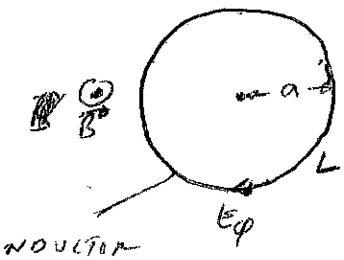
FARADAY'S LAW:

EXAMPLE 1: CONSIDER A MAGNETIC FIELD WHICH INCREASES EXPONENTIALLY WITH TIME

$$\vec{B} = B_0 e^{kt} \vec{z}$$

WHERE B_0 = CONST. CONSIDER A COPPER CONDUCTOR WITH RADIUS a.



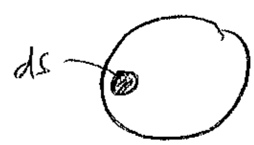
$z = 0$ PLANE

$$EMF = \oint_L \vec{E} \cdot d\vec{L} = 2\pi a E_\varphi \quad \rightarrow (1)$$

($\because E_\varphi$ IS CONST. DUE TO SYMMETRY)

$$\frac{\partial \vec{B}}{\partial t} = k B_0 e^{kt} \vec{z}.$$

$\frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$: IMAGINE A SURFACE $d\vec{S}$ INSIDE L



$|d\vec{S}| = ds$. The direction of $d\vec{S}$ is the same as its normal (in z-direction)

$$\therefore \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} = (k B_0 e^{kt})(ds)(\vec{z} \cdot \vec{z})$$

$$= k B_0 e^{kt} ds$$

$$\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} = \int_S k B_0 e^{kt} \, ds$$

$$= k B_0 e^{kt} \cdot \pi a^2 \rightarrow (2)$$

FARADAY'S LAW:

$$\oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

USING (1) & (2)

$$\text{EMF} = 2\pi a E_\varphi = -k B_0 e^{kt} \pi a^2$$

$$E_\varphi = \frac{-k B_0 e^{kt} a}{2} \rightarrow (3)$$

~~NOW~~ WE HAVE USED THE INTEGRAL FORM TO ~~OBTAIN~~ OBTAIN EQ-(3). NOW WE USE THE DIFFERENTIAL FORM TO GET THE SAME RESULT.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \rightarrow (4)$$

IN CYLINDRICAL CO-ORDINATES, Z-COMPONENT OF CURL OF $\vec{E}$ IS

$$(\nabla \times \vec{E})_z = \frac{1}{\rho} \frac{\partial (\rho E_\varphi)}{\partial \rho}$$

USING (4), WE FIND

$$\frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho E_\varphi) = -k B_0 e^{kt} \rightarrow (5)$$

~~INTEGRATING EQ. (5) w.r.t.~~

$$\frac{d(\rho E_\varphi)}{d\rho} = -k B_0 \rho e^{kt} \rightarrow (6)$$

INTEGRATING EQ. (6) w.r.t. ρ, WE FIND

$$\rho E_\varphi \Big|_0^a = -k B_0 e^{kt} \int_0^a \rho \, d\rho$$

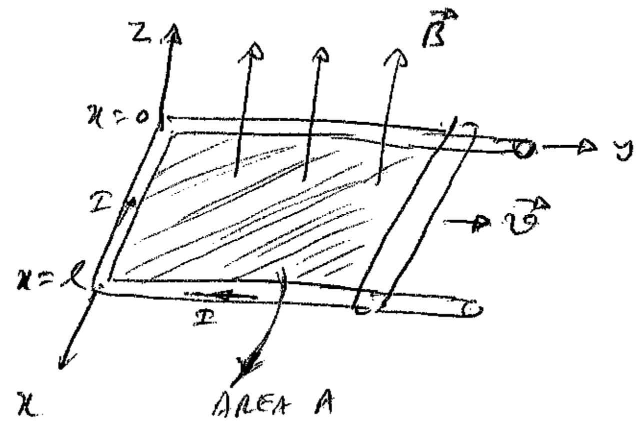
$$E_\varphi \cdot a = -k B_0 e^{kt} \frac{\rho^2}{2} \Big|_0^a$$

$$= -k B_0 e^{kt} \cdot \frac{a^2}{2}$$

$$E_\varphi = \frac{-k B_0 e^{kt} a}{2}, \rightarrow (7)$$

WHICH IS THE SAME AS EQ. (3). TO SOLVE A PROBLEM, YOU COULD EITHER USE THE INTEGRAL FORM OR THE DIFFERENTIAL FORM.

EXAMPLE 2: CONSIDER A CLOSED CIRCUIT CONSISTING OF CONDUCTORS & A SLIDING BAR MOVING AT VELOCITY $\vec{v}$. THE MAGNETIC FLUX DENSITY $\vec{B}$ IS CONSTANT (IN SPACE & TIME) & IS NORMAL TO THE PLANE OF CONDUCTORS.



LET THE POSITION OF THE MOVING BAR AT t BE $y(t)$.

THE AREA $\perp^r$ TO FLUX FLOW AT t

$$A(t) = l\, y(t)$$

MAGNETIC FLUX = MAG. FLUX DENSITY × AREA $\perp^r$ to FLUX ($\because \vec{B}$ IS UNIFORM)

$$\Phi(t) = B\, l\, y(t)$$

$$emf = -\frac{d\Phi}{dt} = -Bl\frac{dy}{dt} = -Blv$$

IF THE ~~AREA~~ TOTAL RESISTANCE IN THE CIRCUIT IS R, THE CURRENT IS

$$I = \frac{emf}{R} = \frac{-Blv}{R}$$

NOTE : (i) v IS NOT VOLTAGE, IT IS SPEED.

(ii) If v IS CONST, emf IS CONST & WE HAVE A DC CIRCUIT. OTHERWISE, VOLTAGE IS TIME-VARYING.

(iii) THE DIRECTION OF CURRENT IS DETERMINED BY THE CONDITION THAT THE INDUCED FLUX (AS GIVEN BY RIGHT HAND RULE) OPPOSES THE EXTERNAL FLUX.