PHASORS : WORKED EXAMPLE

THE ELECTRIC FIELD AT $z = 0.15\,m$ IS

$$E_{xs} = 2\angle{-3\pi/2}$$

THE FREQUENCY OF THE EM WAVE IS 1 GHz. & IT IS PROPAGATING IN +VE Z-DIRECTION IN FREE SPACE. FIND (i) ELECTRIC FIELD & MAGNETIC FIELD AS A FUNCTION OF z & t. (ii) SKETCH THE WAVES AT (a) $t = 0$ & (b) ~~...~~ $t = 0.5\,ns$

$$E_x = Re\{E_{xs}\, e^{j\omega t}\}$$

$$= Re\{2 e^{-j3\pi/2 + j\omega t}\}$$

$$= 2\cos(\omega t - 3\pi/2)\ V/m \longrightarrow (1)$$

WE KNOW THAT

$$E_x = 2\cos(\omega t - \beta z + \theta)\ V/m \longrightarrow (2)$$

WHERE θ IS THE PHASE AT $t=0$ & $z=0$.

$$\omega = 2\pi f = 2\pi \times 1 \times 10^9 = 6.28 \times 10^9\ rad/s$$

$$\beta = \frac{\omega}{c} = \frac{6.28\times10^9}{3\times10^8}\ m^{-1} = 20.943\ m^{-1}$$

At $z = 0.15\,m$, $\beta z = 20.943 \times 0.15$ rad

$$= \pi\ rad.$$

$\therefore$ EQ. (2) BECOMES

$$E_x = 2\cos(\omega t - \pi + \theta) \text{ V/m} \rightarrow (3)$$
$$= 2\cos[\omega t - (\pi - \theta)] \text{ V/m}$$

COMPARING (3) & (1), WE FIND

$$3\pi/2 = \pi - \theta$$
$$\theta = -\pi/2$$

FROM (2),

$$E_x = 2\cos(\overbrace{\omega t - \beta z}^{A} - \pi/2) \text{ V/m}$$
$$= 2\sin(\underbrace{\omega t - \beta z}_{A}) \text{ V/m} \quad (\because \cos(A - \pi/2) = \sin A)$$

H_y

$$\frac{E_x}{H_y} = \eta = 377\ \Omega$$

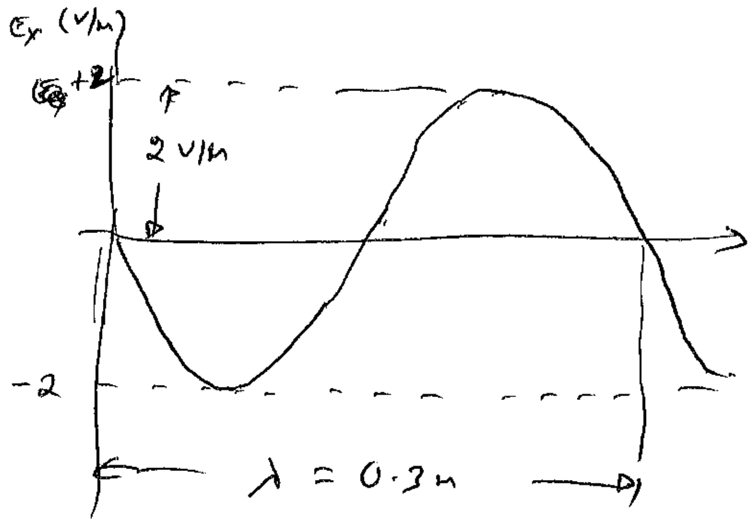
$$\therefore H_y = \frac{E_x}{\eta} = \frac{2}{377}\sin(\omega t - \beta z) \text{ A/m}$$
$$= 5.3 \times 10^{-3}\sin(\omega t - \beta z) \text{ A/m}$$

