TRANSMISSION LINES

TRANSMISSION LINES ARE USED TO TRANSMIT ELECTRIC SIGNALS ~~ENERGY~~ FROM ONE POINT TO ANOTHER. FOR EXAMPLE, THE CONNECTION BETWEEN A TRANSMITTER & AN ANTENNA IS A TRANSMISSION LINE. THE OTHER EXAMPLES INCLUDE THE CONNECTION BETWEEN A HYDROELECTRIC GENERATING PLANT & A SUBSTATION SEVERAL ~~HUN~~ HUNDRED KILOMETERES AWAY, AND THE CONNECTION BETWEEN THE RECEIVER ANTENNA & THE TELEVISION SET.

IN BASIC CIRCUIT ANALYSIS METHODS, THE LENGTH OF THE CONNECTION BETWEEN CIRCUIT ELEMENTS (RESISTORS, INDUCTORS & CAPACITORS) IS MUCH SMALLER THAN THE WAVELENGTH & HENCE, THE PROPAGATION DELAY (OR EQUIVALENTLY PHASE-SHIFT IN PHASOR ANALYSIS) CAN BE IGNORED. TYPICALLY, TRANSMISSION LINES ARE USED TO CONNECT SOURCES AND LOADS THAT ARE SEPARATED BY DISTANCES ON THE ORDER OF A WAVELENGTH OR LARGER.

~~LET US ASSUME THAT THE INPUT VOLTAGE IS SINUSOIDAL & CARRY OUT THE PHASOR ANALYSIS.~~

TRANSMISSION LINE EQUIVALENT CIRCUIT:

WE DEVELOP A CIRCUIT MODEL THAT IS VALID FOR ANY TRANSMISSION LINE. FOR A COAXIAL TRANSMISSION LINE, THE INNER & OUTER CONDUCTORS HAVE A HIGH CONDUCTIVITY, σ_c

