DESIGN OF BLOCK TRANSCEIVERS WITH MMSE DECISION FEEDBACK DETECTION

Fang Xu, Timothy N. Davidson, Jian-Kang Zhang, Scarlett S. Chan, and K. Max Wong

Department of Electrical and Computer Engineering,
McMaster University, Hamilton, Ont., Canada.

ABSTRACT

This paper presents a method for jointly designing the transmitter-receiver pair in a block-by-block transmission system that employs minimum mean square error intra-block decision feedback detection. We provide a recursive closed-form expression for a transceiver which maximizes the Gaussian mutual information and also minimizes the bit error rate at moderate-to-high signal-to-noise ratios (in the absence of error propagation). The proposed design generates uncorrelated inputs to the decision device with equal signal-to-interference-and-noise ratios. These properties suggest that one can approach the capacity of the block transmission system using (independent instances of) the same (Gaussian) code for each element of the block. Our simulation studies indicate that the proposed transceiver performs significantly better than standard transceivers, and that it retains its performance advantages in the presence of error propagation.

1. INTRODUCTION

In block-by-block communication schemes, blocks of data are transmitted in a manner that avoids interference between the received blocks, and hence the detector need only operate on a block-by-block basis. Such schemes arise naturally in narrowband quasi-static multiple antenna systems, and they are effective schemes for the transmission of data over dispersive media; e.g., OFDM and DMT. In general, an optimal detector for a block transmission system must make a decision on the received data block as a whole. (In certain cases, such as OFDM and DMT, the elements of that block can be decoupled.) A useful compromise between performance and complexity in this task can be obtained by employing minimum mean square error intra-block decision feedback detection (MMSE-BDFD) [3, 5]. In fact, in the absence of error propagation, these so-called MMSE generalized decision feedback equalizers are "canonical" receivers [3] in the sense that employing an MMSE-BDFD in place of the maximum likelihood detector does not reduce the achievable data rate.

Our goal is to jointly design the transmitter and receiver matrices so as to optimize the performance of a block communication system with an MMSE-BDFD. The design is based on knowledge of the channel, and hence is an appropriate choice for systems in which there is timely, reliable feedback from the receiver to the transmitter. Our initial design objective is to minimize the arithmetic mean of the squared errors at the decision point (the MSE). That problem has previously been deemed to be difficult, and hence several authors have suggested minimizing the geometric mean of the squared errors (the geometric MSE) [8], which is a lower bound on the MSE. It is reasonably well known [3, 8] that any transmitter which minimizes the geometric MSE of an MMSE-BDFD also maximizes the Gaussian mutual information. The set of such transmitters is parameterized by a unitary matrix. Unfortunately, the standard choice from this set of transmitters does not minimize the (arithmetic) MSE. (It only minimizes the lower bound on the MSE.) Furthermore, it produces potentially different decision point signal-to-noise ratios (SNRs) for each element of the block. Therefore, in order to achieve reliable communication at rates which approach the capacity of the block transmission system, different (Gaussian) codes may need to be applied for each element of the block [3].

In this paper we provide a recursive closed-form expression for a choice of the above-mentioned unitary matrix degree of freedom that results in the minimization of the arithmetic MSE. The resulting transceiver has many desirable properties. In addition to maximizing the mutual information between transmitter and receiver for Gaussian signals, and minimizing the arithmetic and geometric MSEs, it (essentially) minimizes the bit error rate (BER) of a uniformly bit-loaded system employing QAM signalling at moderate-to-high signal to noise ratios (SNRs). The proposed design also generates uncorrelated inputs to the decision device, maximizes the minimum decision point signal-to-interference-and-noise ratio (SINR) over the block, and results in each element of the block having the same SINR. In particular, from within the set of transceivers which maximize the Gaussian mutual information we obtain a transceiver which provides uncorrelated inputs to the decision device which have identical (and maximized) SINRs. Since the MMSE-BDFD is a canonical receiver [3], this suggests that by using our design, reliable communication at rates approaching the capacity of the block transmission system can be achieved using (independent instances of) the same (Gaussian) code for each element of the block.

