TRANSIENT ANALYSIS

SO FAR WE CONSIDERED THE OPERATION OF TRANSMISSION LINES UNDER STEADY-STATE CONDITIONS WITH AC INPUT. IN THIS SECTION, WE CONSIDER THE TRANSIENTS IN A TRANSMISSION LINE EXCITED WITH DC INPUT. LET US FIRST ANALYZE THE CASE IN WHICH THE LINE IS TERMINATED BY A MATCHED LOAD, $R_L = Z_0$. NOTE THAT FOR A LOSSLESS LINE, $R=0$ & $G=0$ & HENCE,

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{L}{C}} \text{ IS REAL.}$$

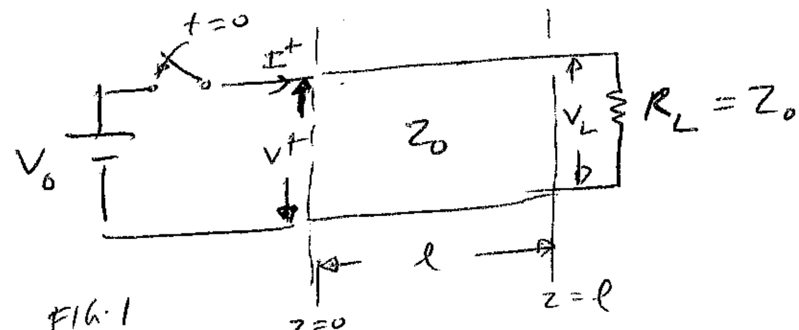


FIG. 1

AT $t=0$, THE SWITCH IS CLOSED & THE LINE VOLTAGE AT THE INPUT (@z=0), V^+ BECOMES EQUAL TO THE BATTERY VOLTAGE, V_0. AT $t=0$, THE LOAD VOLTAGE V_L IS ZERO BECAUSE IT TAKES A CERTAIN TIME FOR THE VOLTAGE WAVE TO APPEAR AT THE LOAD.

(a) $z = 0$

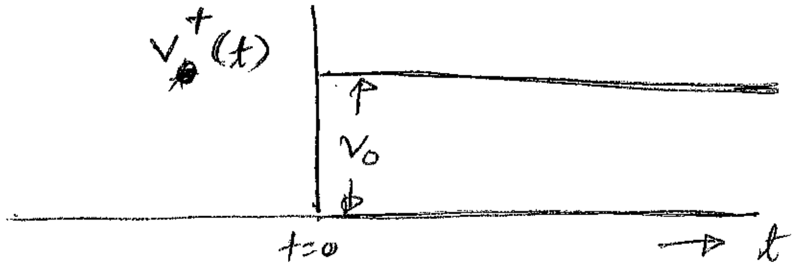


FIG. 2

AS CAN BE SEEN THE LINE VOLTAGE AT THE INPUT IS A STEP FUNCTION. THE VOLTAGE WAVE PROPAGATES AT SPEED v. THE VOLTAGE AT A z_1 IN THE LINE IS SHOWN BELOW.

(a) $z = z_1$

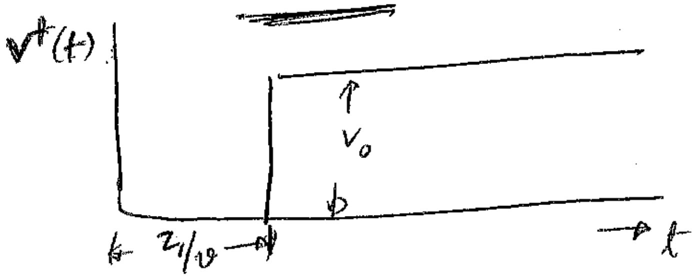


FIG. 3

THE TIME TAKEN BY THE VOLTAGE WAVE TO TRAVERSE THE LINE IS l/v & HENCE THE LOAD VOLTAGE ~~AS A FUNCTION~~ OF ~~TIME~~ IS DELAYED BY l/v AS SHOWN BELOW