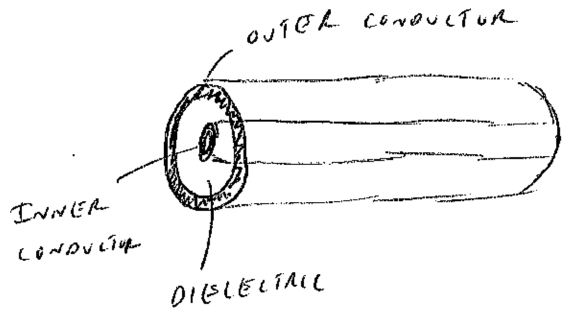


FIG. 1

& INDUCTANCE, L. THE SERIES RESISTANCE R ACCOUNTS FOR

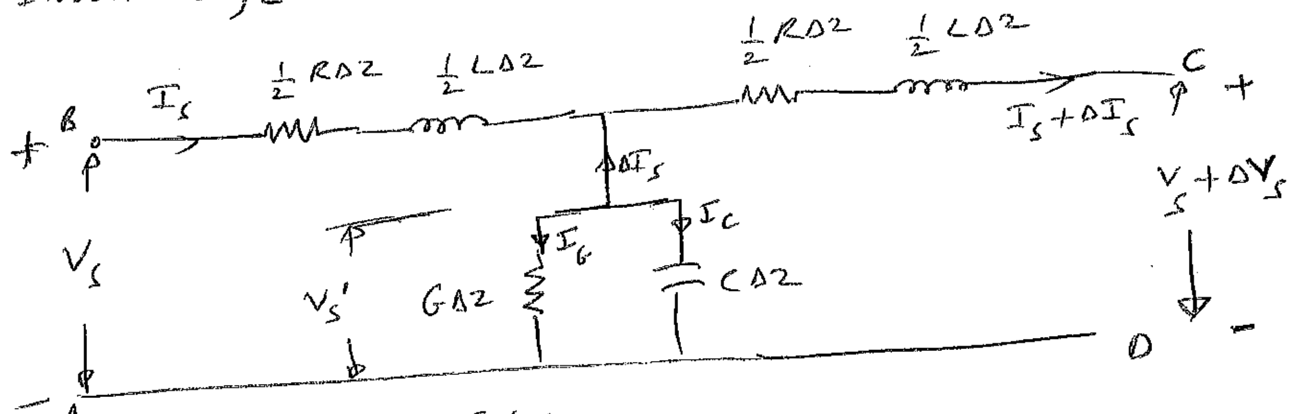


FIG. 2

THE CONDUCTIVITY, σ_c $\left(\because R = \frac{\ell}{\sigma_c A}\right)$. THE SHUNT CONDUCTANCE, G IS USED TO MODEL THE LEAKAGE CURRENT THROUGH THE DIELECTRIC THAT MAY OCCUR THROUGHOUT THE TRANSMISSION LINE. THE CAPACITANCE C ACCOUNTS FOR THE CAPACITOR FORMED DUE TO TWO CONDUCTING CYLINDERS & THE DIELECTRIC BETWEEN THEM. SINCE THE SECTION OF THE TRANSMISSION LINE LOOKS THE SAME FROM EITHER END,

WE DIVIDE THE SERIES ELEMENTS IN HALF TO PRODUCE A SYMMETRIC NETWORK. NOTE THAT THIS IS NOT THE ONLY POSSIBLE NETWORK — WE COULD AS WELL PLACE HALF THE CONDUCTANCE & HALF THE CAPACITANCE AT EACH END.

LET US ASSUME THAT THE INPUT VOLTAGE IS SINUSOIDAL & CARRY OUT ~~HE~~ THE PHASOR ANALYSIS.

$$V = \mathrm{Re}\{V_s(z) \cdot e^{j\omega t}\} \longrightarrow (1)$$
$$I = \mathrm{Re}\{I_s(z)\, e^{j\omega t}\}$$

HERE, V_s & I_s ARE PHASORS. A TRANSMISSION LINE IS DIVIDED INTO SECTION OF LENGTH, Δz & FIG. 2 SHOWS THE EQUIVALENT CIRCUIT OF THE TRANSMISSION LINE OF LENGTH Δz. HERE, R, L, G & C HAVE VALUES SPECIFIED PER UNIT LENGTH. (i.e. UNIT OF R is Ω/m).

APPLYING KVL TO LOOP ABCDA, WE FIND

$$V_s - \frac{1}{2}(R+j\omega L)\Delta z\, I_s - \frac{1}{2}(I_s + \Delta I_s)(R+j\omega L)\Delta z - (V_s + \Delta V_s) = 0$$

$$\frac{\Delta V_s}{\Delta z} = -(R+j\omega L) I_s - \frac{1}{2}\Delta I_s (R+j\omega L) \longrightarrow (2)$$

AS $\Delta z \to 0$, $\Delta I_s \to 0$ & THE ~~LAST~~ TERM IN EQ. (2)

VANISHES.

$$\lim_{\Delta z \to 0} \frac{\Delta V_s}{\Delta z} = \frac{dV_s}{dz} = -(R + j\omega L)\, I_s \longrightarrow (3)$$