Our design is based on the standard assumption [3, 8] that the previous symbols were correctly detected. However, error propagation is not catastrophic in block-by-block communication schemes because errors can only propagate over a single block. Our simulation studies verify that statement by indicating that the proposed transceivers perform significantly better than standard transceivers, and that they retain their performance advantages in the presence of error propagation.

2. BLOCK-BY-BLOCK TRANSMISSION

We consider a generic block transmission model in which a block of M data symbols, $\mathbf{s}$, is linearly precoded to construct a block of $K \geq M$ channel symbols, $\mathbf{u} = \mathbf{F}\mathbf{s}$, which is transmitted over the channel. The receiver independently processes a block of $P \geq M$ received samples in order to detect the data vector $\mathbf{s}$. The received block, $\mathbf{y}$, can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{F}\mathbf{s} + \mathbf{v}, \qquad (1)$$

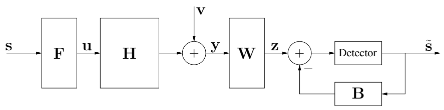


Fig. 1. A block diagram for (2).

where $\mathbf{v}$ is a length P vector of zero-mean circular and additive Gaussian noise samples with positive definite correlation matrix $\mathbf{R}_{vv}$, and $\mathbf{H}$ is the $P \times K$ channel matrix. We will assume that the data symbols have zero mean and are white, of unit energy, and not correlated with the noise, (i.e., $E[\mathbf{s}\mathbf{s}^H] = \mathbf{I}$ and $E[\mathbf{s}\mathbf{v}^H] = 0$).

The MMSE-BDFD first preprocesses $\mathbf{y}$ with an $M \times P$ "feed-forward" matrix $\mathbf{W}$ (given in (4) below) to form $\mathbf{z} = \mathbf{W}\mathbf{y}$. The detection of the transmitted symbols $s_m = [\mathbf{s}]_m$ then proceeds sequentially, starting from $m = M$, by making a scalar decision on $\hat{s}_m = z_m - \sum_{\ell=m+1}^{M} b_{m\ell}\tilde{s}_\ell$, where $\tilde{s}_\ell$ denotes the decision made on the ℓth symbol. If the the coefficients $b_{m\ell}$ are arranged in a strictly upper triangular $M \times M$ matrix $\mathbf{B}$, the operation of the block transceiver is equivalent to successively solving the rows of the following equation, starting from the Mth row (see Fig. 1),

$$\hat{\mathbf{s}} = \mathbf{W}\mathbf{H}\mathbf{F}\mathbf{s} + \mathbf{W}\mathbf{v} - \mathbf{B}\tilde{\mathbf{s}}. \tag{2}$$

Under the assumption of correct past decisions (i.e., when deciding s_m, $\tilde{s}_\ell = s_\ell$ for all $m+1 \leq \ell \leq M$), the error between the input to the decision device and the original transmitted symbol is

$$\mathbf{e} = \hat{\mathbf{s}} - \mathbf{s} = (\mathbf{W}\mathbf{H}\mathbf{F} - \mathbf{I} - \mathbf{B})\mathbf{s} + \mathbf{W}\mathbf{v}. \tag{3}$$

If we let $\mathbf{R}_{ee}$ denote the covariance matrix of $\mathbf{e}$, then the (arithmetic) MSE of the detector input is $\bar{e}^2 = \operatorname{tr}(\mathbf{R}_{ee})/M$, and the feedforward matrix that minimizes the MSE is

$$\mathbf{W}_{\text{MMSE}} = (\mathbf{B}+\mathbf{I})(\mathbf{F}^H\mathbf{H}^H\mathbf{R}_{vv}^{-1}\mathbf{H}^H\mathbf{F}^H + \mathbf{I})^{-1}\mathbf{F}^H\mathbf{H}^H\mathbf{R}_{vv}^{-1}. \tag{4}$$