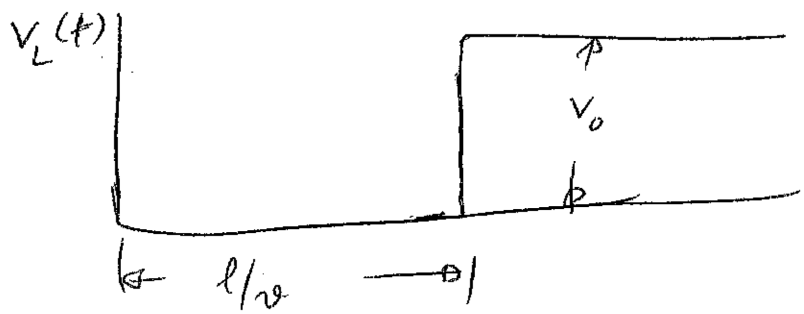


FIG. 4

THE CURRENT IN THE LINE IS

$$I^+ = \frac{V^+}{Z_0}$$

THE TIME-DEPENDENCE OF I^+ IS THE SAME AS THAT OF V^+ SHOWN IN FIGS. 2 & 3 (WITH V_0 REPLACED BY V_0/Z_0 IN FIGS. 2 & 3). THE LOAD CURRENT AT $t = l/v$ IS $I_L = V_0/Z_0 = V_0/R_L$. TO SUMMARIZE,

$$V_L = V_0 \quad \text{FOR} \quad t > l/v \qquad \rightarrow (1)$$
$$= 0 \quad \text{OTHERWISE}$$

$$I_L = V_0/R_L \quad \text{FOR} \quad t > l/v \qquad \rightarrow (2)$$
$$= 0 \quad \text{OTHERWISE}$$

NEXT, LET US CONSIDER A MORE GENERAL CASE IN WHICH THE LOAD IS NOT MATCHED TO THE LINE ($R_L \neq Z_0$). LET US ALSO INTRODUCE A SOURCE RESISTANCE R_g.

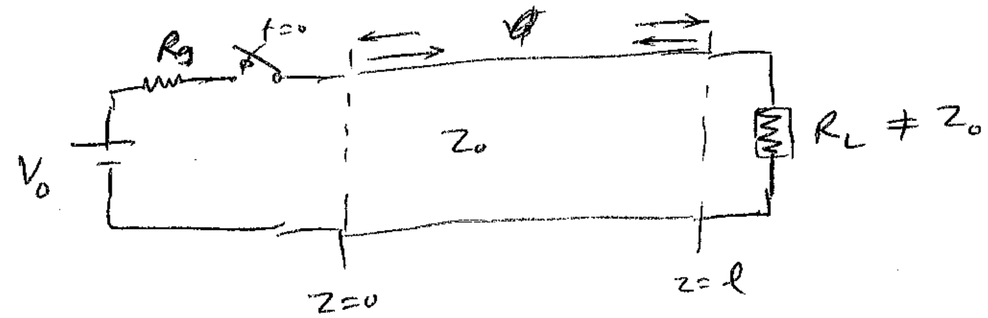


FIG. 5

AT $t=0$, THE SWITCH IS CLOSED & A VOLTAGE WAVE V_1^+ PROPAGATES TO THE RIGHT. AFTER REACHING THE LOAD, THE VOLTAGE WAVE WILL REFLECT, PRODUCING A BACK-PROPAGATING WAVE V_1^-.

$$\frac{V_1^-}{V_1^+} = \Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} \quad \rightarrow (3)$$

HERE, Γ_L IS THE REFLECTION COEFFICIENT AT THE LOAD.
(NOTE: THE SUBSCRIPT L IS ADDED BECAUSE WE HAVE REFLECTIONS at THE SOURCE, TOO).

