PLANE WAVE:

$$E_x = E_{x0}^{+} \cos(\omega t - \beta z) \rightarrow (1)$$

$$= E_{x0}^{+} \cos\left(2\pi f t - \frac{2\pi}{\lambda} z\right)$$

$$= E_{x0}^{+} \cos\left[2\pi f\left(t - z/v\right)\right]$$

FORWARD PROP. PLANE WAVE

E_{x0}^{+} is A CONST. (+ INDICATES FORWARD PROP.).

$$E_x = E_{x0}^{-} \cos(\omega t + \beta z) \rightarrow (2)$$

BACKWARD PROP. PLANE WAVE

THE MAGNETIC FIELD H_y ALSO SATISFIES THE WAVE EQUATION (REFER TO THE PREVIOUS NOTES)

$$\frac{\partial^2 H_y}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 H_y}{\partial t^2}$$

PROCEEDING AS BEFORE (FOR E_x)

$$H_y = H_{y0}^{+} \cos(\omega t - \beta z) \rightarrow (3)$$: FORWARD PROP. WAVE

$$H_y = H_{y0}^{-} \cos(\omega t + \beta z) \rightarrow (4)$$: BACKWARD " "

$$\vec{E} = E_x \vec{x}\,; \quad \vec{H} = H_y \vec{y}$$

$$\nabla \times \vec{H} = \epsilon \frac{\partial E_x}{\partial t}\vec{x}.$$

$$\therefore \left(\nabla \times \vec{H}\right)_x = \frac{-\partial H_y}{\partial z}$$

$$\therefore \quad \frac{\partial H_y}{\partial z} = -\epsilon \frac{\partial E_x}{\partial t} \longrightarrow 5$$

~~USING 1 & 3~~

LET US FIRST CONSIDER THE FORWARD PROP. WAVE.

USING 1 & 3 IN 5, WE FIND

$$-H_{y0}^{+} \sin(\omega t - \beta z)(-\beta) = -\epsilon\left(-E_{x0}^{+} \sin(\omega t - \beta z) \cdot \omega\right)$$

$$\Longrightarrow \quad \frac{E_{x0}^{+}}{H_{y0}^{+}} = \frac{\beta}{\omega \epsilon}$$

$$\frac{\omega}{\beta} = v = \frac{1}{\sqrt{\mu \epsilon}} \Rightarrow \frac{\beta}{\omega \epsilon} = \sqrt{\frac{\mu}{\epsilon}}$$

$$\therefore \boxed{\frac{E_{x0}^{+}}{H_{y0}^{+}} = \sqrt{\frac{\mu}{\epsilon}} = \eta} \longrightarrow 6$$

η is THE INTRINSIC IMPEDANCE OF THE MEDIUM.

NOTE: E_x IS THE ANALOG OF VOLTAGE
H_y " " " " CURRENT

UNIT OF E_x = V/m
" " H_y = A/m

$\therefore$ THE RATIO OF E_{x0}^{+} & H_{y0}^{+} IS IMPEDANCE (UNIT Ω)

FOR FREESPACE,

$$\epsilon = \epsilon_0 = 8.854 \times 10^{-12}\ C^2/Nm^2$$

$$\mu = \mu_0 = 4\pi \times 10^{-7}\ N/A^2$$

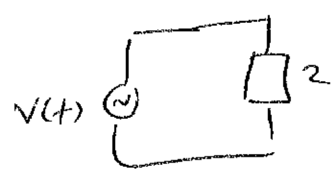
$$\eta = \sqrt{\frac{\mu}{\epsilon}} \cong 377\,\Omega$$

USING 2 & 4 IN 5, YOU CAN VERIFY THAT

$$\frac{E_{x0}^{-}}{H_{y0}^{-}} = -\sqrt{\frac{\mu}{\epsilon}} = -\eta \quad \rightarrow 7$$

COMPLEX NOTATION :

IN CIRCUITS, COMPLEX NOTATION (PHASORS) IS USED TO FACILITATE MATHEMATICAL MANIPULATIONS



$$V(t) = V_0 \cos(2\pi f t)\ ; \ V_0 \Rightarrow \text{REAL}$$

$$\tilde{V}(t) = V_0\, e^{j2\pi f t}$$

$$V(t) = Re\{\tilde{V}(t)\}$$

$$= Re\{V_0\, e^{j2\pi f t}\}$$

$$= V_0 \cos(2\pi f t)$$

WE FOLLOW THE SAME APPROACH FOR FIELDS.

