EXAMPLE: IN A METALLIC CONDUCTOR, THE CURRENT DENSITY, $J = 2 \sin(2\pi f_0 t)\ MA/m^2$. FIND THE CONDUCTION CURRENT & DISPLACEMENT CURRENT IF (a) $f_0 = 60\ Hz$ (b) $f_0 = 10\ GHz$.

THE PARAMETERS OF THE CONDUCTOR, $\epsilon_r = \mu_r = 1$; $\sigma = 5.8 \times 10^7\ S/m$;

CROSS-SECTIONAL AREA = $1\ mm^2$.

(a) $f_0 = 60\ Hz$.

$$\vec{J} = \sigma \vec{E}$$

$$J = 2 \times 10^6 \sin(2\pi f_0 t)\ ;\quad \sigma = 5.8 \times 10^7\ S/m$$

$$E = \frac{J}{\sigma} = \frac{2 \times 10^6}{5.8 \times 10^7} \sin(2\pi \times 60 t)\ V/m$$

$$= 3.45 \times 10^{-2} \sin(2\pi \times 60 t)\ V/m$$

$$D = \epsilon E = \epsilon_0 \epsilon_r E$$

$$\epsilon_0 = 8.85 \times 10^{-12}\ F/m;\quad \epsilon_r = 1$$

$$\therefore D = 8.85 \times 10^{-12} \times 3.45 \times 10^{-2} \sin(2\pi \times 60 t)\ C/m^2$$

$$\cancel{= 1.1498 \times 10^{-10} \sin(2\pi f_0 t)\ C/m^2}$$

$$\cancel{J_d = \frac{\partial D}{\partial t} =}$$

$$\cancel{J_d = \frac{\partial D}{\partial t} = 2\pi f_0 \times 1.1498 \times 10^{-10} \sin(2\pi f_0 t)\ A/m^2}$$

$=$

$$D = 3.05\times10^{-13}\sin(2\pi\times60\,t)\ \mathrm{C/m^2}$$

~~DISPLACEMENT CURRENT~~

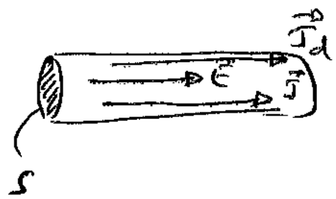
$$J_d = \frac{\partial D}{\partial t} = 2\pi\times60\times3.05\times10^{-13}\cos(2\pi\times60\,t)\ \mathrm{A/m^2}$$

$$= 1.1498\times10^{-10}\cos(2\pi\times60\,t)\ \mathrm{A/m^2}$$

DISPLACEMENT CURRENT, I_d:

$$I_d = \int_S \vec{J}_d\cdot d\vec{S} = J_d\,S$$

($\because$ S IS THE CROSS-SECTIONAL AREA $\perp^r$ TO $\vec{J}_d$ OR $\vec{E}$)



$$I_d = 1.1498\times10^{-10}\times(1\times10^{-6})\cos(2\pi\times60\,t)\ \mathrm{A}$$

$$= 1.1498\times10^{-16}\cos(2\pi\times60\,t)\ \mathrm{A}$$

CONDUCTION CURRENT, I:

$$I = \int \vec{J}\cdot d\vec{S} = J\,S$$

$$= 2\times10^{6}\times(1\times10^{-6})\sin(2\pi\times60\,t)$$

$$= 2\sin(2\pi\times60\,t)\ \mathrm{A}$$

NOTE THAT $I_d \ll I$. THIS IS THE REASON THE DISPLACEMENT CURRENT WAS NOT OBSERVED BEFORE MAXWELL PREDICTED IT FROM THEORETICAL CONSIDERATIONS.

(b) $f_0 = 10\ \mathrm{GHz}$:

CONDUCTION CURRENT DOES NOT DEPEND ON f_0.

SO, $I = 2\sin(2\pi\times10^{10}\,t)$ A

NOTE THAT $J_d = \frac{\partial D}{\partial t} = 2\pi f_0 \times 3.05 \times 10^{-13} \cos(2\pi f_0 t)$ A/m².

If f_0 INCREASES FROM 60 Hz to 10 GHz, J_d would INCREASE BY A FACTOR OF $\frac{10 \times 10^9}{60} = 1.667 \times 10^8$.

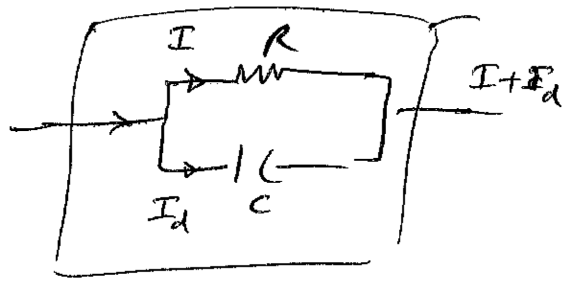
So, $I_d = 1.1498 \times 10^{-16} \times 1.667 \times 10^8 \cos(2\pi \times 10^{10} t)$ A

$= 1.9163 \times 10^{-8} \cos(2\pi \times 10^{10} t)$ A.

NOTE THAT DISPLACEMENT CURRENT INCREASES AS WE GO FROM RADIO FREQUENCY TO MICROWAVE FREQUENCIES.

~~Note~~

THE DISPLACEMENT CURRENT IS OUT OF PHASE WITH CONDUCTION CURRENT BY 90°. THIS CAN BE INTERPRETED AS THE CONDUCTOR HAVING A CAPACITANCE IN PARALLEL WITH AN IDEAL RESISTOR, AS SHOWN



EQUIVALENT CIRCUIT OF A CONDUCTOR

FOR AN IDEAL CONDUCTOR, $\sigma = \infty$.

$$J = \sigma E \Rightarrow E = \frac{J}{\sigma} = 0$$

$$D = 0$$

$$J_d = 0$$

$$\therefore \quad I_d = 0$$

~~AN IDEAL CONDUCTOR HAS ZERO RESISTANCE~~

2~~Ex~~