WAVE REFLECTION FROM MULTIPLE INTERFACES

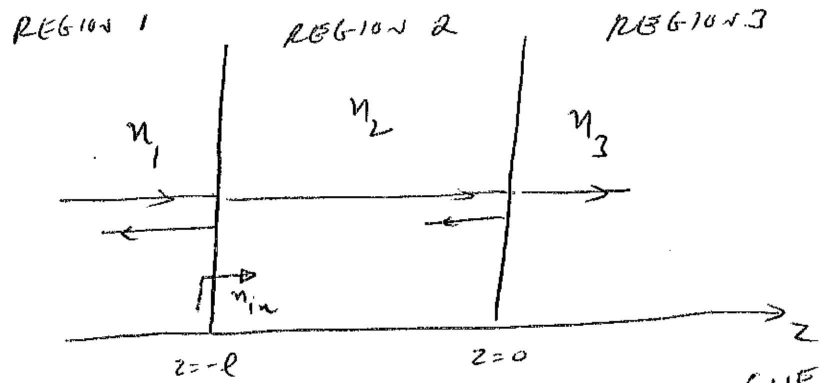


FIG. 1.

(WE ASSUME THAT THE REGIONS ARE LOSSLESS) $jk = j\beta$

THE ELECTRIC FIELD IN REGION 2 IS

$$E_{xs2} = A_2^+ e^{-j\beta_2 z} + A_2^- e^{j\beta_2 z} \rightarrow (1)$$

$$H_{ys2} = \frac{A_2^+}{\eta_2} e^{-j\beta_2 z} - \frac{A_2^-}{\eta_2} \cdot e^{j\beta_2 z} \rightarrow (2)$$

LET THE REFLECTION COEFFICIENT AT THE SECOND INTERFACE BE Γ_{23}.

$$\Gamma_{23} = \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2} = \frac{A_2^-}{A_2^+} \rightarrow (3)$$

THE <u>WAVE IMPEDANCE</u> IS DEFINED AS

$$\eta_w(z) = \frac{E_{xs2}}{H_{ys2}} = \frac{A_2^+ e^{-j\beta_2 z} + A_2^- e^{j\beta_2 z}}{\left(A_2^+ e^{-j\beta_2 z} - A_2^- \cdot e^{j\beta_2 z}\right)/\eta_2} \rightarrow (4)$$

NOTE THAT THIS DEFINITION OF WAVE IMPEDANCE IS THE SAME AS WAVE IMPEDANCE IN TRANSMISSION LINES WITH $E_{xs} \rightarrow V_s$ & $H_{ys} \rightarrow I_s$. FOLLOWING THE SAME PROCEDURE AS IN TRANSMISSION LINES, EQ. (4) MAY BE SIMPLIFIED AS

②

$$\eta_w(z) = \eta_2 \left[\frac{\eta_3 \cos \beta_2 z - j\eta_2 \sin \beta_2 z}{\eta_2 \cos \beta_2 z - j\eta_3 \sin \beta_2 z} \right] \rightarrow (5)$$

(NOTE $\eta_2 \rightarrow Z_0$ & $\eta_3 \rightarrow Z_L$)

THIS WAVE IMPEDANCE AT $z = -l$ IS KNOWN AS INPUT IMPEDANCE, η_{in}.

$$\boxed{\eta_{in} = \eta_w(-l) = \eta_2 \left[\frac{\eta_3 \cos(\beta_2 l) + j\eta_2 \sin \beta_2 l}{\eta_2 \cos(\beta_2 l) + j\eta_3 \sin \beta_2 l} \right]} \rightarrow (6)$$

η_{in} MAY BE INTERPRETED AS THE EFFECTIVE IMPEDANCE "SEEN" AT THE FIRST INTERFACE ($z = -l$) DUE TO REGION 2 & REGION 3.

