WAVE REFLECTION

~~GENERAL SOLUTION~~

For a forward propagating wave,

$$\left.\begin{aligned} V_s(z) &= V_0^+ e^{-\gamma z} \\ I_s(z) &= \frac{V_0^+}{Z_0} e^{-\gamma z} \end{aligned}\right\} \rightarrow (1)$$

$\gamma = \alpha + j\beta$; Z_0 = CHARACTERISTIC IMPEDANCE

FOR A BACKWARD PROP. WAVE

$$\left.\begin{aligned} V_s(z) &= V_0^- e^{\gamma z} \\ I_s(z) &= -\frac{V_0^-}{Z_0} e^{\gamma z} \end{aligned}\right\} \rightarrow (2)$$

THE GENERAL SOLUTION IS (ADD (1) & (2))

$$\left.\begin{aligned} V_s(z) &= V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \\ I_s(z) &= \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z} \end{aligned}\right\} \rightarrow (3)$$

HERE V_0^+ & V_0^- ARE UNKNOWNS TO BE DETERMINED FROM THE BOUNDARY CONDITIONS. AS AN EXAMPLE, CONSIDER THE WAVE REFLECTION AT THE END OF TRANSMISSION LINES.

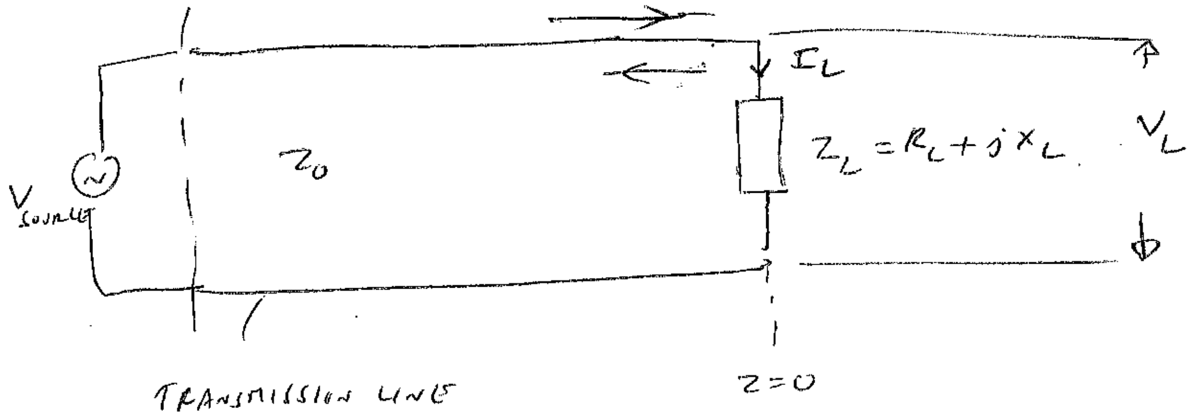

FIG. 1

IF THE TRANSMISSION LINE IS INFINITELY LONG, IT CAN BE SHOWN THAT THERE IS ONLY FORWARD PROPAGATING WAVE IF THE VOLTAGE SOURCE IS ON THE LEFT. HOWEVER, IF THE TRANSMISSION LINE OF FINITE LENGTH IS TERMINATED WITH A LOAD IMPEDANCE, THE REFLECTED WAVE (BACKWARD PROPAGATING WAVE) COULD EMERGE. ~~DEPENDING ON THE DIF~~ THE MAGNITUDE OF THE REFLECTED WAVE DEPENDS ON THE **DIFFERENCE** BETWEEN THE CHARACTERISTIC IMPEDANCE, Z_0 OF THE TRANSMISSION LINE & THE LOAD IMPEDANCE, Z_L. THE REFLECTIONS COULD ALSO OCCUR AT LOCATIONS AT WHICH TWO DISSIMILAR TR. LINES ARE CONNECTED TO EACH OTHER, AS SHOWN IN FIG. 2.

