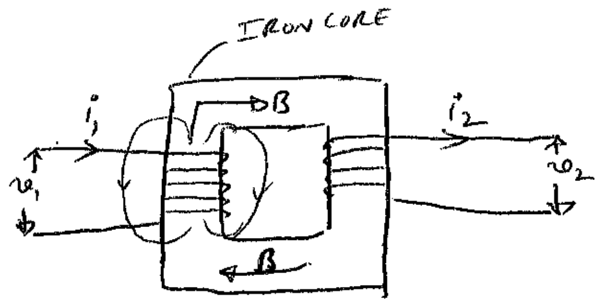
FARADAY'S LAW: ENGINEERING APPLICATIONS

(i) TRANSFORMER :



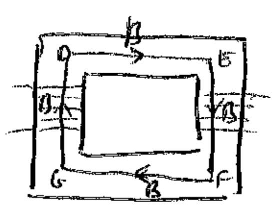
LET v_1 BE THE AC VOLTAGE APPLIED TO THE PRIMARY WINDING. TRANSFORMER ACTION CAN BE UNDERSTOOD QUALATATIVELY AS FOLLOWS: THE TIME-VARYING CURRENT i_1 ~~TO~~ LEADS TO TIME-VARYING MAGNETIC FLUX DENSITY B IN THE IRON CORE. SINCE THE MAGNETIC FLUX LINKED WITH THE SECONDARY WINDING (DUE TO B) CHANGES WITH TIME, IT INDUCES VOLTAGE v_2 ACROSS THE ~~...~~ TERMINALS OF THE SECONDARY WINDING. THIS IS HOW THE ELECTRICAL POWER IS TRANSMITTED FROM TRANSFORMER INPUT TO ITS OUTPUT.

USING AMPERE'S LAW ~~WE HAVE~~ AT THE PRIMARY WINDING, WE HAVE

$$\oint_L \vec{H}\cdot d\vec{l} = i_1 N_1 \longrightarrow (1)$$

($\because$ THE LOOP L ENCLOSES $N_1 i_1$ SINCE THERE ARE N_1 LOOPS CARRYING CURRENT, i_1)

THE MAGNETIC FIELD IS ROUGHLY CONSTANT (AS A FUNCTION OF DISTANCE, NOT TIME) IN THE IRON CORE. LET THE MAGNETIC FIELD LOOP L BE 'DEFG'. (SEE THE FOLLOWING FIG.)



LET THE LENGTH OF THE LOOP 'DEFG' BE l. N_1 IS THE

NUMBER OF TURNS IN THE ~~PRIMAR~~ PRIMARY WINDING.

SINCE H DOES NOT CHANGE (ALTHOUGH IT CHANGES AS A FUNCTION OF TIME) AS A FUNCTION OF DISTANCE, WE CAN TAKE IT OUT OF THE INTEGRAL,

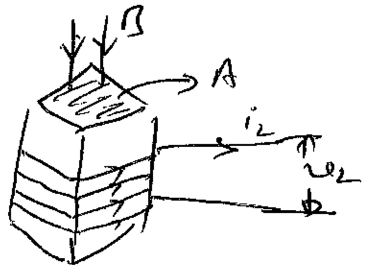
$$H \oint dl = H l = N_1 i_1 \quad \rightarrow (2)$$

MAGNETIC FLUXDENSITY, $B = \mu H = \mu N_1 i_1 / l \quad \rightarrow (3)$

~~SINCE i_1 IS A TIME-VARYING~~

LET THE CROSS-SECTION OF THE IRON CORE BE A. THE MAGNETIC FLUX LINKED TO ONE OF THE COILS OF THE SECONDARY WINDINGS IS

$$\psi_M = BA \quad \rightarrow (4)$$



HERE, A IS $\perp^r$ TO $\vec{B}$. FROM FARADAY'S LAW, THE INDUCED emf IN A COIL IS

(USE EQ. (3))

$$emf_1 = -\frac{d\psi_M}{dt} = -A\frac{dB}{dt} = -\frac{A\mu N_1}{l}\cdot\frac{di_1}{dt} \quad \rightarrow (4)$$