THE ~~ELECTRIC~~ FIELDS IN REGION 1 ARE $\rightarrow (7)$

$$E_{xs1} = A_1^+ e^{-j\beta_1 z} + A_1^- e^{j\beta_1 z}$$

$$H_{ys1} = \frac{A_1^+}{\eta_1} \cdot e^{-j\beta_1 z} - \frac{A_1^-}{\eta_1} \cdot e^{j\beta_1 z} \rightarrow (8)$$

SINCE E_x & H_y SHOULD BE CONTINUOUS AT THE FIRST INTERFACE, ($z = -l$)

$$A_1^+ e^{j\beta_1 l} + A_1^- e^{-j\beta_1 l} = E_{xs2}(z = -l) \rightarrow (9)$$

$$\frac{A_1^+ e^{j\beta_1 l} - A_1^- e^{-j\beta_1 l}}{\eta_1} = \frac{E_{xs2}(z = -l)}{\eta_{in}} \rightarrow (10)$$

$$\left(\because \; \eta_w(-l) = \eta_{in} = E_{xs2} / H_{ys2} \right)$$

THE REFLECTION COEFFICIENT AT THE FIRST INTERFACE,

$$\Gamma_{12} = \frac{\text{AMPLITUDE OF THE REFLECTED WAVE AT } z=-l}{\text{AMPLITUDE OF THE INCIDENT WAVE AT } z=-l}$$

$$= \frac{A_1^- e^{-j\beta_1 l}}{A_1^+ e^{j\beta_1 l}}$$

$$A_1^- e^{-j\beta_1 l} = \Gamma_{12} A_1^+ e^{j\beta_1 l} \longrightarrow (11)$$

$\therefore$ (9) & (10) CAN BE WRITTEN AS

$$A_1^+ e^{j\beta_1 l}(1 + \Gamma_{12}) = E_{xs2}(-l) \longrightarrow (12)$$

$$A_1^+ e^{j\beta_1 l}(1 - \Gamma_{12}) = E_{xs}(-l)\frac{\eta_1}{\eta_{in}} \longrightarrow (13)$$

(12) $\div$ (13) $\Rightarrow$

$$\frac{1 + \Gamma_{12}}{1 - \Gamma_{12}} = \frac{\eta_{in}}{\eta_1}$$

$$\eta_1 + \Gamma_{12}\eta_1 = \eta_{in} - \eta_{in}\Gamma_{12}$$

$$\Gamma_{12}(\eta_1 + \eta_{in}) = \eta_{in} - \eta_1$$

$$\boxed{\Gamma_{12} = \frac{\eta_{in} - \eta_1}{\eta_{in} + \eta_1}} \longrightarrow (14)$$

SINCE η_{in} IS THE IMPEDANCE "SEEN" AT $z=-l$ DUE TO REGION 2 & REGION 3, IT CAN BE REGARDED AS THE EFFECTIVE IMPEDANCE OF REGION 2 & REGION 3, AS SHOWN IN FIG. 2. THUS,

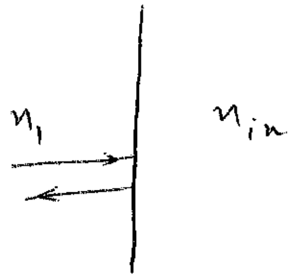
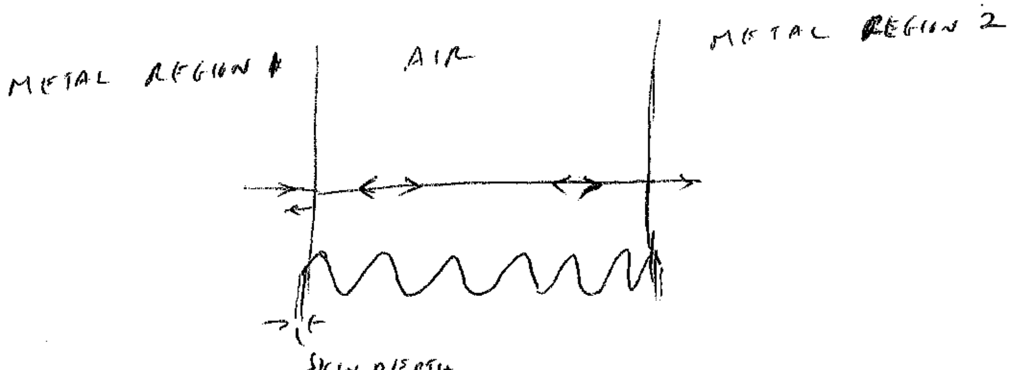


FIG. 2.