THE VOLTAGE IN THE LINE AFTER THE (FIRST) REFLECTION IS $V_1^+ + V_1^-$. ~~AT THE SOURCE~~ THE REFLECTED VOLTAGE V_1^- HEADS TOWARDS THE SOURCE. AT THE SOURCE, LET US SUPPOSE $R_g \neq Z_0$. BECAUSE OF THE IMPEDANCE MISMATCH, THE V_1^- WAVE IS REFLECTED AT THE SOURCE TO PRODUCE A NEW FORWARD WAVE V_2^+.

$$\frac{\overset{\text{REFLECTED}}{V_2^+}}{\underset{\text{INCIDENT}}{V_1^-}} = \Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} \quad \rightarrow (4)$$

($\because$ AS "SEEN" BY V_1^-, THE LOAD IS R_g)

THE NEW FORWARD WAVE V_2^+ PROPAGATES TOWARDS THE LOAD. AT THIS TIME, THE VOLTAGE IN THE LINE IS

$$V_1^+ + V_1^- + V_2^+ = V_1^+ + \Gamma_L V_1^+ + \Gamma_g V_1^-$$

$$= V_1^+ + \Gamma_L V_1^+ + \Gamma_L \Gamma_g \cdot V_1^+ \quad \rightarrow (5)$$

V_2^+ IS REFLECTED AT THE LOAD TO PRODUCE A NEW BACKWARD WAVE V_2^-.

$$\frac{V_2^-}{V_2^+} = \Gamma_2$$

AFTER MANY ROUNDTRIPS (i.e. STEAD STATE), THE LINE/LOAD VOLTAGE IS

$$V_L = V_1^+ + V_1^- + V_2^+ + V_2^- + V_3^+ + V_3^- + \cdots$$

$$= V_1^+ \left(1 + \Gamma_L + \Gamma_L \Gamma_g + \Gamma_L^2 \Gamma_g + \Gamma_L^2 \Gamma_g^2 + \cdots \right)$$

$$= V_1^+ \left(1 + \Gamma_g \Gamma_L + \Gamma_g^2 \Gamma_L^2 + \cdots \right) + \Gamma_L \left(1 + \Gamma_L \Gamma_g + \Gamma_g^2 \Gamma_L^2 + \cdots \right)$$

$$= V_1^+ (1 + \Gamma_L)(1 + \Gamma_g \Gamma_L + \Gamma_g^2 \Gamma_L^2 + \cdots) \quad \rightarrow (6)$$

$$= \frac{V_1^+ (1 + \Gamma_L)}{(1 - \Gamma_g \Gamma_L)} \quad \rightarrow (7)$$

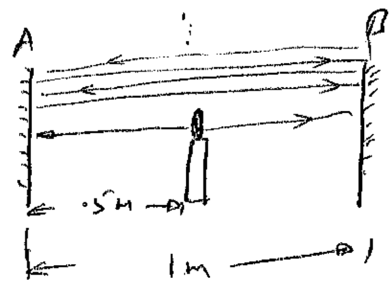
($\because$ FOR A GEOMETRIC SERIES, $1 + a + a^2 + \cdots$

$= \frac{1}{1-a}$ if $|a| < 1$ }

$|\Gamma_g| < 1$ & $|\Gamma_L| < 1$

~~IT CAN BE SHOWN THAT $\Gamma_g \Gamma_L < 1$~~)

CONSIDER THE FOLLOWING ANALOGY: AN OBJECT IS PLACED BETWEEN THE MIRRORS THAT ARE 1m APART.



AS THE LIGHT FROM THE OBJECT FALLS ON THE LEFT MIRROR, ONE COULD SEE A VIRTUAL IMAGE OF THE OBJECT (APPEARS TO BE AT DISTANCE .5m BEHIND THE MIRROR A), WHICH I CALL THE FIRST VIRTUAL IMAGE. THE LIGHT REFLECTED FROM THE MIRROR A IS ALSO REFLECTED FROM MIRROR B & PROPAGATES TOWARDS THE MIRROR A, PRODUCING A SECOND VIRTUAL IMAGE (APPEARS TO BE AT DISTANCE 2.5m BEHIND THE MIRROR A). THE BRIGHTNESS OF THE SECOND IMAGE IS LESS THAN THAT OF THE FIRST IMAGE SINCE THE REFLECTIONS COEFFICIENTS Γ_A & Γ_B OF THE MIRRORS IS ALWAYS LESS THAN UNITY. THIS PROCESS CONTINUES & THERE ARE

(7)

INFINITE NUMBER OF VIRTUAL IMAGES IN BOTH MIRRORS UNDER THE STEADY-STATE CONDITIONS. HOWEVER, WE MAY BE ABLE TO SEE THREE OR FOUR IMAGES DEPENDING ON THE QUALITY OF THE MIRRORS. SIMILARLY, IN THE CASE TRANSMISSION LINE,

$$\ldots\ |V_2^-| < |V_2^+| < |V_1^-| < |V_1^+|$$ & HENCE THE

SUM IN EQ. (6) CONVERGES.