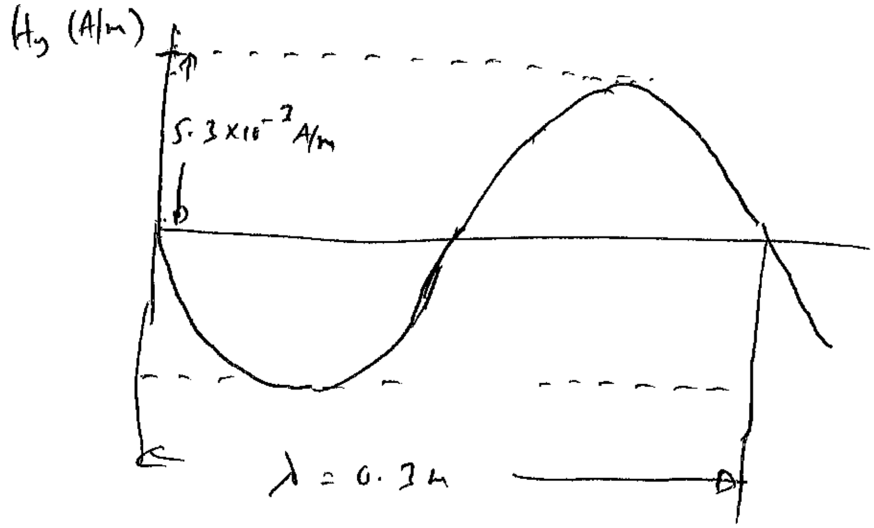
(a) At $t = 0$,

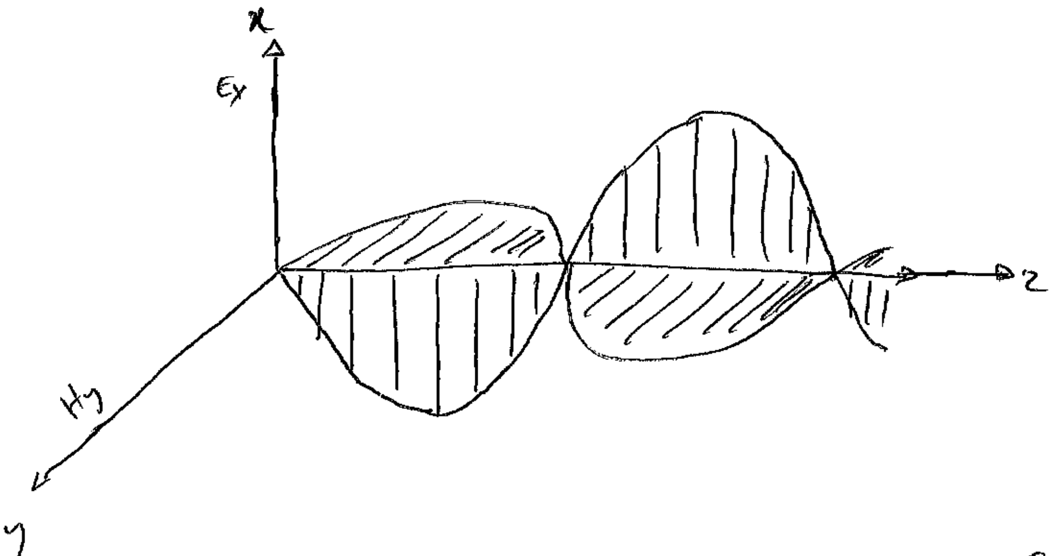
$$E_x = 2\sin(-\beta z) = -2\sin(\beta z) \text{ V/m}$$
$$H_y = -5.3 \times 10^{-3}\sin(\beta z) \text{ A/m}$$

$$\beta = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{\beta} = 0.3 \text{ m}$$

At $t=0$







NOTE THAT E_x & H_y ARE IN PHASE IN FREE SPACE. THIS IS NOT THE CASE ~~WHEN~~ FOR THE EM WAVE IN A CONDUCTOR OR A LOSSY DIELECTRIC.

(b) At $t = 0.5$ ns

$$\omega t = 2\pi f t = 2\pi \times 10^9 \times 0.5 \times 10^{-9} \text{ rad} = \pi \text{ rad}$$

$\therefore$

$$E_x = 2 \sin(\pi - \beta z) \text{ V/m}$$

$$= 2 \sin(\beta z) \text{ V/m}$$

$$H_y = 5.3 \times 10^{-3} \sin(\beta z)$$

<u>At $t = 0.5$ ns</u>