$$\mathcal{E}_x = \mathcal{E}_{x0}^{+} \cos(\omega t - \beta z) \rightarrow (8)$$

LET $\tilde{\mathcal{E}}_x = \mathcal{E}_{x0}^{+} e^{j(\omega t - \beta z)} \rightarrow (9)$. $\mathcal{E}_{x0}^{+}$ IS ASSUMED TO BE REAL.

$$\begin{aligned} Re\left[\tilde{\mathcal{E}}_x\right] &= \mathcal{E}_{x0}^{+} \, Re\left\{ e^{j(\omega t - \beta z)} \right\} \\ &= \mathcal{E}_{x0}^{+} \left\{ \cos(\omega t - \beta z) + j \sin(\omega t - \beta z) \right\} \\ &= \mathcal{E}_{x0}^{+} \cos(\omega t - \beta z) \\ &= \mathcal{E}_x \end{aligned}$$

$$\therefore \quad \mathcal{E}_x = Re\left[\tilde{\mathcal{E}}_x\right] \rightarrow (10)$$

IN THIS COURSE, WE MOSTLY DEAL WITH HARMONICALLY VARYING ($e^{j\omega t}$) FIELDS. SO, WE LIKE TO ~~SEPARAT~~ SEPARATE SPATIAL & TEMPORAL PARTS. REWRITE EQ. (9) AS

$$\tilde{\mathcal{E}}_x(z,t) = \mathcal{E}_{xs}^{+}(z)\, e^{j\omega t} \rightarrow (11)$$

WHERE

$$\mathcal{E}_{xs}^{+}(z) = \mathcal{E}_{x0}^{+} e^{-j\beta z} \rightarrow (12)$$

FROM (10), $\mathcal{E}_x = Re\left[\tilde{\mathcal{E}}_x\right] = Re\left[\mathcal{E}_{xs}^{+}(z)\, e^{j\omega t}\right] \rightarrow (13)$

$\mathcal{E}_{xs}^{+}$ IS THE PHASOR CORRESPONDING TO $\mathcal{E}_x$.

A BACKWARD-PROPAGATING WAVE:

$$\mathcal{E}_x = \mathcal{E}_{x0}^- \cos(\omega t + \beta z)$$

$$\tilde{\mathcal{E}}_x = \mathcal{E}_{x0}^- e^{j(\omega t - \beta z)}$$

$$\mathcal{E}_x = Re[\tilde{\mathcal{E}}_x]$$

$$= Re[E_{xs}^-(z) e^{j\omega t}] \longrightarrow (14)$$

$E_{xs}^-(z) = E_{x0}^- \cdot e^{j\beta z}$ IS THE PHASOR CORRESPONDING TO $\mathcal{E}_x$.

III'y

$$H_y = H_{y0}^+ \cos(\omega t - \beta z) \quad : \text{FORWARD}$$

$$H_y = Re[H_{ys}^+(z) e^{j\omega t}]$$

$$H_{ys}^+ = H_{y0}^+ e^{-j\beta z} \longrightarrow (15)$$

~~From~~ From (13), WE HAVE

COMPLEX CONJUGATE

$$\mathcal{E}_x = Re[E_{xs}^+(z) e^{j\omega t}]$$

$$= \frac{[E_{xs}^+ e^{j\omega t} + (E_{xs}^+)^* e^{-j\omega t}]}{2} = \frac{1}{2}[E_{xs}^+ e^{j\omega t} + \text{c.c.}]$$

PROOF: CONSIDER A COMPLEX NUMBER, $z = x + jy$

$$\frac{z + z^*}{2} = \frac{(x + jy) + (x - jy)}{2} = x$$

$$= Re[z]$$

TRANSVERSE ELECTROMAGNETIC WAVE (TEM):

IN FREE SPACE OR PERFECT DIELECTRIC MEDIUM, EM WAVE IS DESCRIBED BY

$$E_x = E_{x0} \cos(\omega t - kz)$$

$$H_y = H_{y0} \cos(\omega t - kz)$$

$$H_{y0} = \frac{E_{x0}}{\eta} \; ; \; \eta = \text{IMPEDANCE OF THE MEDIUM.}$$

ELECTRIC FIELD → X DIRECTION
MAGNETIC FIELD → Y DIRECTION
PROPAGATION → Z "

MUTUALLY PERPENDICULAR → POLARIZATION X

OR

ELECTRIC FIELD → Y DIRECTION
MAGNETIC " → X DIRECTION
PROPAGATION " → Z "

→ POLARIZATION Y

IN FREE SPACE, ELECTRIC & MAGNETIC FIELDS ARE $\perp^r$ TO THE DIRECTION OF PROPAGATION. HENCE, THEY ARE CALLED TRANSVERSE ELECTRO-MAGNETIC (TEM) WAVES.