AS AN EXAMPLE, LET $R_g = 0$;

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = -1$$

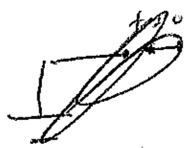
AT $t=0$, IN THIS CASE, VOLTAGE AT THE INPUT OF THE LINE, $V_1^+ = V_0$. FROM (7), THE STEADY-STATE LOAD VOLTAGE IS

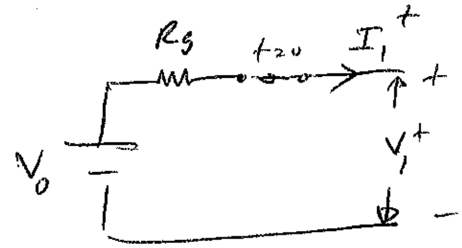
$$V_L = \frac{V_0(1+\Gamma_L)}{1-(-1)\Gamma_L} = V_0$$

SO, UNDER THE STEADY-STATE CONDITIONS, THE LOAD IS CHARGED TO THE BATTERY VOLTAGE. NOTE THAT THIS IS A LOSSLESS TRANSMISSION LINE & THERE IS NO VOLTAGE

LOSS IN THE LINE.

NEXT, LET US CONSIDER A MORE GENERAL CASE $R_g \neq 0$. AT $t=0$.





APPLYING KVL, $V_1^+ = V_0 - I_1^+ R_g$

FOR A FORWARD WAVE: $I_1^+ = V_1^+/Z_0$

$$V_1^+\left(1 + R_g/Z_0\right) = V_0$$

or

$$V_1^+ = \left(\frac{Z_0}{Z_0+R_g}\right) V_0$$

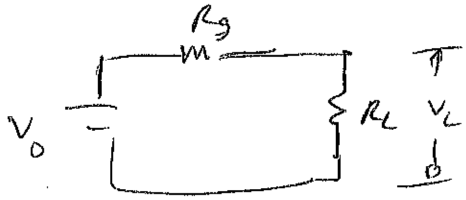
$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} \qquad \Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}$$

FROM 7,

$$V_L = \frac{\left(\frac{Z_0}{Z_0+R_g}\right) V_0 \times \left(1 + \frac{R_L - Z_0}{R_L+Z_0}\right)}{1 - \left(\frac{R_L - Z_0}{R_L+Z_0}\right)\left(\frac{R_g - Z_0}{R_g+Z_0}\right)}$$

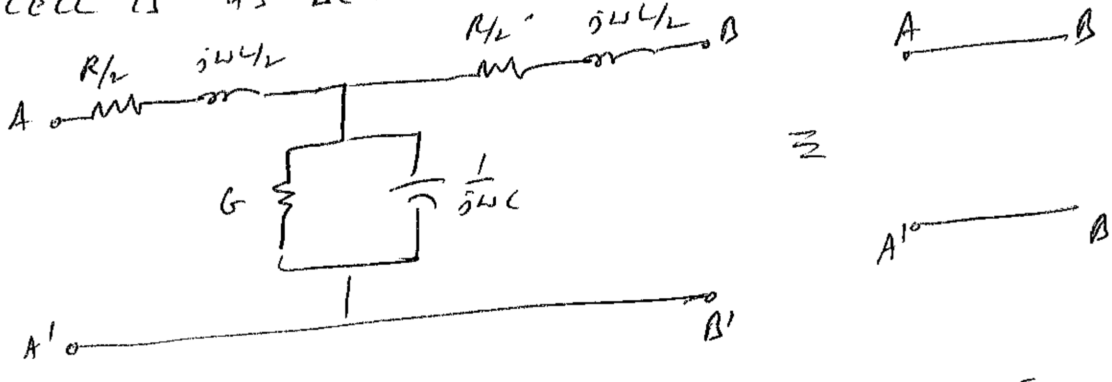
$$V_L = \left(\frac{Z_0}{Z_0+R_g}\right) V_0 \times \frac{\left(\dfrac{2R_L}{R_L+\cancel{Z_0}}\right)}{\dfrac{(R_L+Z_0)(R_s+Z_0) - (R_L-Z_0)(R_g-Z_0)}{(\cancel{R_L+Z_0})(R_s+Z_0)}}$$