With the choice $\mathbf{W} = \mathbf{W}_{\text{MMSE}}$, we have

$$\mathbf{R}_{ee} = (\mathbf{B}+\mathbf{I})(\mathbf{I} + \mathbf{F}^H\mathbf{H}^H\mathbf{R}_{vv}^{-1}\mathbf{H}\mathbf{F})^{-1}(\mathbf{B}+\mathbf{I})^H. \tag{5}$$

3. PRECODER DESIGN

The goal of our design is to find matrices $\mathbf{F}$ and $\mathbf{B}$ that minimize the (arithmetic) MSE subject to an average transmitted power constraint. Letting $\mathbf{U} = \mathbf{B} + \mathbf{I}$, that design problem is

$$\min_{\mathbf{F},\mathbf{U}} \quad \operatorname{tr}(\mathbf{U}(\mathbf{I} + \mathbf{F}^H\mathbf{H}^H\mathbf{R}_{vv}^{-1}\mathbf{H}\mathbf{F})^{-1}\mathbf{U}^H) \tag{6a}$$

$$\text{subject to} \quad \operatorname{tr}(\mathbf{F}\mathbf{F}^H) \leq p_0,\ \mathbf{U} \text{ is monic upper triangular.} \tag{6b}$$

We will solve this problem in two stages. First we will obtain a lower bound on the objective, and will minimize that lower bound. We will then obtain matrices $\mathbf{F}$ and $\mathbf{B}$ for which the original objective achieves the minimized lower bound.

The first stage begins by observing that the arithmetic-geometric mean inequality and the structure of $\mathbf{U}$ imply that

$$\begin{aligned}\operatorname{tr}(\mathbf{U}(\mathbf{I} &+ \mathbf{F}^H\mathbf{H}^H\mathbf{R}_{vv}^{-1}\mathbf{H}\mathbf{F})^{-1}\mathbf{U}^H) \\ &\geq M|\mathbf{I} + \mathbf{F}^H\mathbf{H}^H\mathbf{R}_{vv}^{-1}\mathbf{H}\mathbf{F}|^{-1/M},\end{aligned} \tag{7}$$

where $|\cdot|$ denotes the determinant. Therefore, minimizing the lower bound on the MSE reduces to

$$\max_{\mathbf{F}} \quad |\mathbf{I} + \mathbf{F}^H\mathbf{H}^H\mathbf{R}_{vv}^{-1}\mathbf{H}\mathbf{F}| \quad \text{s.t.} \quad \operatorname{tr}(\mathbf{F}\mathbf{F}^H) \leq p_0. \tag{8}$$

Although the objective in (8) was derived as the reciprocal of the geometric MSE in (7), it is also [3, 8] the Gaussian mutual information. Hence, minimizing the lower bound on the arithmetic MSE is equivalent to maximizing the Gaussian mutual information. Therefore, the solution to (8) involves a "waterfilling" power allocation over the eigenvectors of $\mathbf{H}^H\mathbf{R}_{vv}^{-1}\mathbf{H}$, [6]. More formally, if $\mathbf{H}^H\mathbf{R}_{vv}^{-1}\mathbf{H} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^H$ denotes an eigen decomposition of $\mathbf{H}^H\mathbf{R}_{vv}^{-1}\mathbf{H}$ with eigenvalues λ_i arranged in non-increasing order, the solution depends on a parameter $r \leq K$ which is the largest integer satisfying $1/\lambda_r < \left(p_0 + \sum_{j=1}^{r}\lambda_j^{-1}\right)/r$. If we define $q = \min\{r, M\}$, then the following set of precoders minimize the lower bound [6]: $\mathbf{F} = \tilde{\mathbf{V}}_q\left[\mathbf{\Phi} \quad \mathbf{0}_{q\times(M-q)}\right]\mathbf{\Psi}$, where $\tilde{\mathbf{V}}_q$ is the first q columns of $\mathbf{V}$, $\mathbf{\Phi}$ is a $q \times q$ diagonal matrix with diagonal elements satisfying

