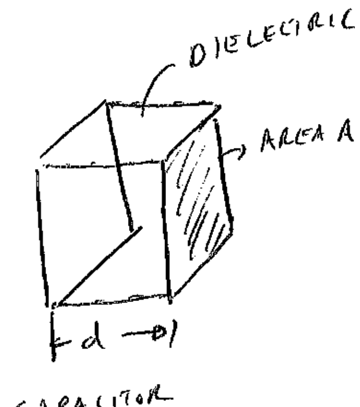
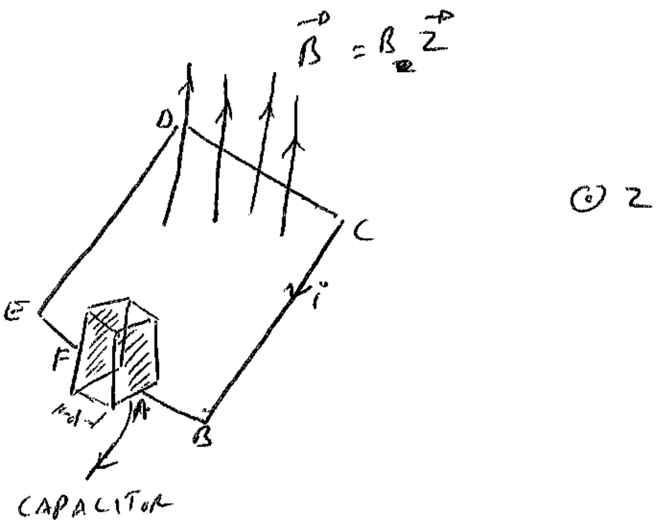
# WORKED EXAMPLE: DISPLACEMENT CURRENT & FARADAY'S LAW




CAPACITOR




THE MAGNETIC FLUX DENSITY LINKED WITH THE CONDUCTING LOOP 'ABCDEFA' IS GIVEN BY

$$\vec{B} = B_0 e^{\alpha t} \vec{z} \ \mathrm{T}$$

WHERE $\alpha = 2 \ s^{-1}$, $B_0 = 500$ T. THE AREA OF THE LOOP 'ABCDEFA' IS 20 $cm^2$. THE LOOP IS IN THE XY PLANE. THE CAPACITANCE OF THE CAPACITOR IS 5 F. ASSUME THAT THE DIELECTRIC INSIDE THE CAPACITOR IS PERFECT (ZERO RESISTANCE). FIND (a) INDUCED EMF

(b) CONDUCTION CURRENT DENSITY & CONDUCTION CURRENT IN THE CIRCUIT. ~~ASSUME THA~~

ASSUME THAT THE CONDUCTORS ARE PERFECT (ZERO RESISTANCE)

(c) DISPLACEMENT CURRENT ~~INSIDE~~ THE DIELECTRIC.

(d) CONDUCTION CURRENT & DISPLACEMENT IN THE CONDUCTOR & DISPLACEMENT CURRENT IN THE DIELECTRIC IF ~~FOR~~ CONDUCTIVITY OF THE CONDUCTOR IS $5.5 \times 10^7$ S/m. ASSUME THAT THE LENGTH OF THE CONDUCTORS IS 3 m & CROSS-SECTIONAL AREA IS 0.5 $mm^2$; $\epsilon_r = \mu_r = 1$.

SOLUTION:

(a) THE MAGNETIC FLUX,

$$\psi_m = \int_S \vec{B} \cdot d\vec{s} = B_0 e^{\alpha t} \times (20 \times 10^{-4}) \text{ Wb}$$

$$= e^{2t} \text{ Wb} \quad (\because B_0 = 500\,T, \; \alpha = 2\, s^{-1})$$

$$\text{emf} = v = \frac{-d\psi_m}{dt} = -2e^{2t} \text{ V}$$

$$= -2e^{2t} \text{ V}$$

(b) FOR A CAPACITOR,

$$q = Cv \; ; \; C = 5F$$

$$\therefore \; i = \frac{dq}{dt} = C\frac{dv}{dt} = C\left[-4 \cdot e^{2t}\right] \text{ A}$$

$$= -20e^{2t} \text{ A}$$

NOTE THAT IN THIS CASE, THE CONDUCTOR IS PERFECT & HENCE, THERE IS NO VOLTAGE DROP ACROSS THE CONDUCTOR. THE EMF DEVELOPED DUE TO TIME-CHANGING MAGNETIC FIELD APPEARS AT THE CAPACITOR TERMINALS.

