CASE (ii) LOSSY DIELECTRIC (PERMITTIVITY IS COMPLEX)

$$\frac{d^2 E_{xs}}{dz^2} = -k^2 E_{xs} \quad \longrightarrow (1)$$

$$k^2 = -j\omega\mu(\sigma + j\epsilon\omega)$$

FOR DIELECTRICS, $\sigma = 0$,

$$\therefore \quad k^2 = -j\omega\mu(j\epsilon\omega)$$

$$= \omega^2\mu\epsilon \quad \longrightarrow (2)$$

~~LET~~, LET

$$\epsilon = \epsilon' - j\epsilon'' = \epsilon_0(\epsilon_r' - j\epsilon_r'') \quad \longrightarrow (3)$$

ϵ_r' IS THE REAL PART OF ϵ_r & $-\epsilon_r''$ IS THE IMAGINARY PART WHICH IS RESPONSIBLE FOR THE LOSS OF WAVE AMPLITUDE IN A DIELECTRIC MEDIUM. SINCE ϵ IS COMPLEX, k^2 IS ALSO COMPLEX (SEE EQ. (2)).

IT IS CUSTOMARY TO WRITE k IN TERMS OF ITS REAL & IMAGINARY PARTS:

$$jk = \alpha + j\beta \quad \longrightarrow (4)$$

$$\cancel{E_{xs}(z) = A e^{jkz} + B e^{-jkz}}$$

SOLN OF (1) IS (REFER TO THE PREVIOUS LECTURE NOTES)

$$E_{xs}(z) = A e^{jkz} + B e^{-jkz} \quad \longrightarrow (5)$$

CONSIDER ONE OF THE TERMS (SAY THE SECOND TERM)

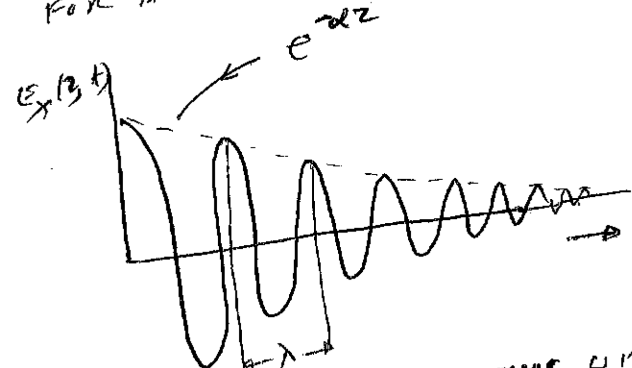
$$E_{xs}(z) = B e^{-jkz} = B e^{-(\alpha + j\beta) z}$$

$$E_x = Re\{ E_{xs} e^{j\omega t} \}$$

$$= Re\{ B e^{-(\alpha + j\beta) z + j\omega t} \}$$

$$= B e^{-\alpha z} \cos(\omega t - \beta z) \qquad (6)$$

IF $\alpha > 0$, IT IS CALLED ATTENUATION COEFFICIENT. TYPICALLY, α is +VE FOR A DIELECTRIC MEDIUM.

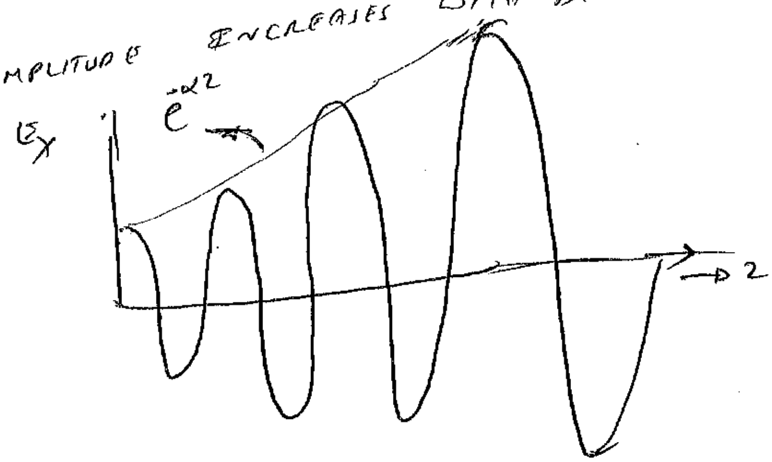


$\alpha > 0$

$\beta = \frac{2\pi}{\lambda}$

AS CAN BE SEEN, THE AMPLITUDE DECAYS WITH DISTANCE.

IF THE MEDIUM IS SPECIALLY PREPARED, α CAN BE NEGATIVE (FOR EXAMPLE, LASER AMPLIFIER). IN THIS CASE, THE AMPLITUDE INCREASES WITH DISTANCE.