$$|\phi_{ii}|^2 = \frac{1}{q}\Big(p_0 + \sum_{j=1}^{q}\lambda_j^{-1}\Big) - \lambda_i^{-1}, \tag{9}$$

and $\mathbf{\Psi}$ is an arbitrary $M \times M$ unitary matrix. (Since the rank of the resulting product $\mathbf{H}\mathbf{F}$ is q, if M were a design variable rather than a parameter of the problem, a natural choice for M would be $M = r$.) The minimized lower bound on the MSE is

$$\bar{e}^2 \geq q^{q/M}\Big(p_0 + \sum_{j=1}^{q}\lambda_j^{-1}\Big)^{-q/M}\prod_{j=1}^{q}\lambda_j^{-1/M}. \tag{10}$$

Moving to the second stage of our design approach, we now determine a transceiver whose arithmetic MSE achieves the minimized lower bound in (10). Defining $\breve{\mathbf{\Phi}} = \left[\begin{smallmatrix}\mathbf{\Phi} & \mathbf{0}_{q\times(M-q)}\end{smallmatrix}\right]$. and substituting $\mathbf{F} = \tilde{\mathbf{V}}_q\breve{\mathbf{\Phi}}\mathbf{\Psi}$ into (5) and (6a), we have that

$$\mathbf{R}_{ee} = \mathbf{U}\mathbf{\Psi}^H(\mathbf{I}_M + \breve{\mathbf{\Phi}}^T\tilde{\mathbf{\Lambda}}_M\breve{\mathbf{\Phi}})^{-1}\mathbf{\Psi}\mathbf{U}^H, \tag{11}$$

where $\tilde{\mathbf{\Lambda}}_M$ is the upper left $M \times M$ block of $\mathbf{\Lambda}$. From the conditions for equality in the arithmetic-geometric mean inequality we know that for the MSE to achieve its minimized lower bound, we must choose $\mathbf{\Psi}$ and $\mathbf{U}$ so that $\mathbf{R}_{ee} = \bar{\mu}\mathbf{I}$, where $\bar{\mu}$ is equal to the right hand side of (10). That is, we must choose $\mathbf{\Psi}$ and $\bar{\mathbf{U}} = 1/\sqrt{\bar{\mu}}\,\mathbf{U}$ so that

$$(\mathbf{I}_M + \breve{\mathbf{\Phi}}^T\tilde{\mathbf{\Lambda}}_M\breve{\mathbf{\Phi}})^{1/2}\mathbf{\Psi} = \mathbf{P}\bar{\mathbf{U}}, \tag{12}$$

where $\mathbf{P}$ is unitary and $\bar{\mathbf{U}}$ has all its diagonal elements equal to $1/\sqrt{\bar{\mu}}$. The following result, which is a special case of a more general result in [9], provides a solution to (12).

Lemma 1 *Let $\mathbf{\Gamma}$ be an $M \times M$ positive definite diagonal matrix. There exists a unitary matrix $\mathbf{S}$ such that $\mathbf{\Gamma}\mathbf{S} = \mathbf{Q}\mathbf{R}$, where $\mathbf{Q}$ is an $M \times M$ unitary matrix and $\mathbf{R}$ is an upper-triangular matrix with equal diagonal elements $[\mathbf{R}]_{ii} = \left(\prod_{k=1}^{M}\gamma_k\right)^{1/M}$, $i \in [1, M]$, where γ_k is the kth diagonal element of $\mathbf{\Gamma}$. Such an $\mathbf{S}$ can be obtained by using the algorithm in the Appendix.*

Our transceiver design can now be summarized as follows:

Proposition 1 *The mean-square error $\operatorname{tr}(\mathbf{R}_{ee})/M$ achieves its minimized lower bound (10) when $\mathbf{F} = \tilde{\mathbf{V}}_q\left[\begin{smallmatrix}\mathbf{\Phi} & \mathbf{0}\end{smallmatrix}\right]\mathbf{\Psi}$, where $\mathbf{\Phi}$ satisfies (9), and $\mathbf{\Psi}$ is obtained by applying the algorithm in the Appendix to $(\mathbf{I}_M + \breve{\mathbf{\Phi}}^T\tilde{\mathbf{\Lambda}}_M\breve{\mathbf{\Phi}})^{1/2}$. The corresponding feedback matrix $\mathbf{B} = \mathbf{U} - \mathbf{I}$, where $\mathbf{U} = \sqrt{\bar{\mu}}\,\bar{\mathbf{U}}$, and $\bar{\mathbf{U}}$ is obtained from the QR-decomposition in* (12). *Substituting such $\mathbf{F}$ and $\mathbf{B}$ into (4) yields $\mathbf{W}$.*

From the derivation it is apparent that our precoder, which minimizes the arithmetic MSE, lies in the set of precoders which minimize the geometric MSE (and maximize the Gaussian mutual information). However, a precoder chosen arbitrarily from that set does not necessarily minimize the arithmetic MSE. This observation provides a connection between the proposed design and that in [8]. When simplified to the current context, the design in [8] corresponds to choosing $\mathbf{\Psi} = \mathbf{I}_M$, rather than choice of $\mathbf{\Psi}$ in Prop. 1. While the choice of $\mathbf{\Psi} = \mathbf{I}_M$ results in a system that minimizes the geometric MSE, it does not minimize the arithmetic MSE in the general case. In addition, the SINR for each element of the block may be different. In contrast, the choice of $\mathbf{\Psi}$ in Prop. 1 minimizes the geometric MSE and the arithmetic MSE, and provides an equal SINR for each element of the block.

The choice of $\mathbf{\Psi}$ also has an impact on the nature of coding strategies for approaching the capacity of the block-by-block transmission system. From the discussion following (8) it is evident that the Gaussian mutual information is maximized by choosing $M = r$ and employing a transmitter matrix of the form $\mathbf{F} = \tilde{\mathbf{V}}_r\mathbf{\Phi}\mathbf{\Psi}$, where $\mathbf{\Phi}$ satisfies (9) and $\mathbf{\Psi}$ is an arbitrary $r \times r$ unitary matrix. The choice $\mathbf{\Psi} = \mathbf{I}_r$ results in a "vector coding" scheme [3] in which the feedback component of the MMSE-BDFD is inactive; i.e., $\mathbf{B} = \mathbf{0}$. Vector coding induces an equivalent system with r parallel Gaussian subchannels, each with a possibly different SNR ρ_i, and one can approach the capacity of the block transmission scheme by simply choosing the code for the ith element of the block to be one that approximates the ideal Gaussian code of rate $b_i = \log_2(1 + \rho_i)$ bits per channel use. The choice of $\mathbf{\Psi}$ in Prop. 1 results in a system in which the feedback component of the MMSE-BDFD is active, and the inputs to the decision device are uncorrelated and have identical SINRs ρ. Since the MMSE-BDFD is a canonical receiver, this suggests that one can also approach the capacity of the block transmission system by employing an independent instance of the same approximation of the ideal Gaussian code of rate $b = \log_2(1 + \rho)$ for each element of the block.

