) when SNR is high.

4. CONCLUSION

In this paper, we proposed a novel blind channel identification technique for Alamouti space-time coded channel by properly designing and transmitting signals. Using our strategy we proved that in the noise-free case, only two distinct pairs of symbols are needed to uniquely determine the channel coefficients, while in the complex Gaussian noise case when pth-order and qth-order statistics of the received signals are available, we are still able to uniquely determine the channel coefficients. In both cases, simple closed-form solutions were derived. When only finite received data are given and under Gaussian noise environment, we employed the semi-definite relaxation algorithm to approximate ML detection so that the joint estimation of the channel and symbols can be efficiently implemented. Simulation results showed that our signal design and transmission method provides much better mean square error performance of channel estimation when compared to the method transmitting only one constellation and that the average symbol error rate approaches that of the coherent detector.

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