$$= \left(\frac{\cancel{Z_0}}{\cancel{Z_0+R_g}}\right) V_0 \times \frac{2R_L \times (R_s+Z_0)}{\cancel{2}Z_0 (R_L+R_s)} = \frac{V_0 R_L}{R_L+R_s}$$



$$V_L = \frac{V_0 R_L}{R_L+R_s}$$

UNDER THE STEADY STATE CONDITIONS, VOLTAGE ACROSS THE LOAD CAN BE FOUND ~~FROM THE~~ ~~VOLTA~~ THROUGH SIMPLE VOLTAGE DIVISION RULES, AS IF THE LINE IS A SHORT CIRCUIT. THIS IS BECAUSE, THE EQUIVALENT CIRCUIT OF THE TRANSMISSION LINE UNIT CELL IS AS BELOW



FOR A LOSSLESS LINE, $R = G = 0$. REACTANCE DUE TO INDUCTANCE IS ZERO ($\because \omega = 2\pi f = 0$ for DC).

THE CONDUCTANCE $G = 0 \Rightarrow$ SHUNT RESISTANCE $= \frac{1}{G} = \infty$.

$\Rightarrow$ OPEN CIRCUIT.

$\omega = 0 \Rightarrow \frac{1}{j\omega C} = \infty \Rightarrow$ OPEN CIRCUIT

(i.e. CAPACITOR BLOCKS THE DC VOLTAGE)

FIG. 6 SHOWS THE VOLTAGE REFLECTION DIAGRAM. TIME IS PLOTTED ON THE VERTICAL AXIS & DISTANCE Z IS PLOTTED ON THE HORIZONTAL AXIS.

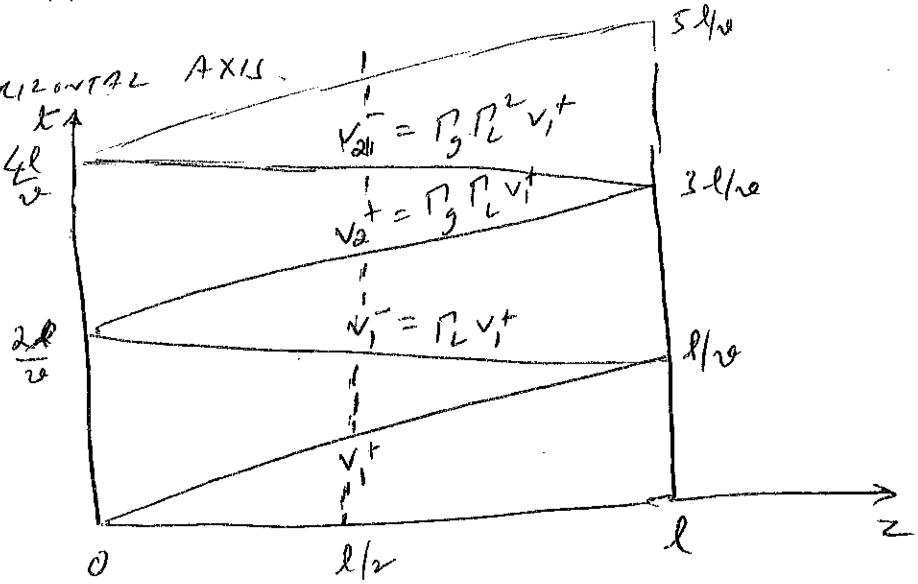


FIG. 6

AT $t=0$, INITIAL VOLTAGE WAVE V_1^+ STARTS AT THE ORIGIN ($t=z=0$). IT REACHES THE LOAD ($z=l$) AT $t = l/v$. THE SLOPE OF THE FIRST DIAGONAL LINE IS $1/v$. V_1^+ IS REFLECTED AT THE LOAD TO PRODUCE ~~A~~ NEW BACKWARD WAVE WITH THE AMPLITUDE $V_1^- = \Gamma_L V_1^+$, WHICH REACHES THE SOURCE AT $t = 2l/v$. V_1^- WAVE IS REFLECTED AT THE SOURCE