4. BIT ERROR RATE PERFORMANCE

In this section, we show that the $(\mathbf{F}, \mathbf{B})$ pair in Prop. 1 also minimizes the (dominant components of the) bit error rate (BER) of a system with uniform bit loading at moderate-to-high SNRs. We define the average BER of the detected signal to be the average of the probability of error of each element of the block; i.e. $P_e = (1/M)\sum_{i=1}^{M} P_{ei}$, where P_{ei} denotes the BER of the ith symbol s_i. If all the previous decisions are correct, for square QAM signalling with $2b_i$ bits per symbol, P_{ei} is closely approximated by [2]

$$P_{ei} \approx \tilde{P}_{ei} = \alpha_i \operatorname{erfc}\left(\sqrt{\beta_i\rho_i}\right) + \zeta_i \operatorname{erfc}\left(3\sqrt{\beta_i\rho_i}\right), \tag{13}$$

where $\operatorname{erfc}(x) = (2/\sqrt{\pi})\int_x^\infty e^{-z^2}\,dz$, ρ_i is the ith decision point SINR, $\alpha_i = \frac{\sqrt{4^{b_i}}-1}{b_i\sqrt{4^{b_i}}}$, $\beta_i = \frac{3}{2(4^{b_i}-1)}$, and $\zeta_i = \frac{\sqrt{4^{b_i}}-2}{b_i\sqrt{4^{b_i}}}$. The expression in (13) involves the approximation of the residual intra-block interference by a Gaussian random variable. This approximation is (almost surely) sufficiently accurate for all by the last few elements of the block; c.f., [7]. Under our assumptions, we have that $\rho_i = -1 + 1/[\mathbf{R}_{ee}]_{ii}$. Hence,

$$P_e \approx \tilde{P}_e = \frac{1}{M}\sum_{i=1}^{M} \alpha_i \operatorname{erfc}\Big(\sqrt{\beta_i\big(([\mathbf{R}_{ee}]_{ii})^{-1} - 1\big)}\Big) + \zeta_i \operatorname{erfc}\Big((3\sqrt{\beta_i\big(([\mathbf{R}_{ee}]_{ii})^{-1} - 1\big)}\Big). \tag{14}$$

Since our design generates equal ρ_i's (recall that the optimal system results in $\mathbf{R}_{ee} = \bar{\mu}\mathbf{I}$), we will assume uniform bit-loading in the remainder of this section, and therefore we will drop the element index i in α_i, β_i and ζ_i. When $[\mathbf{R}_{ee}]_{ii} < 2\beta/3$, which corresponds to moderate-to-high SINRs, $\tilde{P}_e$ is a convex function of $[\mathbf{R}_{ee}]_{ii}$, [1, 4]. By applying Jensen's inequality to (14), we obtain the lower bound

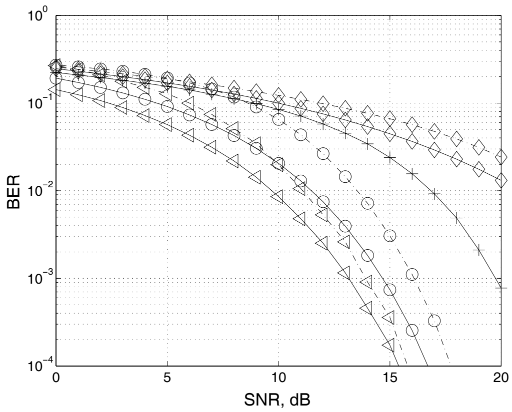
$$\tilde{P}_e \geq \alpha \operatorname{erfc}\Big(\sqrt{\beta\big(M/\operatorname{tr}(\mathbf{R}_{ee}) - 1\big)}\Big) + \zeta \operatorname{erfc}\Big(3\sqrt{\beta\big(M/\operatorname{tr}(\mathbf{R}_{ee}) - 1\big)}\Big). \tag{15}$$

Equality in (15) holds if and only if the diagonal elements of $\mathbf{R}_{ee}$ are equal. Equation (15) exposes an intriguing relationship between the (arithmetic) MSE and the BER. Since minimizing $\operatorname{tr}(\mathbf{R}_{ee})$ minimizes both terms on the right hand side of (15), minimizing the lower bound on the BER in (15) is equivalent to minimizing the MSE. However, for the actual BER to achieve the minimized lower bound (i.e., for (15) to hold with equality), the diagonal elements of $\mathbf{R}_{ee}$ must be identical. Fortunately, the diagonal elements of $\mathbf{R}_{ee}$ are equal for our design in Prop. 1. Therefore, we can conclude that at moderate-to-high SINRs, the system in Prop. 1 is a minimum BER system in the sense that it minimizes $\tilde{P}_e$ in (14).