VOLTAGE ACROSS ~~THE~~ G & C IS

$$V_s' = V_s - I_s \frac{(R + j\omega L)\Delta z}{2}$$

$$\cong V_s \quad \text{AS} \quad \Delta z \to 0.$$

$$I_G = V_s' G \Delta z$$

$$I_C = V_s' (j\omega C) \Delta z$$

$$\Delta I_s = -(I_G + I_C) = -V_s' (G + j\omega C) \Delta z$$

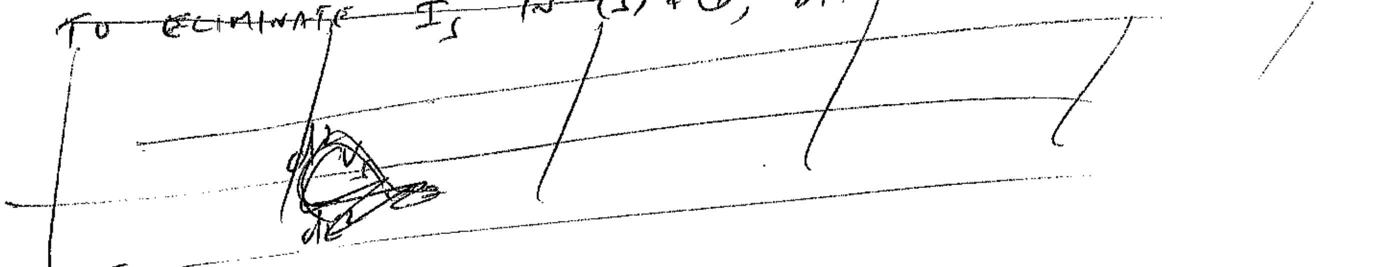
$$\cong -V_s (G + j\omega C) \Delta z$$

or

$$\frac{dI_s}{dz} = -(G + j\omega C) V_s \longrightarrow (4)$$

TRANSMISSION LINES		FIELDS
V_s	$\rightarrow$	E_{xs}
I_s	$\rightarrow$	H_{ys}
$\frac{dI_s}{dz} = -(G + j\omega C) V_s$	$\rightarrow$	$\frac{dH_{ys}}{dz} = -(\sigma + j\omega \epsilon) E_{xs}$
$\frac{dV_s}{dz} = -(R + j\omega L) I_s$	$\rightarrow$	$\frac{dE_{xs}}{dz} = -j\omega\mu H_{ys}$
G	$\rightarrow$	σ
C	$\rightarrow$	ϵ
$R + j\omega L$	$\rightarrow$	$j\omega\mu$

~~TO ELIMINATE I_s IN (3) + (4), DIFFERENTIATE (3) w.r.t. z,~~



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$$Z = R + j\omega L \qquad \text{SERIES IMPEDANCE}$$

$$Y = G + j\omega C \qquad \text{SHUNT ADMITTANCE}$$

(3) $\rightarrow$ $\frac{dV_s}{dz} = -Z I_s$ $\rightarrow$ (3′)

(4) $\rightarrow$ $\frac{dI_s}{dz} = -Y V_s$ $\rightarrow$ (4′)

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TO ELIMINATE I_s IN (3') & (4'), DIFFERENTIATE (3') w.r.t. z,

from (4')

$$\frac{d^2 V_s}{dz^2} = -Z\frac{dI_s}{dz} = -Z(-Y V_s)$$

$$\frac{d^2 V_s}{dz^2} = \gamma^2 V_s$$

$$\gamma^2 = YZ = (R + j\omega L)(G + j\omega C)$$

$$V_s = A e^{\gamma z} + B e^{-\gamma z} \longrightarrow (5)$$

CONSIDER THE SECOND TERM,

$$V_s = B e^{-\gamma z}$$

LET $\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$

$$V_s = B \cdot e^{-(\alpha + j\beta)z}$$

$$V = Re\{V_s e^{j\omega t}\}$$

$$= B \cdot e^{-\alpha z} \cdot Re\{e^{j(\omega t - \beta z)}\}$$

$$V(z) = B e^{-\alpha z} \cos(\omega t - \beta z)$$

FORWARD PROPAGATING WAVE

THE SPEED v IS

$$v = \frac{\omega}{\beta}.$$

FOR THE LOSSLESS CASE, $R = 0$, $G = 0$.

$$\gamma = \sqrt{-\omega^2 LC} = \pm j\sqrt{LC}\,\omega = \alpha + j\beta$$

$$\beta = \pm\,\omega\sqrt{LC} \longrightarrow (6)$$

PICK THE +VE SIGN FOR FORWARD PROPAGATION

$$v = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$$

$$\therefore \boxed{v = \frac{1}{\sqrt{LC}}} \longrightarrow (7)$$

$$L \rightarrow \mu$$
$$C \rightarrow \epsilon$$
$$\frac{1}{\sqrt{LC}} \rightarrow \frac{1}{\sqrt{\mu\epsilon}}$$

~~NOTE: GEOMETRY~~ $\cancel{C = \frac{\epsilon A}{l}}$ ~~cross-section, length~~ $\cancel{L = \frac{\mu l}{A}}$

$\cancel{\frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\epsilon\mu}} = v}$ → ~~THE SAME EQUATION WE HAVE OBTAINED~~ FOR FIELDS.