CONDUCTION CURRENT DENSITY,

$$J = \frac{i}{S},$$

WHERE $S$ = CROSS-SECTIONAL AREA OF THE CONDUCTOR.

$$J = \frac{-20\, e^{2t}}{0.5 \times (10^{-3})^2} = -4 \times 10^{7}\, e^{2t} \ A/m^2$$

FOR A PERFECT CONDUCTOR, $\sigma = \infty$

$$J = \sigma E$$

SINCE $\sigma = \infty$, $E = 0$ & $D = 0$ IN THE CONDUCTOR.

THIS IMPLIES THAT THERE IS NO DISPLACEMENT CURRENT IN THE CONDUCTOR ($\because \ J_d = \partial D/\partial t$).

ⓒ IN THE CAPACITOR,

$$E = V/d$$

$$D = \epsilon E = \epsilon V/d$$

$$J_d = \frac{\partial D}{\partial t} = \frac{\epsilon}{d}\frac{dv}{dt}$$

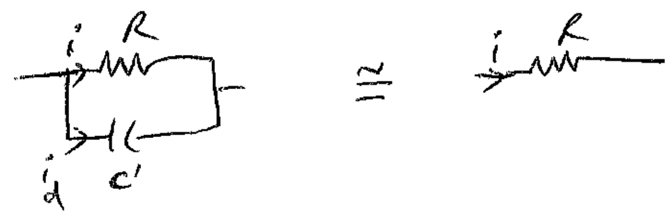
THE DISPLACEMENT CURRENT $i_d = \int \vec{J}_d \cdot d\vec{s} = \left(\frac{\epsilon}{d}\frac{dv}{dt}\right) \cdot A$

$$= C\frac{dv}{dt} \quad \left(\because C = \frac{\epsilon A}{d}\right)$$

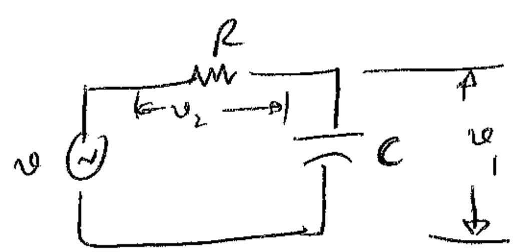
$= i =$ CONDUCTION CURRENT IN THE CONDUCTORS

$$= -20\, e^{2t} \ A.$$

(d) WHEN THE CONDUCTOR IS NOT PERFECT, THERE EXISTS A SMALL DISPLACEMENT CURRENT IN THE CONDUCTORS & THE EQUIVALENT CIRCUIT OF THE CONDUCTOR MAY BE WRITTEN AS




HOWEVER, $i_d << i$. HENCE, THE CAPACITANCE OF THE CONDUCTOR MAY BE IGNORED. NOW, THE EQUIVALENT CIRCUIT OF THE LOOP 'ABCDEFA' MAY BE WRITTEN AS




FOR THE CASE OF (b) & (c), $\sigma = \infty \Rightarrow R = 0$. HENCE, THE GENERATED emf, $v$ APPEARS ACROSS C. NOW, WE NEED TO TAKE INTO ACCOUNT THE VOLTAGE DROP IN CONDUCTORS.