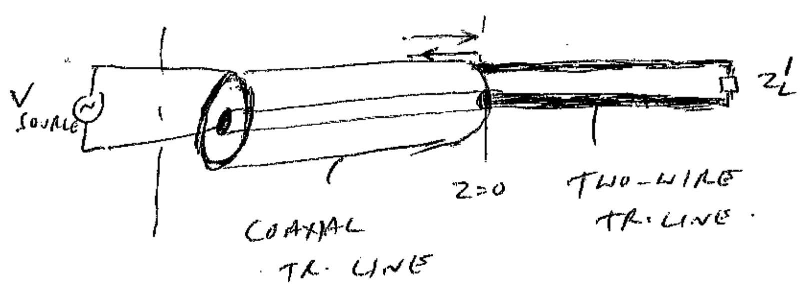

FIG. 2.

IN FIG. 2, REFLECTION OCCURS AT THE ~~INTERFACE OF~~ THE POINTS WHERE COAXIAL & TWO-WIRE LINES ARE CONNECTED TO EACH OTHER. IN THIS CASE, THE EFFECTIVE LOAD TO COAXIAL TR. LINE IS

$$Z_L = \text{IMPEDANCE OF TWO-WIRE LINE} + \text{ACTUAL LOAD, } Z_L'.$$

FOR CONVENIENCE, WE CHOOSE THE LOCATION OF LOAD (FIG. 1) OR THE LOCATION AT WHICH TWO (FIG. 2) DISSIMILAR TR. LINES ARE CONNECTED AS $z=0$.

FROM (3), @ $z=0$,

$$V_s(0) = V_L = V_0^+ + V_0^- \quad \rightarrow (4)$$

$$I_s(0) = I_L = (V_0^+ - V_0^-)/Z_0 \quad \rightarrow (5)$$

NOTE ~~THAT~~ V_0^+ = AMPLITUDE OF THE FORWARD WAVE

V_0^- = " " " BACKWARD OR REFLECTED WAVE.

THE RATIO OF THE ~~RE~~ AMPLITUDE OF THE REFLECTED WAVE TO THE FORWARD (INCIDENT) WAVE AMPLITUDE IS CALLED THE REFLECTION COEFFICIENT, Γ

$$\Gamma = \frac{V_0^-}{V_0^+} \quad \rightarrow (6)$$

FROM (4) & (5),

$$Z_L = \frac{V_L}{I_L} = \frac{V_0^+ + V_0^-}{(V_0^+ - V_0^-)/Z_0} = \frac{\cancel{V_0^+}(1 + V_0^-/V_0^+)\, Z_0}{\cancel{V_0^+}(1 - V_0^-/V_0^+)}$$

$$\frac{Z_L}{Z_0} = \frac{1+\Gamma}{1-\Gamma}$$

$$Z_L(1-\Gamma) = Z_0(1+\Gamma)$$

$$Z_L - Z_0 = \Gamma(Z_L + Z_0)$$

or

$$\boxed{\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}} \longrightarrow 7$$

IN GENERAL, Γ IS COMPLEX

$$\Gamma = |\Gamma| e^{j\varphi_r}$$

WHEN, $Z_L = Z_0$, $\Gamma = 0 \Rightarrow V_0^- = 0$ (FROM 6)

$\therefore$ NO REFLECTED WAVE WHEN $Z_L = Z_0$. THIS IS KNOWN AS <u>IMPEDANCE MATCHING</u>. NOW, THE LOAD IS MATCHED TO THE LINE.

FROM 4,

$$V_L = V_0^+ + V_0^-$$

$$= V_0^+\left(1 + \frac{V_0^-}{V_0^+}\right) = V_0^+(1+\Gamma)$$

$$\boxed{\tau \equiv \frac{V_L}{V_0^+} = 1 + \Gamma} \quad \text{—— (8)}$$

τ IS KNOWN AS TRANSMISSION COEFFICIENT.

FROM 7,

$$\tau = 1 + \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L + \cancel{Z_0} + Z_L - \cancel{Z_0}}{Z_L + Z_0}$$

$$\boxed{\tau = \frac{2Z_L}{Z_L + Z_0} = |\tau| e^{j\varphi_L}}$$