IF THERE ARE N_2 TURNS IN THE SECONDARY WINDING, THE EMF INDUCED IN COILS ADD UP ~~LEA~~ LEADING TO

$$v_2 = N_2\, emf_1 = \frac{-A\mu N_1 N_2}{l}\frac{di_1}{dt} \quad \rightarrow (5)$$

$$v_2 = -M \frac{di_1}{dt} \rightarrow (6)$$

$M = \frac{A\mu N_1 N_2}{l}$ IS KNOWN AS <u>MUTUAL INDUCTANCE.</u>

THUS, THE EMF INDUCED IN THE SECONDARY WINDING IS PROPORTIONAL TO THE RATE OF CHANGE OF CURRENT IN THE PRIMARY WINDING. NOTE THAT IF THE PRIMARY ~~WINDING~~ CURRENT v_1 IS A CONST. (DC), THERE WOULD BE NO VOLTAGE ACROSS THE SECONDARY WINDING.

THE FLUX LINKED WITH THE PRIMARY WINDING ALSO CHANGES WITH TIME DUE TO i_1 & AN EMF IS INDUCED IN THE PRIMARY WINDING AS WELL.

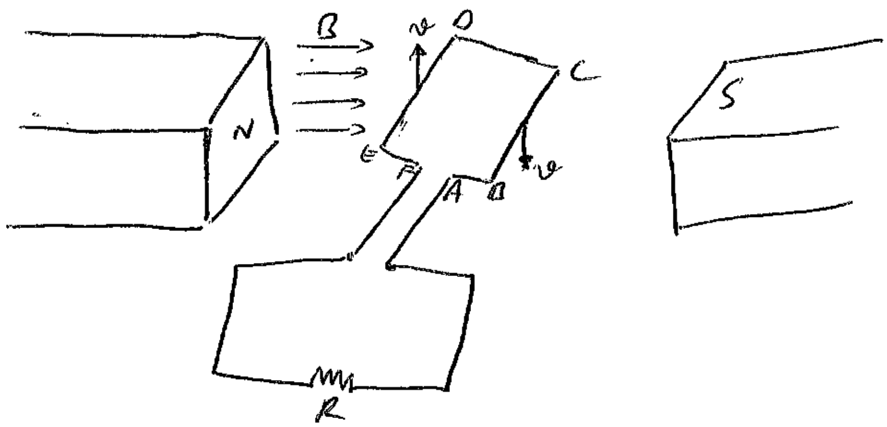
FOLLOWING THE SAME ARGUMENT, YOU CAN SHOW THAT

$$v_{Lind} = -L \frac{di_1}{dt}, \quad L = \frac{A\mu N_1^2}{l}$$

(TRY TO PROVE IT) SELF-INDUCTANCE

NOTE THAT ~~THE~~ EQ. (6) PROVIDES THE CONTRIBUTION TO THE SECONDARY EMF DUE TO PRIMARY CURRENT i_1. THERE ARE OTHER CONTRIBUTIONS TO v_2 WHICH ARE IGNORED IN EQ. (6). FOR EXAMPLE, TIME-CHANGING i_2 PRODUCES FLUX, WHICH LEADS TO emf ($\propto di_2/dt$) & THIS IS IGNORED IN (6).

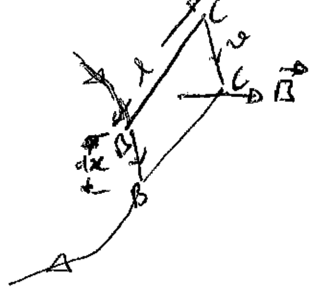
(ii) ELECTRIC GENERATOR



CONSIDER A COIL ~~ROTATING~~ ROTATING IN A UNIFORM MAGNETIC FIELD $\vec{B}$. LET THE VELOCITY OF THE COIL BE $\vec{v}$. WHEN THE COIL IS IN THE HORIZONTAL POSITION (AS SHOWN IN FIG.), THE ARM 'BC' MOVES ~~UPWARD~~ DOWNWARDS AT SPEED v WHILE THE ARM 'DE' MOVES UPWARDS " " ". LET THE LENGTH OF $\vec{BC}$ OR $\vec{DE}$ BE l. THE MAGNETIC FLUX CUT BY THE ARM 'BC' IN A SMALL TIME INTERVAL dt IS

$$d\psi_m = BA \quad \text{where } A \text{ is } \perp^r \text{ to } \vec{B}$$
$$= Bl\,dx, \rightarrow (1)$$