THE PROBLEM WITH TWO INTERFACES (FIG. 1) IS REDUCED TO A PROBLEM WITH SINGLE INTERFACE (FIG. 2). THIS METHOD CAN BE APPLIED RECURSIVELY IF THERE ARE MORE THAN TWO INTERFACES.

TYPICALLY, A METALLIC WAVEGUIDE CONSISTS OF TWO METAL REGIONS SEPARATED BY A DIELECTRIC / AIR. THE INCIDENT EM WAVE UNDERGOES

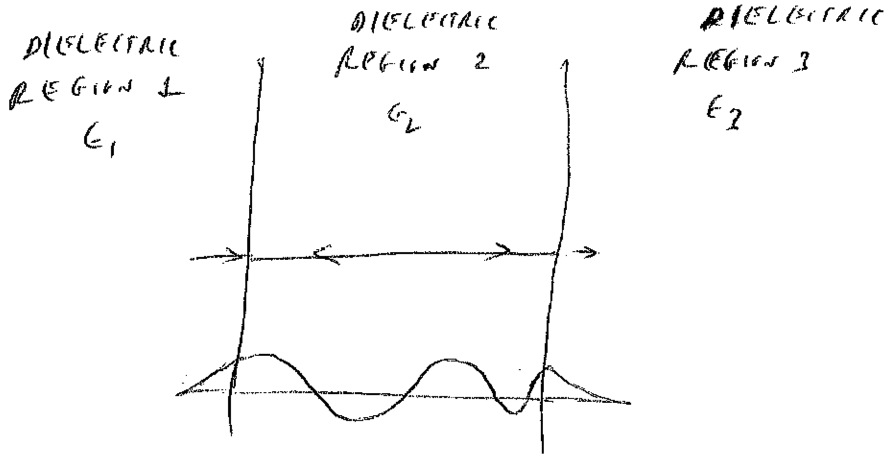


REFLECTIONS AT BOTH THE INTERFACES & FORMS STANDING WAVES. IF THE METAL IS A GOOD CONDUCTOR, ($\sigma/\omega\epsilon >> 1$), SKIN DEPTH IS CLOSE TO ZERO, & NEARLY FULL REFLECTION ($\Gamma \cong -1$) OCCURS AT THE INTERFACES. THE STANDING WAVES ARE

CALLED <u>MODES</u> & PLAY AN IMPORTANT ROLE IN THE TRANSMISSION OF MICROWAVES.

ANOTHER EXAMPLE IS OPTICAL FIBER/OPTICAL WAVEGUIDES.

A DIELECTRIC OPTICAL WAVEGUIDE, CONSISTS OF A CENTRAL REGION WITH THE PERMITTIVITY HIGHER THAN THE SURROUNDING REGIONS.



AS IN THE CASE OF METAL WAVEGUIDES, THE EM WAVE IS REFLECTED AT BOTH THE INTERFACES. HOWEVER, THE REFLECTIONS ARE PARTIAL & AS A RESULT, THERE ~~IS A FIELD IN A~~ EXIST NON-ZERO FIELDS IN REGION 1 & REGION 3.

<u>QUARTER WAVE MATCHING:</u>

WHEN

$$\beta_2 l = (2m+1)\pi/2 \quad , \quad m = 0, 1, 2, \ldots \quad \rightarrow (7)$$

$$\beta_2 = \frac{2\pi}{\lambda_2} \; ; \; \lambda_2 = \text{WAVELENGTH IN REGION 2.}$$

$$\frac{2\pi}{\lambda_2} l = (2m+1)\pi/2 \Rightarrow l = (2m+1)\frac{\lambda_2}{4} \quad \rightarrow (8)$$

WHEN $m = 0$, $l = \lambda_2/4$, i.e. THE MIDDLE LAYER IS QUARTER-WAVELENGTH LONG, ~~PATH~~

$$\cos(\beta_2 l) = 0$$

$$\sin(\beta_2 l) = 1$$

FROM EQ. (6), WE FIND

$$\eta_{in} = \eta_2 \left(\frac{j\eta_2}{j\eta_3} \right) = \frac{\eta_2^2}{\eta_3} \longrightarrow (9)$$