$\alpha < 0$

USING (3) IN (2), WE FIND

$$k^2 = \omega^2 \mu (\epsilon' - j\epsilon'')$$

$$= \omega^2 \mu \epsilon' \left(1 - j\frac{\epsilon''}{\epsilon'}\right)$$

$$k = \omega\sqrt{\mu\epsilon'}\sqrt{1 - j\epsilon''/\epsilon'} \quad \rightarrow (7)$$

FROM (4),

$$jk = \alpha + j\beta$$

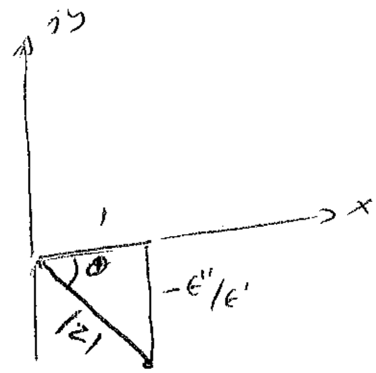
$$\beta = \operatorname{Im}\{jk\} = \omega\sqrt{\frac{\mu\epsilon'}{2}}\left(\sqrt{1+\left(\frac{\epsilon''}{\epsilon'}\right)^2} + 1\right)^{1/2} \quad \rightarrow (8)$$

$$\alpha = \operatorname{Re}\{jk\} = \omega\sqrt{\frac{\mu\epsilon'}{2}}\left(\sqrt{1+\left(\frac{\epsilon''}{\epsilon'}\right)^2} - 1\right)^{1/2} \quad \rightarrow (9)$$

PROOF: LET

$$z = 1 - j\epsilon''/\epsilon'$$

$$|z| = \sqrt{1+\left(\frac{\epsilon''}{\epsilon'}\right)^2}.$$



POLAR FORM $z = |z| e^{j\theta}$;

$$\therefore \sqrt{1 - j\epsilon''/\epsilon'} = \sqrt{z} = \sqrt{|z|}\, e^{j\theta/2}$$

$$K = \omega\sqrt{\mu\epsilon'}\sqrt{Z}$$

$$(\because e^{j\pi/2} = j)$$

$$\beta = \mathrm{Im}\{jK\} = \mathrm{Im}\{e^{j\pi/2}\,\omega\sqrt{\mu\epsilon'}\cdot\sqrt{Z}\}$$

$$= \cancel{\mathrm{Im}\{}$$

$$= \omega\sqrt{\mu\epsilon'}\;\mathrm{Im}\{\sqrt{|Z|}\,e^{j(\theta/2+\pi/2)}$$

$$= \omega\sqrt{\mu\epsilon'}\cdot(\sqrt{|Z|})\cdot\sin(\theta/2+\pi/2)$$

$$= \omega\sqrt{\mu\epsilon'}\,\sqrt{|Z|}\cdot\cos(\theta/2)$$

FROM THE FIGURE,

$$\cos\theta = \frac{1}{|Z|}$$

SINCE $\cos^2\theta/2 = \dfrac{1+\cos\theta}{2}$

$$\cos\theta/2 = \sqrt{\frac{1+\cos\theta}{2}} = \sqrt{\frac{1+1/|Z|}{2}}$$

$$= \frac{1}{\sqrt{2}}\sqrt{\frac{1+|Z|}{|Z|}}$$

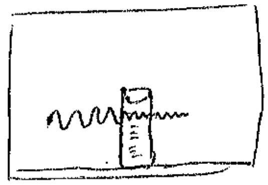
$$\therefore\quad \beta = \mathrm{Im}\{jK\} = \frac{\omega\sqrt{\mu\epsilon'}}{\sqrt{2}}\sqrt{1+|Z|}$$

$$= \omega\sqrt{\frac{\mu\epsilon'}{2}}\sqrt{1+\left\{1+\left(\frac{\epsilon''}{\epsilon'}\right)^2\right\}^{1/2}}$$

IIIly PROVE (9) → (HOMEWORK)

EXAMPLES FOR EM WAVE ATTENUATION:

EX. 1: SUPPOSE YOU ARE HEATING UP WATER IN A MICROWAVE. ASSUME

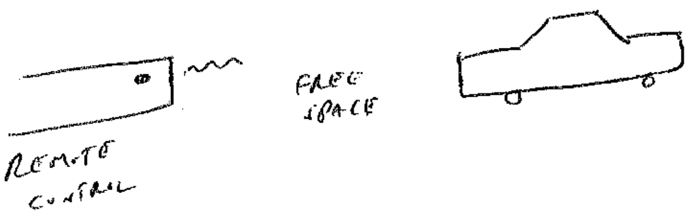


THE MICROWAVE IS IMPINGING ON THE WATER CUP FROM THE LEFT.

