EXAMPLE A: 1 MHz PLANE WAVE IS PROPAGATING IN FRESH WATER.

AT THIS FREQUENCY, LOSS IN WATER IS NEGLIGIBLE WHICH MEANS THAT $\epsilon_r'' = 0$; ~~[illegible]~~ FOR WATER $\mu_r = 1$ & $\epsilon_r' = 81$. FIND THE ~~PEAK AMPLITUDES OF ELECTRIC & MAGNETIC FIELDS~~. EXPRESSIONS FOR ELECTRIC & MAGNETIC FIELDS. ASSUME THAT PEAK ELECTRIC FIELD INTENSITY IS 0.1 V/m.

$$E_x = E_{x0} e^{-\alpha z} \cos(\omega t - \beta z)$$

SINCE $\epsilon_r'' = 0$, ~~LOSS~~ THERE IS NO LOSS IN WATER $\Rightarrow \alpha = 0$.

$$E_{x0} = 0.1 \text{ V/m}$$

$$\omega = 2\pi \times 1 \times 10^6 \text{ rad/m}$$

$$\beta = \frac{\omega}{v} \; ; \; v = \frac{1}{\sqrt{\mu \epsilon}} \Rightarrow \beta = \omega\sqrt{\mu\epsilon}$$

$$\mu = \mu_0 \mu_r = 4\pi \times 10^{-7} \times 1 \text{ H/m}$$

$$\epsilon = \epsilon_0 \epsilon_r = 8.854 \times 10^{-12} \times (\epsilon_r' - j\epsilon_r'') \text{ F/m}$$

$$= 8.854 \times 10^{-12} \times 81 \text{ F/m}$$

$$= 7.17 \text{ F/m}.$$

$$\beta = 2\pi \times 1 \times 10^6 \times \sqrt{4\pi \times 10^{-7} \times 8.854 \times 10^{-12} \times 81} \text{ rad/m}$$

$$= 0.19 \text{ rad/m}$$

IMPEDANCE OF FRESH WATER $\rightarrow$ $$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{4\pi \times 10^{-7}}{8.854 \times 10^{-12} \times 81}} = 42\,\Omega$$

$\therefore \quad E_x = 0.1 \cos(2\pi \times 10^6 t - 0.19z)$ V/m

$H_y = \frac{E_{x0}}{\eta} \cos(\omega t - \beta z)$ → (VALID ONLY IF η IS REAL)

$= \frac{0.1}{42} \cos(2\pi \times 10^6 t - 0.19z)$ A/m

$= 0.0024 \cos(2\pi \times 10^6 t - 0.19z)$ A/m

NOTE THAT E_x & H_y ARE IN PHASE AT THIS FREQUENCY.

EXAMPLE B: REPEAT EXAMPLE A AT 2.5 GHz. AT THIS FREQUENCY, WATER IS LOSSY, i.e. $\epsilon_r'' \neq 0$. ASSUME $\epsilon_r' = 81$, $\epsilon_r'' = 7$ & $\mu_r = 1$.

$$\alpha = \omega \sqrt{\frac{\mu \epsilon'}{2}} \left(\sqrt{1 + \left(\frac{\epsilon''}{\epsilon'}\right)^2} - 1 \right)^{1/2}$$

$$= 2\pi \times 2.5 \times 10^9 \sqrt{\frac{4\pi \times 10^{-7} \times 81 \times \epsilon_0}{2}} \left(\sqrt{1 + \left(\frac{7}{81}\right)^2} - 1 \right)^{1/2}$$

$= 20.35$ Np/m

$\beta \cong 0.19$ rad/m [YOU CAN CALCULATE THE EXACT VALUE USING THE FORMULA $\beta = \omega \sqrt{\frac{\mu \epsilon'}{2}} \left(\sqrt{1 + \left(\frac{\epsilon''}{\epsilon'}\right)^2} + 1 \right)^{1/2}$]

$E_x = E_{x0} e^{-\alpha z} \cos(\omega t - \beta z)$

$= 0.1 e^{-20.35z} \cos(2\pi \times 2.5 \times 10^9 t - 0.19z)$ V/m

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{4\pi \times 10^{-7}}{(81 - j7)\epsilon_0}} = 41.78 \angle 2.46^\circ \ \Omega$$

LET THE AMPLITUDE OF E_x BE $A(z)$, i.e.

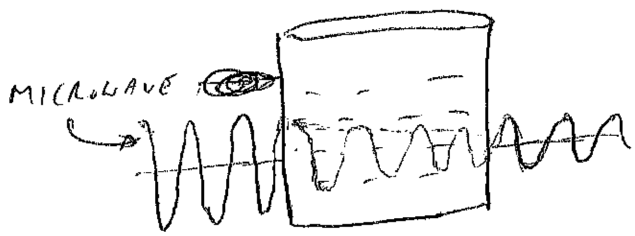
$$A(z) = E_{x0}\, e^{-\alpha z}$$

$$A(0) = E_{x0}$$

AT $z = 1/\alpha$, $A(z) = E_{x0} \cdot e^{-1}$

THE DISTANCE $1/\alpha$ IS CALLED THE PENETRATION DEPTH. AT THIS DISTANCE, THE AMPLITUDE IS DECAYED BY A FACTOR e. WITH $\alpha = 20.35$ Np/m, $1/\alpha = 4.8$ cm.

CONSIDER THE HEATING OF WATER IN A MICROWAVE.



AS THE MICROWAVE PROPAGATES IN WATER, IT IS ABSORBED BY WATER LEADING TO HEATING OF WATER. THUS, THE ELECTRICAL ENERGY IS CONVERTED TO THERMAL ENERGY — OPERATING PRINCIPLE OF MICROWAVE. LOSS OF MICROWAVE IS REPRESENTED BY $e^{-\alpha z}$.

$$H_y = \frac{E_{x0}}{\eta} e^{-\alpha z} \cos(\omega t - \beta z)$$

$$= \frac{0.1}{41.78\angle 2.46^\circ} e^{-20.35 z} \cos(2\pi \times 2.5 \times 10^9 t - 0.19 z) \text{ A/m}$$

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$$H_{ys} = H_{yo}\, e^{-(\alpha + j\beta) z}$$

$$H_{yo} = \frac{E_{xo}}{\eta} = \frac{0.1}{41.78 \angle 2.46^\circ} \text{ A/m}$$

$$= 0.0024 \angle -2.46^\circ \times e^{-\alpha z} \text{ A/m}$$

$$H_y = Re\{ H_{ys}\, e^{j\omega t} \}$$

$$= Re\{0.0024\, e^{-\alpha z} e^{j(\omega t - \beta z)}\} \text{ A/m}$$

$$= Re\{ 0.0024 \angle -2.46^\circ\, e^{-\alpha z}\, e^{j(\omega t - \beta z)} \} \text{ A/m}$$

$$= 0.0024\, e^{-20.35 z} \cos(2\pi \times 2.5 \times 10^{9} t - 0.192 - 2.46^\circ) \text{ A/m}$$

H_y lags E_x by 2.46°.