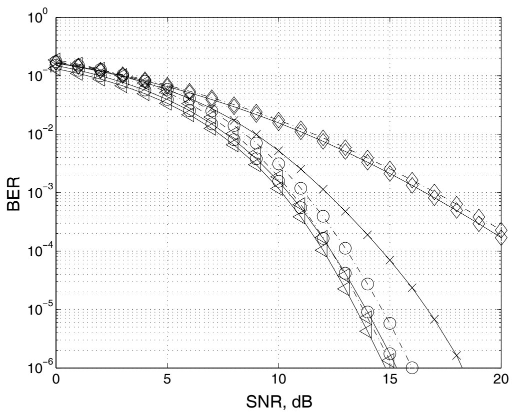
5. PERFORMANCE ANALYSIS

We now compare the performance of our design to that of existing designs, and demonstrate its graceful performance degradation in the presence of error propagation. We consider zero-padded block transmission (e.g., [5]) over a scalar channel. (A multiple antenna example appears in [7].) We will consider the transmitter in Prop. 1, denoted by $\mathbf{F}_{\text{OPT-MMSE-BDFD}}$, the "single-carrier zero-padded" (SCZP) transmission scheme, $\mathbf{F}_{\text{I}} = \sqrt{p_0/M}\,\mathbf{I}$, and the "zero-padded OFDM" (ZP-OFDM) scheme, $\mathbf{F}_{\text{ZP-OFDM}} = \sqrt{p_0/M}\,\mathbf{D}$, where $\mathbf{D}$ is a normalized discrete Fourier transform matrix. In both those additional cases, the receiver matrices $\mathbf{B}$ and $\mathbf{W}$ are chosen according to an existing design procedure [5]. As a benchmark, we will also consider the system with a linear MMSE equalizer that provides the minimum BER [1, 4]. We will consider two scenarios. In both scenarios, the noise is white (i.e., $\mathbf{R}_{vv} = \sigma^2\mathbf{I}$), the channel has an impulse response of length 5, and the block sizes are $M = 16$ and $P = 20$. Each element of $\mathbf{s}$ is an independent 4–QAM symbol with equally likely signalling points. We will plot the BER against the block SNR, $\operatorname{tr}(\mathbf{F}\mathbf{F}^H)/\operatorname{tr}(\mathbf{R}_{vv}) = p_0/(P\sigma^2)$.

Single channel scenario: In Fig. 2(a) we provide the BER performance of each scheme in a single channel with zeros at $1, 0.9j, -0.9j$, and $1.3\exp(j\pi5/8)$. From the solid curves it is clear that in the absence of error propagation the proposed scheme performs significantly better than the SCZP scheme, with an SNR gain of almost 1 dB at a BER of 10^{-4}. Furthermore, the dashed curves demonstrate that this performance advantage is maintained in the presence of error propagation.



(a) Single channel scenario



(b) Random channel scenario

Fig. 2. BER performance in the two channel scenarios in Section 5. Legend—solid curves: correct decisions fed back; dashed curves: actual decisions fed back; $\lhd$: optimized scheme, $\mathbf{F}_{\text{OPT-MMSE-BDFD}}$; $\circ$: SCZP scheme, $\mathbf{F}_1$; $\diamond$: ZP-OFDM scheme, $\mathbf{F}_{\text{ZP-OFDM}}$; $+$: optimized scheme with linear MMSE detection [1,4].