TO PRODUCE A NEW FORWARD WAVE WITH THE AMPLITUDE $V_2^+ = \Gamma_S V_1^-$, WHICH REACHES THE LOAD AT $t = 3\ell/v$ & SO ON.

THE VOLTAGE AT ANY POINT ON THE LINE AT A time t_1 CAN BE FOUND BY ADDING THE VOLTAGES AT THAT POINT FROM $t = 0$ to t_1. FOR EXAMPLE, TO DETERMINE THE VOLTAGE AT THE MIDDLE OF THE LINE ($z = \ell/2$) AT $t = 4\ell/v$, ADD THE VOLTAGES AT $z = \ell/2$ (VERTICAL DASHED LINE) from $t = 0$ to $4\ell/v$, i.e.

$$V(\ell/2, 4\ell/v) = V_1^+ + V_1^- + V_2^+ + V_2^-.$$

EXAMPLE 1

SUPPOSE $R_L = R_g = \frac{1}{3} Z_0$.

$$\Gamma_L = \Gamma_g = \frac{R_L - Z_0}{R_L + Z_0} = \frac{(1/3 - 1) Z_0}{(1/3 + 1) Z_0} = -\frac{1}{2}$$

THE STEADY-STATE LOAD VOLTAGE IS

$$V_L = \frac{V_0 \times R_L}{R_L + R_g} = \frac{V_0}{2} .$$

$$V_1^+ = \frac{V_0 \times Z_0}{R_L + Z_0} = \frac{3}{4} V_0 .$$

$$V_1^- = \Gamma_L V_1^+ = -\frac{3}{8} V_0$$

$$V_1^+ + V_1^- = \frac{3}{8} V_0 .$$

$$V_2^+ = \Gamma_g . V_1^- = +\frac{3}{16} V_0$$

$$V_1^+ + V_1^- + V_2^+ = \left(\frac{3}{8} + \frac{3}{16}\right) V_0 = \frac{9}{16} V_0$$