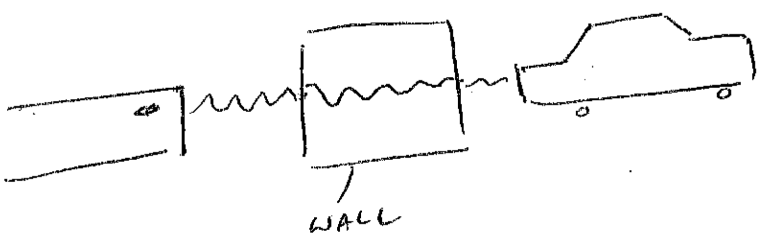
THE MICROWAVE IS STRONGLY ATTENUATED INSIDE THE CUP & ITS ENERGY IS TRANSFERRED TO WATER MOLECULES. LOSS OF ~~ENERGY~~ MICROWAVE ENERGY APPEARS AS HEAT.

EX. 2: WHEN A RADIO REMOTE CONTROLLER IS USED TO ALARM THE CAR, RADIO WAVES ARE TRANSMITTED BY THE REMOTE CONTROLLER, WHICH IS RECEIVED BY THE RECEIVER ANTENNA IN THE CAR. WHEN THERE IS

CASE (i)



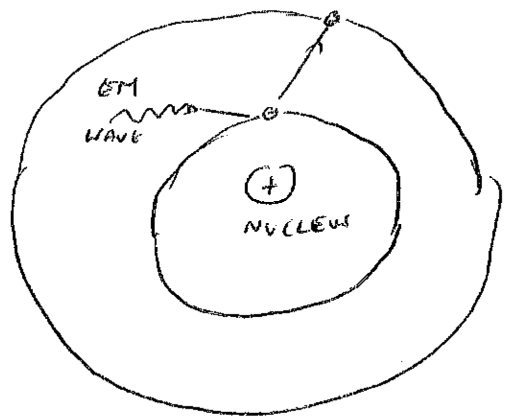
CASE (ii)



A WALL BETWEEN THE CAR & THE REMOTE CONTROL (CASE ii), ~~THE ATTENUAL~~ THE AMPLITUDE OF THE EM WAVE AT THE CAR IS WEAKER THAN THAT IN THE CASE OF FREE SPACE (CASE (i)). THIS IS BECAUSE

THE PERMITTIVITY OF THE WALL IS COMPLEX (IN CONTRAST, THE PERMITTIVITY OF THE FREESPACE IS REAL) & THE IMAGINARY PART OF ϵ IS RELATED TO THE ATTENUATION COEFFICIENT α. (SEE EQ.(9)). ALSO NOTE FROM EQ. (9) THAT, IF $\epsilon'' = 0$, $\alpha = 0$.

~~THE PHYSICAL ORIGIN O~~ ONE OF THE REASONS FOR THE EM WAVE ATTENUATION IS ABSORPTION, WHICH CAN BE UNDERSTOOD AS FOLLOWS.



IN A CLASSICAL MODEL OF ATOMS, ELECTRONS ORBIT AROUND THE NUCLEUS. IF AN ELECTRON ABSORBS EM WAVE, IT GOES TO A HIGHER ORDER STATE (OF HIGHER ENERGY). THUS, THE ENERGY OF THE EM WAVE IS TRANSFERRED TO THE ATOMIC SYSTEM & THE EM WAVE IS ATTENUATED.

AS A REVERSE PROCESS, THE ELECTRON IN THE EXCITED STATE COULD EMIT EM WAVES & THEREBY AMPLIFIES THE EXISTING EM WAVE. THIS IS CALLED STIMULATED EMISSION & OCCURS IN LASERS. HOWEVER, THIS DOES NOT HAPPEN

NATURALLY & YOU NEED TO PREPARE THE SYSTEM TO AMPLIFY THE INCIDENT EM WAVE. IN THIS CASE, THE ENERGY OF THE ATOMIC SYSTEM IS TRANSFERRED TO THE EM WAVE.

THE ELECTRIC FIELD & MAGNETIC FIELD INTENSITIES ARE RELATED BY

$$H_{ys} = \frac{E_{xs}}{\eta}$$

$$\eta = \text{INTRINSIC IMPEDENCE} = \sqrt{\frac{\mu}{\epsilon}}$$

IN FREESPACE/PERFECT DIELECTRIC, ϵ IS REAL & η IS REAL. SO, ELECTRIC & MAGNETIC ~~FIELD~~ FIELDS ARE IN PHASE. FOR THE CASE OF LOSSY DIELECTRIC, ϵ IS COMPLEX, & HENCE η IS COMPLEX.

$$\eta = \sqrt{\frac{\mu}{\epsilon' - j\epsilon''}} = |\eta| \angle \delta$$

NOW, ELECTRIC & MAGNETIC FIELDS ARE NO LONGER IN PHASE.

$$E_{xs} = B \cdot e^{-(\alpha + j\beta)z}$$

$$H_{ys} = \frac{B}{\eta} e^{-(\alpha + j\beta)z}$$

$$H_{ys} = \frac{B \cdot e^{-\alpha z}}{|\eta|} \cdot e^{j(\beta z + \delta)}$$