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v IS THE SPEED AT WHICH VOLTAGE (OR ELECTRIC FIELD) MOVES. v IS <u>NOT</u> THE SPEED AT WHICH ELECTRONS ARE MOVING.

FROM THE FIRST TERM OF EQ. (5),

$$V_s = A e^{\gamma z} = A e^{(\alpha + j\beta) z}$$

$$V = Re\{V_s e^{j\omega t}\}$$

$$= A e^{\alpha z} \cos(\omega t + \beta z)$$

BACKWARD PROPAGATING WAVE

NOTE: WHEN $\alpha > 0$, IT IS CALLED THE ATTENUATION COEFFICIENT. $e^{\alpha z}$ DOES NOT IMPLY THAT IT GROWS EXPONENTIALLY. THE REASON IS THAT THE WAVE IS PROPAGATING BACKWARDS. IF THE INITIAL FIELD PEAK IS AT $z=0$, AT A LATER TIME ~~WAVE MOVES~~ THE PEAK WILL BE IN THE NEGATIVE z DIRECTION &

HENCE $e^{\alpha z}$ IMPLIES EXPONENTIAL DECAY.

FOR A LATER CONVENIENCE, EQ. (5) IS RE-WRITTEN AS

$$V_s = V_0^- e^{\gamma z} + V_0^+ e^{\gamma z} \longrightarrow (5')$$

$V_0^+ = B =$ AMPLITUDE OF THE FORWARD PROP. WAVE

$V_0^- = A =$ " " " BACKWARD " "

HOMEWORK: USING (3') & (4'), SHOW THAT

$$\frac{d^2 I_s}{dz^2} = \gamma^2 I_s \rightarrow (8)$$

THE SOLUTION OF (8) IS

$$I_s(z) = I_0^- e^{\gamma z} + I_0^+ e^{-\gamma z} \rightarrow (9)$$

CONSIDER THE FORWARD PROPAGATING WAVE

$$V_s(z) = V_0^+ e^{-\gamma z} \rightarrow (10)$$

$$I_s(z) = I_0^+ e^{-\gamma z} \rightarrow (11)$$

DIFFERENTIATING (10) & USING (3'), WE FIND

$$\frac{dV_s}{dz} = V_0^+(-\gamma)\, e^{-\gamma z} = -Z\, I_s$$

$$= -Z \cdot I_0^+ e^{-\gamma z}$$

$$\therefore \quad \frac{V_0^+}{I_0^+} = \frac{Z}{\gamma} \, ;$$

$$\gamma^2 = YZ = (R + j\omega L)(G + j\omega C)$$ — SEE PAGE-6.

THE RATIO OF V_0^+ & I_0^+ IS CALLED THE CHARACTERISTIC IMPEDANCE, Z_0

$$\boxed{Z_0 = \frac{V_0^+}{I_0^+} = \frac{Z}{\sqrt{YZ}} = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}}$$

FOR THE LOSSLESS CASE, $(R = G = 0)$,

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}}$$

~~CHARA~~

NOTE: CHARACTERISTIC IMPEDANCE IS THE ANALOG OF INTRINSIC IMPEDANCE, η FOR FIELDS.

$$\eta = \frac{E_{xo}^{+}}{H_{yo}^{+}} = \sqrt{\frac{\mu}{\epsilon}} \rightarrow$$ PERFECT ~~LOSSLESS~~ DIELECTRIC

BACKWARD PROPAGATING WAVE:

$$V_s(z) = V_0^{-} e^{\gamma z}$$

$$I_s(z) = I_0^{-} e^{\gamma z}$$

SHOW THAT, $$Z_0 \equiv \frac{-V_0^{-}}{I_0^{-}} = \sqrt{\frac{Z}{Y}}$$