Random channel scenario: In Fig. 2(b) we provide the average BER performance over 500 randomly generated channels, whose taps were generated independently from a zero-mean circular complex Gaussian distribution and then normalized. Similar trends are apparent. In particular, the precoder in Prop. 1 provides an SNR gain approaching 0.5 dB over the SCZP scheme, and this advantage is maintained in the presence of error propagation.

6. CONCLUSION

In this paper, we have jointly designed the precoder and the feedback matrix of a block transmission scheme equipped with a minimum mean-square error intra-block decision feedback detector (MMSE-BDFD). The design minimizes the arithmetic mean of the expected squared errors at the decision point in the absence of error propagation, and also maximizes the Gaussian mutual information. The covariance matrix of the minimized error is white, and hence the proposed design also minimizes the (dominant components of the) bit error rate of a uniformly bit-loaded transmission system. In our simulations, the proposed system performed better than standard precoding systems, and retained its performance advantage in the presence of error propagation. Furthermore, the fact that the MMSE-BDFD is a canonical receiver suggests that by using the proposed design, one can approach the capacity of the block transmission system by using independent instances of the same (Gaussian) code for each element of the block.

A. ALGORITHM FOR LEMMA 1

Arrange $\mathbf{\Gamma}$ so that $\gamma_k \geq \gamma_{k+1}$ and let $g = \left(\prod_{k=1}^{M}\gamma_k^2\right)^{1/M}$. Let $\mathbf{s}_k$ be the kth column of $\mathbf{S}$ and $s_{\ell k}$ its elements. Let $\mathbf{S}_k$ be the first k columns of $\mathbf{S}$ and $\mathbf{S}_k^{\perp}$ its orthogonal complement. Define $\mathcal{P}_{\mathbf{A}} = \mathbf{I} - \mathbf{A}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H$, and $\mathbf{A}^{(k)} = \left(\mathbf{\Gamma}\mathbf{S}_k^{\perp}\right)^H \mathcal{P}_{(\mathbf{\Gamma}\mathbf{S}_k^{\perp})}\mathbf{\Gamma}\mathbf{S}_k^{\perp}$. The algorithm proceeds as follows:

1. *Initialization:* Set $k = 1$. An explicit solution for $\mathbf{s}_1$ is $s_{11} = \sqrt{\frac{g-\gamma_M^2}{\gamma_1^2-\gamma_M^2}}$, $s_{M1} = \sqrt{\frac{\gamma_1^2-g}{\gamma_1^2-\gamma_M^2}}$, $s_{\ell 1} = 0$ for $\ell = 2, 3, \cdots, M-1$.
2. Construct $\mathbf{A}^{(k)}$ and its eigen decomposition, $\mathbf{A}^{(k)} = \mathbf{V}^{(k)}\mathbf{\Lambda}^{(k)}\left(\mathbf{V}^{(k)}\right)^H$.
3. Set $\mathbf{s}_{k+1} = \mathbf{S}_k^{\perp}\mathbf{V}^{(k)}\mathbf{y}^{(k)}$, where $y_1^{(k)} = \sqrt{\frac{g-\lambda_{M-k}^{(k)}}{\lambda_1^{(k)}-\lambda_{M-k}^{(k)}}}$, $y_{M-k}^{(k)} = \sqrt{\frac{\lambda_1^{(k)}-g}{\lambda_1^{(k)}-\lambda_{M-k}^{(k)}}}$, $y_\ell^{(k)} = 0$ for $\ell = 2, 3, \cdots, M-k-1$.
4. Increment k. If $k \leq M-2$ return to 2. Otherwise, set $\mathbf{s}_M = \mathbf{S}_{M-2}^{\perp}\mathbf{V}^{(M-2)}\mathbf{z}$, where $z_1 = -\sqrt{\frac{g-\lambda_2^{(M-2)}}{\lambda_1^{(M-2)}-\lambda_2^{(M-2)}}}$, $z_2 = \sqrt{\frac{\lambda_1^{(M-2)}-g}{\lambda_1^{(M-2)}-\lambda_2^{(M-2)}}}$.

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