WHERE dx IS THE DISPLACEMENT OF THE ARM 'BC' IN THE INTERVAL dt.



SINCE $v = \frac{dx}{dt} \Rightarrow dx = v\,dt \rightarrow 8$

$$d\psi_m = B l v\, dt \rightarrow \cancel{(8)}$$

$$\frac{d\psi_m}{dt} = B l v \rightarrow (9)$$

USING FARADAY'S LAW, THE emf INDUCED IN THE ARM 'BC' IS

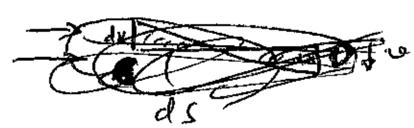
$$e_{CB} = -\frac{d\psi_m}{dt} = -Blv \rightarrow (10)$$

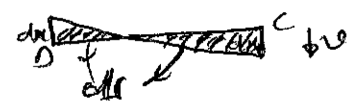
THE MAGNITUDE OF THE EMF INDUCED IN THE ARM 'DE' IS THE SAME AS THAT IN 'BC'. THESE TWO EMFS ARE IN SERIES (IN PHASE) LEADING TO

$$e_{FA} = \overset{=0}{e_{FE}} + e_{ED} + \overset{=0}{e_{DC}} + e_{CB} + \overset{=0}{e_{BA}}$$

$$= -2Blv \rightarrow (11)$$

YOU CAN SHOW THAT ~~THE~~ THE EMFs INDUCED IN THE ARM CD, AB & EF ARE ZEROS SINCE THEY DO NOT CUT THE FLUX, i.e. THE AREA ds IS NOT $\perp^r$ TO $\vec{B}$ (SEE FIG.)

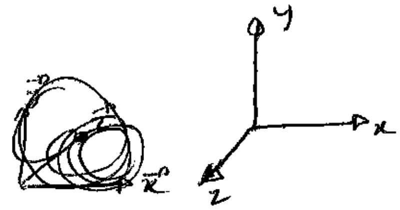


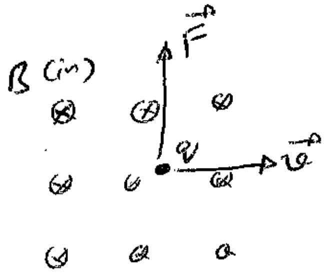


NOTE THAT EQ. (11) PROVIDES THE EMF INDUCED WHEN THE COIL IS IN THE HORIZONTAL POSITION. WHEN THE COIL IS IN THE VERTICAL POSITION, NO EMF IS INDUCED.

LORENTZ FORCE: FROM EXPERIMENTS, IT IS FOUND THAT A CHARGE q MOVING WITH THE VELOCITY IN A MAGNETIC FIELD $\vec{B}$ EXPERIENCES A FORCE $\vec{F}$ GIVEN BY

$$\vec{F} = q(\vec{v} \times \vec{B}) \longrightarrow (12)$$





(THIS MEANS INTO THE PAGE)

IN THE FIG., $\vec{B}$ IS $\perp^r$ TO THE PAGE (SHOWN AS $\otimes$).

IF $\vec{v} = v\vec{x}$, $\vec{B} = -B\vec{z}$

$$\vec{F} = q(\vec{v} \times \vec{B}) = -vB(\vec{x} \times \vec{z})$$
$$= vB\,\vec{y}$$

SO, $\vec{F}$ IS IN THE DIRECTION OF $\vec{y}$. $\vec{F}$ IS KNOWN AS <u>LORENTZ FORCE</u>.