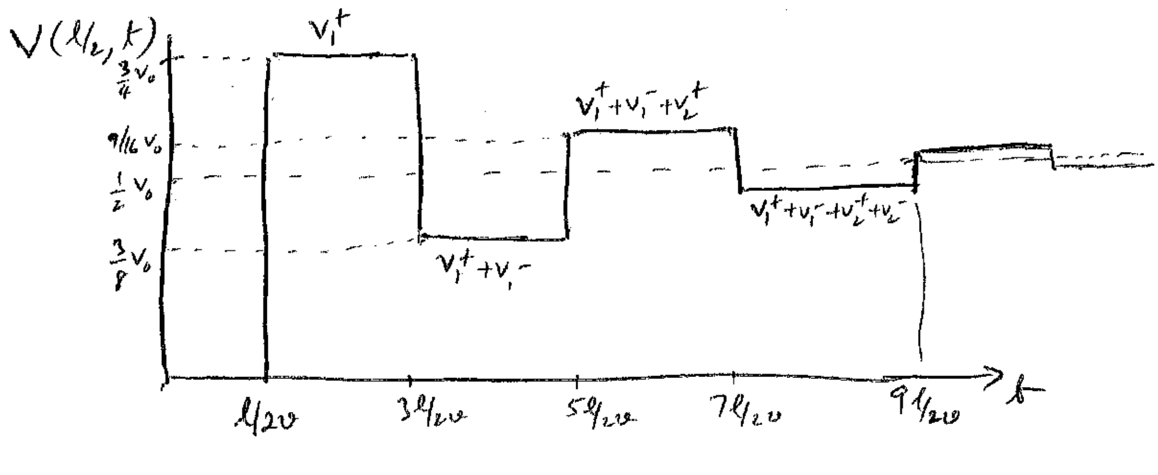


FIG. 7

FIG. 7 SHOWS THE LINE VOLTAGE AT $z = \ell/2$. NOTE THAT THE LINE VOLTAGE AT $z = \ell/2$ IS ZERO for $t < \ell/(2v)$ SINCE IT TAKES $\ell/(2v)$ SECONDS FOR THE VOLTAGE WAVE TO ARRIVE AT $\ell/2$ FROM THE BATTERY. THE REFLECTED WAVE IS GENERATED AT THE LOAD AT $t = \ell/v$ & IT TAKES $\ell/(2v)$ SECONDS TO ARRIVE AT $z = \ell/2$. SO, V_1^- IS ADDED TO V_1^+ AT $t = 3\ell/(2v)$ & SO ON.

LINE CURRENT CAN BE FOUND IN A SIMILAR WAY THROUGH A <u>CURRENT REFLECTION DIAGRAM</u>. THE FORWARD CURRENT IS

$$I^+ = \frac{V^+}{Z_0} \longrightarrow (*)$$

& THE BACKWARD CURRENT IS

DUE TO OPPOSITE DIRECTION

$$I^- = \frac{-V^-}{Z_0} \longrightarrow (**)$$

EQS. (*) & (**) ARE VALID FOR ANY ORDER OF REFLECTIONS, i.e.,

$$I_j^+ = \frac{V_j^+}{Z_0} \quad \& \quad I_j^- = -\frac{V_j^-}{Z_0} \quad \text{FOR } j = 1, 2, \ldots$$

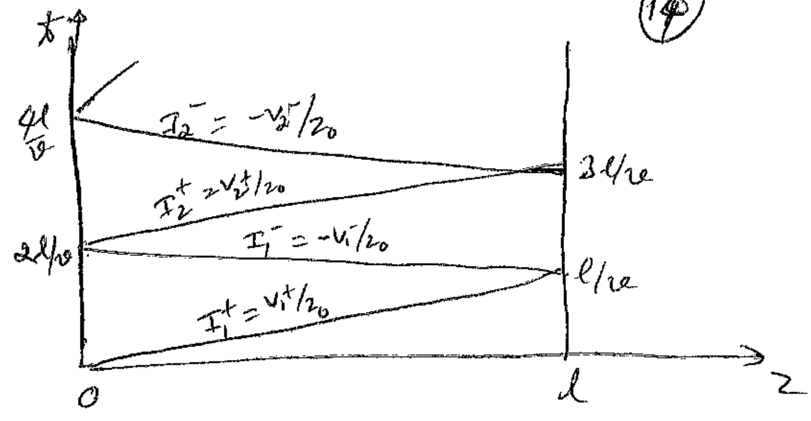
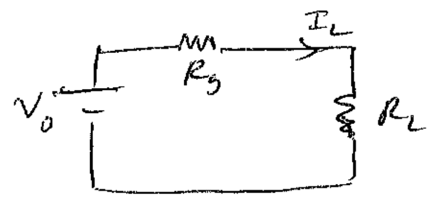


FIG. 8.

IN EXAMPLE 1, (SEE p. 12), $R_L = R_S = 1/3\ Z_0$.

THE STEADY STATE LINE/LOAD CURRENT IS



$$I_L = \frac{V_0}{R_S + R_L} = \frac{3}{2}\frac{V_0}{Z_0}$$

CALL IT I_0

$$= \frac{3}{2} I_0 .$$

$$I_1^+ = V_1^+/Z_0 = \frac{3}{4}\frac{V_0}{Z_0} = \frac{3}{4} I_0 .$$

$$I_1^- = -V_1^-/Z_0 = 3/8\ I_0$$

$$I_2^+ = V_2^+/Z_0 = \frac{3}{16} I_0$$

$$I_1^+ + I_1^- = \frac{9}{8} I_0$$

$$I_1^+ + I_1^- + I_2^+ = \frac{21}{16} I_0$$