EXAMPLE: AN EM WAVE IS PROPAGATING IN A MEDIUM OF RELATIVE PERMITTIVITY, $\epsilon_r = 2.25$. THE PEAK ELECTRIC FIELD INTENSITY IS 10 V/m. THE ELECTRIC FIELD AT $t = 0$ & $z = 0$ m IS 5 V/m. ~~[illegible]~~ THE FREQUENCY, $f = 1$ GHz.

LET $\quad E_x = E_{x0} \cos(\omega t - \beta z + \phi) \longrightarrow (16)$

$$E_{x0} = 10 \text{ V/m}$$

AT $t = 0$, $z = 0$, $\quad E_x = E_{x0} \cos\phi = 5 \text{ V/m}$

$$\cos\phi = \frac{1}{2}$$

$$\phi = \pm \pi/3$$

LET US PICK THE +VE SIGN, FOR CONVENIENCE.

$$\omega = 2\pi f = 2\pi \times 1 \times 10^9 \text{ rad/s}$$

$$\frac{\omega}{\beta} = v \Rightarrow \beta = \frac{\omega}{v}$$

$$v = \frac{c}{n} = \frac{c}{\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{\sqrt{2.25}} \text{ m/s} = 2 \times 10^8 \text{ m/s}$$

$$\beta = \frac{2\pi \times 1 \times 10^9}{2 \times 10^8} = 10\pi$$

$$\therefore E_x = E_{x0} \cos(2\pi \times 1 \times 10^9 t - 10\pi z + \pi/3) \text{ V/m}$$

PHASOR FORM:

$$\cancel{E_x = \mathrm{Re}\{E_{xs} e^{j\omega t}\}}$$

$$E_x(z,t) = \mathrm{Re}\left\{ 10\, e^{j(2\pi \times 10^9 t - 10\pi z + \pi/3)} \right\} \text{ V/m}$$

$$= \mathrm{Re}\left\{ E_{xs}\, e^{j(2\pi \times 10^9 t)} \right\} \text{ V/m}$$

$$\therefore \; E_{xs} = 10\, e^{-j10\pi z + j\pi/3} \text{ V/m} \quad : \text{PHASOR FORM}$$

TO OBTAIN THE PHASOR FORM, DROP 'Re' & SUPPRESS $e^{j(2\pi \times 10^9 t)}$.

AT $z = 1/10$ m, $\quad E_{xs} = 10\, e^{-j\pi + j\pi/3}$ V/m

$$= 10 \angle -2\pi/3 \text{ V/m}$$

FOR EXAMPLE, DIFFERENTIATION OF (16) YIELDS

$$\frac{\partial E_x}{\partial t} = -\omega E_{x0} \sin(\omega t - \beta z + \varphi) \rightarrow (17)$$

IN THE PHASOR FORM

$$E_{xs} = E_{x0}\, e^{-j\beta z + \varphi}$$

$$E_x = \mathrm{Re}\left\{ E_{xs}(z)\, e^{j(\omega t)} \right\}$$

$$\frac{\partial E_x}{\partial t} = \mathrm{Re}\left\{ E_{xs}(z)\, j\omega \cdot e^{j\omega t} \right\} \rightarrow (18)$$

NOTE THAT THE DIFFERENTIAL OPERATOR $\frac{\partial}{\partial t}$ CAN BE TAKEN INSIDE Re{ }. THIS CAN BE VERIFIED BY TAKING THE REAL PART OF $E_{xs}(z)\, j\omega \cdot e^{j\omega t}$ & COMPARING IT WITH EQ. (17).

NOTE THAT THE EFFECT OF $\frac{\partial}{\partial t}$ IS TO MULTIPLY THE PHASOR BY $j\omega$.

REAL WORLD	PHASOR WORLD
E_x	E_{xs}
$\frac{\partial E_x}{\partial t}$	$j\omega E_{xs}$
$\int E_x(t)\,dt$	$E_{xs}/j\omega$

A FAMILIAR EXAMPLE FROM THE CIRCUIT IS

$$v = L\frac{di}{dt} \quad : \text{REAL WORLD}$$

$$V_s = L I_s j\omega \quad : \text{PHASOR WORLD}$$

$$= \underbrace{X_L}_{\text{REACTANCE}} I_s$$