SINCE $\vec{F} = q\vec{E} \longrightarrow (13)$

FROM (12), $\cancel{q}\vec{E} = \cancel{q}(\vec{v} \times \vec{B})$.

THE <u>MOTIONAL ELECTRIC FIELD</u> MAY BE WRITTEN AS

$$\vec{E}_m = \vec{v} \times \vec{B}$$

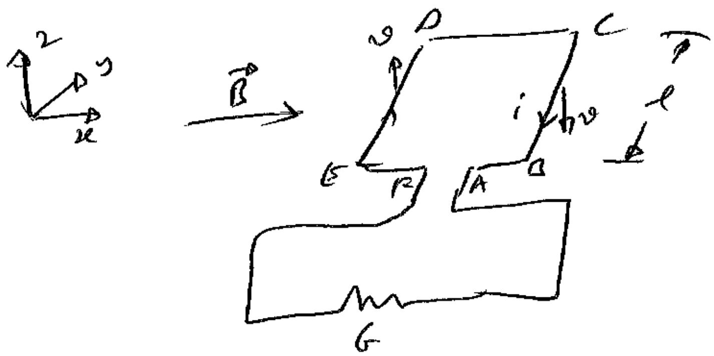
IN THE ABOVE FIG., THE CHARGE q IS MOVING IN $\vec{y}$ DIRECTION DUE TO FORCE $\vec{F}$, IT IS INTERPRETED AS

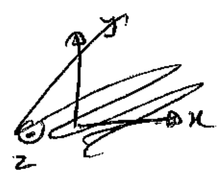
THE CHARGE IS SUBJECTED TO AN EFFECTIVE (MOTIONAL) ELECTRIC FIELD $\vec{E}_M$.

FROM FARADAY'S LAW, WE HAVE

$$emf = \oint_L \vec{E}_M \cdot d\vec{l} = \oint_L (\vec{v} \times \vec{B}) \cdot d\vec{l} \quad \rightarrow \quad (14)$$

THE EMF GENERATED IN A GENERATOR (EQ. (11)) MAY ALSO BE OBTAINED USING EQ. (14). LET THE LOOP L BE 'ABCDEFGA'





CONSIDER THE SEGMENT $\vec{BC}$: (LET THE ORIGIN OF y-co-ordinate be at B.)

$$e_{CB} = \int_B^C (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

$$\vec{v} = -v\vec{z}; \quad \vec{B} = B\vec{x}; \quad d\vec{l} = dl\,\vec{y}$$

$$(\vec{v} \times \vec{B}) \cdot d\vec{l} = -vB\,(\vec{z} \times \vec{x}) \cdot (dl\,\vec{y}) = -vB\,dl\,\underbrace{(\vec{y} \cdot \vec{y})}_{=1}$$

$$\therefore \; e_{CB} = -\int_0^l vB\,dl = -Blv$$

(8)

$$e_{DC} = \int_C^D (\vec{v} \times \vec{B}) \cdot dl = 0$$ ($\because$ $\vec{v} \times \vec{B}$ is in $\pm \vec{y}$ direction & $d\vec{l}$ is in $\vec{x}$ direction)

$$e_{ED} = \int_D^E (\vec{v} \times \vec{B}) \cdot d\vec{l} = \int_0^l vB\,(\vec{z} \times \vec{x}) \cdot (dl\,\vec{y})$$

~~$\vec{v} = v\vec{z}$; $\vec{B} = B\vec{x}$~~ ; $\vec{v} = v\vec{z}$; $\vec{B} = B\vec{x}$

$$= -Blv$$

$e_{EF} = e_{AB} = 0$;

$e_{FG} = e_{GA} = 0$ since $|\vec{B}|$ is assumed to be non-zero only inside the generator. Load is outside the generator & B is zero in the external circuit.

$$emf = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l} = \overset{=0}{e_{GF}} + \overset{=0}{e_{FE}} + e_{ED} + \overset{=0}{e_{DC}} + e_{CB} + \overset{=0}{e_{BA}} + \overset{=0}{e_{AG}}$$

$$= -2Blv$$