$$R = \frac{l}{\sigma S},$$

$l$ = LENGTH OF THE CONDUCTOR
$\sigma$ = CONDUCTIVITY,
$S$ = CROSS-SECTIONAL AREA

$$= \frac{3}{5.5\times10^{7} \times 0.5\times10^{-6}}\ \Omega$$

$$= 0.1091\ \Omega$$

USING KVL, $v_1 + v_2 = v \rightarrow (1)$

$v_2 = iR$

CAPACITOR: $q = C v_1$

$i = \frac{dq}{dt} \Rightarrow q = \int i\,dt$

$v_1 = \frac{1}{C} \int i\,dt$

(1) BECOMES

$$\frac{1}{C}\int i\,dt + iR = v = -2e^{2t}$$

DIFFERENTIATING, WE GET

$$\frac{i}{C} + R\frac{di}{dt} = -4e^{2t}$$

$$\frac{i}{\tau} + \frac{di}{dt} = \frac{-4}{R}e^{2t}$$

$$= -36.66\,e^{2t} \rightarrow (2)$$

WHERE $\tau = RC = 0.1091 \times 5 = 0.5455\,s$

LET $i = x\,e^{-t/\tau}$

$$\frac{di}{dt} = \frac{dx}{dt}e^{-t/\tau} - \frac{x}{\tau}\cdot e^{-t/\tau} \rightarrow (3)$$

$$\frac{i}{\tau} = \frac{x}{\tau}e^{-t/\tau} \rightarrow (4)$$

USING (3) & (4) in (2), WE FIND

$$\frac{dx}{dt} = -36.66\, e^{2t} \times e^{t/\tau}$$

$$x = \frac{-36.66\, e^{(2+1/\tau)t}}{(2+1/\tau)}$$

$$i = x\, e^{-t/\tau} = \frac{-36.66}{2+1/\tau}\, e^{2t}$$

$$= -9.5639\, e^{2t}\ A$$

$$J = \frac{i}{S} = \frac{-9.5639\, e^{2t}}{0.5 \times (10^{-3})^2}\ A/m^2$$

$$= -1.9128 \times 10^{7}\ \cancel{\phantom{xx}}\ e^{2t}\ A/m^2$$

$$J = \sigma E$$

$$E = \frac{J}{\sigma} = \frac{-1.9128 \times 10^{7}\, e^{2t}}{5.5 \times 10^{7}}\ V/m$$

$$= 0.3478\, e^{2t}\ V/m$$

$$D = \epsilon_0 \epsilon_r E = \epsilon_0 E \quad (\because \epsilon_r = 1)$$

$$\epsilon_0 = 8.854 \times 10^{-12}\ F/m$$

$$J_d = \frac{\partial D}{\partial t} = \epsilon_0 \frac{\partial E}{\partial t} = 8.854 \times 10^{-12} \times 0.3478 \times 2 \times e^{2t}\ A/m^2$$

$$= 6.146 \times 10^{-12}\ A/m$$

$$i_d = J_d S = 6.146\times10^{-12} \times 0.5\times(10^{-3})^2 \text{ A}$$
$$= 3.073\times10^{-18} \text{ A}$$

NOTE $i_d \ll i$ IN THE CONDUCTOR

DISPLACEMENT CURRENT IN THE ~~CONDUCTOR~~ CAPACITOR?

$$E = \frac{v_1}{d}$$

$v_1$ = VOLTAGE ACROSS THE CAPACITOR

$$D = \epsilon E = \frac{\epsilon v_1}{d}$$

$$J_d = \frac{\partial D}{\partial t} = \frac{\epsilon}{d}\frac{dv}{dt}$$

$$i_d = J_d A = C\frac{dv_1}{dt}$$

$$v_1 = v - v_2 = v - iR$$
$$= -2e^{2t} + 9.5639 e^{2t} \times 0.1091 \text{ V}$$
$$= -0.9567 e^{2t} \text{ V}$$

$$i_d = 5\times(-0.9567)\times 2 e^{2t} \text{ A}$$
$$= -9.567 e^{2t} \text{ A}$$

Note that ~~the condu~~ conduction current in the conductor is the same as the displacement current in capacitor ( displacement ~~current~~ CURRENT in the conductor may be ignored