FROM FIG. 2, WE FIND THAT THERE WOULD BE NO REFLECTION IF

$$\boxed{\eta_1 = \frac{\eta_2^2}{\eta_3}} \quad \text{or} \quad \boxed{\eta_2 = \sqrt{\eta_1 \eta_3}} \longrightarrow (10)$$

~~CHOOSING THE LENGTH OF~~ IF THE LENGTH & IMPEDANCE OF THE REGION 2 IS CHOSEN SO AS TO SATISFY EQS. (8) & (10), IT IS KNOWN AS QUARTER WAVE MATCHING.

~~THE TECHNIQUE OF~~ THIS TECHNIQUE IS USED IN THE DESIGN OF ANTI-REFLECTION COATING. FOR EXAMPLE, THE ~~LIGHT~~ OUTPUT OF AN OPTICAL FIBER IS LAUNCHED TO A PHOTO-DIODE, AS SHOWN IN FIG. 3.

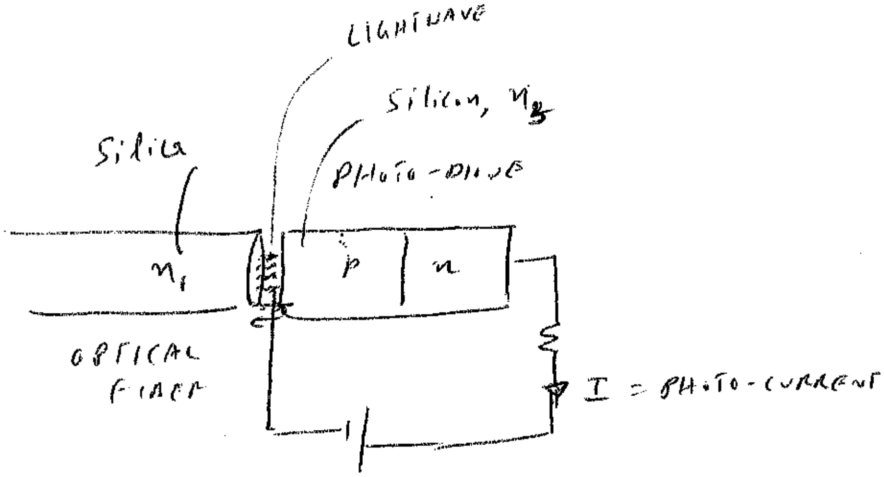
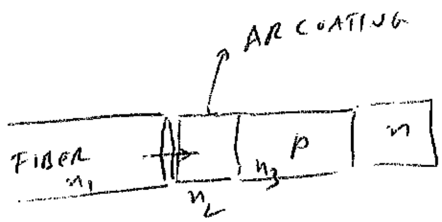


FIG. 3.

THE OPTICAL SIGNAL INCIDENT ON THE PHOTO-DIODE LEADS TO PHOTO-CURRENT IN THE PHOTO-DIODE CIRCUIT. HOWEVER, THE ~~CHARACTERI~~ IMPEDANCES OF OPTICAL FIBER & PHOTO-DIODE ARE DIFFERENT LEADING TO REFLECTION AT THE INTERFACE. TO INCREASE THE EFFICIENCY OF OPTICAL POWER COUPLING TO THE PHOTO-DIODE, ANTI-REFLECTION (AR) COATING IS USED.



LET THE IMPEDANCE OF FIBER & P-SIDE OF THE PHOTO-DIODE BE η_1 & η_3, RESPECTIVELY. THE IMPEDANCE, η_2 OF THE AR COATING IS SO CHOSEN THAT

$$\eta_2 = \sqrt{\eta_1 \eta_3}$$

THE LENGTH OF THE AR COATING IS

$$l = \frac{\lambda_2}{4} \text{ or } (2m+1)\frac{\lambda_2}{4}$$

TYPICALLY THE THICKNESS OF THE AR COATING IS $< 1\mu m$.